Composite Fermion Theory of Collective Excitations in Fractional Quantum Hall Effect

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The low energy neutral excitations of incompressible fractional quantum Hall states are called collective modes or magnetic excitons. This work shows that the interaction between the quasiparticle and quasihole forming the exciton is well described *quantitatively* by the unprojected composite fermion theory, except at very small wave vectors. This allows a study of large systems to determine the collective mode dispersions for general fractions up to an overall additive constant. The positions of various minima are explained by analogy to the integer quantum Hall effect.

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Girvin, MacDonald, and Platzman [1] developed a single mode approximation (SMA) to obtain the dispersion of collective mode (CM) excitation of the $\nu = 1/(2m + 1)$ state. The (unnormalized) SMA takes the wave function of the collective excitation to be

$$\chi_k^{\mathrm{SMA}} = \mathcal{P}\rho_k\psi_L\,, \tag{1}$$

where ψ_L is the Laughlin wave function [2], $\mathcal{P}$ is the lowest-Landau-level projection operator, and $\rho_k = \sum_j e^{i\mathbf{k}\cdot\mathbf{r}_j}$ is the density wave operator with wave vector k. In finite system studies, the SMA was found to work well in an intermediate range of wave vectors near a minimum in the dispersion, called the roton minimum, in analogy with Feynman's theory of superfluid ^{4}He. A generalization of the SMA to other fractions, i.e., with ψ_L in Eq. (1) replaced by other fractional-quantum-Hall-effect (FQHE) ground states, did not give a satisfactory description of their collective excitations [3]. Finite system studies have also failed to provide a satisfactory picture for the collective excitations at other fractions.

In the last few years, there has been a resurgence of interest in these issues for two reasons. First, significant progress has been made on the experimental front. Pinczuk *et al.* have measured the positions of the maxima and minima in the collective modes of the integer-QHE (IQHE) states [4], and also recently their detailed dispersion in modulated density samples [5]. Further, Raman scattering [6] and phonon absorption [7] experiments have reported observation of the collective modes in the FQHE regime. Second, there now exists a new theoretical framework, called the composite fermion (CF) theory [8], for describing all FQHE on an equal footing. The FQHE of electrons is understood as the IQHE of composite fermions, suggesting that it should be possible to describe the collective excitations of the FQHE states as the simple, IQHE-like collective excitations of composite fermions. This work studies the collective modes of several FQHE states using the CF theory, and obtains detailed predictions for their dispersions; in particular, the minima and maxima in the dispersion are identified. These are of experimental relevance, since the CM density of states has peaks at energies corresponding to the extrema of the dispersion curve, which, as a result of a disorder-induced breakdown of the wave vector conservation, are observable in inelastic light scattering experiments [4].

The CF theory [8] is based on the principle that, in a range of filling factors, the electrons in the lowest Landau level (LL) find it energetically favorable to capture an even number ($2m$) of vortices of the many-particle wave function. The bound state of an electron and vortices behaves the same as a particle, called the composite fermion. The vortices produce phases as the composite fermions move around, which partly cancel the Aharonov-Bohm phases originating from the external magnetic field, and, as a result, the composite fermions experience an effective magnetic field given by $B^* = B - 2m\rho\phi_0$, where B is the external field, $\phi_0 = hc/e$ is the flux quantum, and ρ is the electron (or CF) density. The residual interaction between the composite fermions is weak, and the strongly correlated liquid of electrons maps into a weakly interacting gas of composite fermions. An effective single-particle description of the electron state then becomes possible in terms of composite fermions. The energy levels of composite fermions are analogous to the LL's of *noninteracting* electrons in this weaker magnetic field, called quasi- or CF-LL's. Defining the CF filling factor as $\nu^* = \rho\phi_0/B^*$, in analogy to the electron filling factor $\nu = \rho\phi_0/B$, the above equation can also be expressed as $\nu = \nu^*/(2m\nu^* + 1)$. The IQHE of composite fermions at $|\nu^*| = n$ manifests as the FQHE of electrons at $\nu = n/(2mn \pm 1)$.

The CF picture has led to two detailed, microscopic calculational schemes. One constructs explicit trial wave functions [8]. We confine the discussion below to the special filling factors $\nu = n/(2mn + 1)$, where the CF filling factor is $\nu^* = n$. Let us denote the ground state of noninteracting electrons at $\nu^* = n$ by Φ_n. The corresponding wave function for the composite fermions is obtained by attaching $2m$ vortices to each electron in the state Φ_n, which amounts to a multiplication by the Jastrow factor $\prod_{j<k}(z_j - z_k)^{2m}$, where $z_j = x_j + iy_j$ denotes the position of the jth electron. The wave function, $\mathcal{P}\prod_{j<k}(z_j - z_k)^{2m}\,\Phi_n$, thus describes the electron ground state at $\nu = n/(2mn + 1)$. The CF

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structure of this state suggests a new wave function for the collective excitation given by

$$\chi_k^{\rm CF} = \mathcal{P}\prod_{j<k}(z_j - z_k)^{2m}\,[\rho_k^{n\to n+1}\Phi_n] \equiv \mathcal{P}\chi_k^{\rm UP-CF}\ , \tag{2}$$

where $\chi_k^{\rm UP-CF}$ is the *unprojected* CF wave function. In second quantized notation (choosing Landau gauge),

$$\rho_k^{n\to n+1} \equiv \sum_p \langle n+1, p+k|e^{iky}|n,p\rangle c^\dagger_{n+1,p+k}c_{n,p}\ ,$$

where $|n,p\rangle$ denotes the wave vector p state in the nth LL, and the y axis is chosen parallel to $\mathbf{k}$. The operator $\rho_k^{n\to n+1}$ excites a single electron from the topmost filled (nth) LL of Φ_n to the lowest empty, i.e., the $(n+1)$th LL, creating the lowest energy collective mode of the $\nu^* = n$ IQHE state [9]. $\chi^{\rm CF}$ contains a single excited composite fermion in the $(n+1)$th CF-LL (which will be referred to as a quasiparticle below [10]) and a hole left behind in the nth CF-LL (which will be called a quasihole). It can be interpreted either as the collective mode *of composite fermions* or as the CF exciton.

For the $1/(2m+1)$ state, the SMA wave function can be written as

$$\chi_k^{\rm SMA} = \mathcal{P}\prod_{j<k}(z_j - z_k)^{2m}\,\rho_k\Phi_1\ .$$

Kohn's theorem tells us that $\rho_k\Phi_n = \rho_k^{n\to n+1}\Phi_n$ in the limit $k\to 0$. Thus, for the $1/(2m+1)$ state, $\chi_k^{\rm SMA}$ and $\chi_k^{\rm CF}$ become identical in the limit $k\to 0$. At finite k, the two are different. ρ_k excites electrons to arbitrarily high LL's, and hence, in the CF interpretation, $\chi_k^{\rm SMA}$ contains composite fermions excited to arbitrarily high CF-LL's. For other fractions, $\chi_k^{\rm SMA}$ does not yield to a CF-type interpretation, and differs from $\chi_k^{\rm CF}$ at all k.

In the second scheme, the composite fermions are modeled as electrons carrying flux quanta at the mean-field level, where the flux quanta simulate the vortices [8,11]. A perturbation theory around the mean-field solution is then carried out using Chern-Simons (CS) field theoretical techniques. There have been several studies of the collective mode dispersion using the CS approach [12].

We develop here techniques for computing the CM dispersion using the CF wave functions. The standard spherical geometry [13] is used in our calculations below. The total orbital angular momentum L is related to the wave vector of the planar geometry by $kl_0 = L/\sqrt{q}$, where l_0 is the magnetic length, and $2q\phi_0$ is the magnetic flux through the surface of the sphere. If the highest occupied shell in Φ_n has angular momentum ℓ, then the IQHE collective mode has a single excited electron in the $(n+1)$th LL, with angular momentum $\ell+1$, and the hole in the nth LL, with angular momentum ℓ, with the allowed values of L for the collective mode given by $L = 1, 2, \ldots, 2\ell+1$ with precisely one multiplet at each L (with $2L+1$ degenerate states). Let us denote the z component of the angular momentum of the excited electron (hole left behind) by ℓ_z^e (ℓ_z^h), and the corresponding IQHE Slater determinant basis state by $\Phi_n(\ell_z^e, \ell_z^h)$. The CM wave function with a well defined L and $L_z = \ell_z^e + \ell_z^h$ is an appropriate linear combination of the basis states. We restrict our study to the $L_z = 0$ sector, with no loss of generality. The CM wave function of the IQHE state is given by

$$\rho_L^{n\to n+1}\Phi_n = \sum_{\ell_z=-\ell}^{\ell}\langle \ell+1, \ell_z; \ell, -\ell_z|L,0\rangle \times \Phi_n(\ell_z, -\ell_z)\ .$$

It contains no adjustable parameters. The same is true of the CF-CM wave function, $\chi_L^{\rm CF}$, obtained according to Eq. (2). We have computed the CM energy from exact diagonalization in the lowest LL ($\Delta V_{\rm ex}$) from the projected CF wave function $\chi^{\rm CF}$ (ΔV_p) and from the unprojected CF wave function $\chi^{\rm UP-CF}$ (ΔV). In each case, the CM energy is measured relative to the corresponding ground state energy.

A comparison with finite-size exact-diagonalization studies has shown that ΔV_p provides a good quantitative description of the collective modes of various FQHE states [14,15]. However, our brute force projection method (for details, see Ref. [15]) allows us to carry out the projection for general states only for up to $\sim$10 electrons. We now generalize a Monte Carlo projection technique used by Bonesteel [16] to obtain the energy of the collective mode of the 1/3 state for large systems. This relies on the special feature of the unprojected 1/3-CM wave function that it contains no more than one electron in the second LL, and none in the higher LL's. The projected wave function can then be written as [16]

$$\chi^{\rm CF} \propto (T - E_1)\chi^{\rm UP-CF}, \tag{3}$$

where T is the kinetic energy operator and E_1 is the energy separation between the two lowest LL's, equal to $(1+q^{-1})\hbar eB/mc$ in the spherical geometry. Figure 1 shows the CM energy ΔV_p as a function of k for a 20-electron system. There is a deep minimum at $kl_0 \approx 1.4$. Our estimate for the thermodynamic value of the energy at the minimum $0.063(3)e^2/\epsilon l_0$ (see the inset in Fig. 1), which should be compared to the SMA value $0.078e^2/\epsilon l_0$ [1]. (Note that both approximations use the same ground state.) Additional minima are clearly visible at $kl_0 \approx 2.7$ and 3.5. Are they real? We believe so. A direct confirmation of the genuineness of the former minimum is seen in the nine-electron exact diagonalization calculation of Fano, Ortolani, and Colombo [17], reproduced in Fig. 2(b).

This method, however, is not applicable to other FQHE states. We now show that the unprojected CF wave function, $\chi^{\rm UP-CF}$, can itself be used to investigate the collective excitations. The relevance of the unprojected theory for the CM dispersion can be motivated by the following consideration. At large wave vectors, the CM state contains a far separated pair of a quasiparticle and a quasihole,

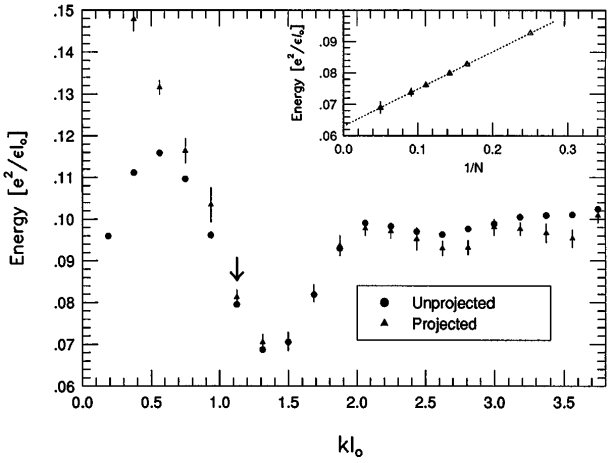

FIG. 1. The energy of the collective mode computed with the projected CF wave functions (V_p, triangles) and with the unprojected wave functions ($\Delta V + \Gamma$, circles, $\Gamma = 0.064 e^2/\epsilon l_0$) for $N = 20$ electrons. The error bar for the latter is smaller than the size of the circles. All energies are in units of $e^2/\epsilon l_0$, where ϵ is the background dielectric constant. The inset shows the energy at the minimum as a function of N^{-1}; these have been determined in each case by fitting the lowest few points to a smooth curve.

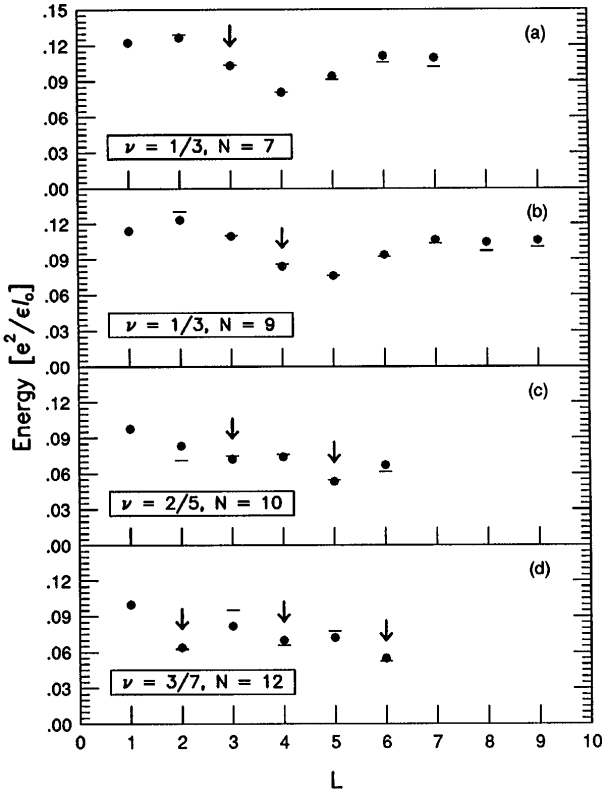

FIG. 2. The collective mode energies from the exact diagonalization ($\Delta V_{\rm ex}$, dashes) shown together with those obtained from the unprojected CF theory ($\Delta V + \Gamma$, filled circles). The exact energies in (b) are taken from Fano *et al.* [17]; in (c) from N. d'Ambrumenil and R. Morf, [Phys. Rev. B **40**, 6108 (1989)], and in (d) from He *et al.* [12].

which approach one another as k is reduced. So long as the distance between them is not too small, the CM energy can be viewed as the sum of (i) the creation energy of an isolated quasihole, (ii) the creation energy of an isolated quasiparticle, and (iii) their interaction energy. The creation energy, computed with the unprojected CF wave functions, differs from the actual energy by as much as a factor of 2, indicating that the small amount of admixture with higher LL's, present in the unprojected CF wave functions, builds very good short distance correlations. The unprojected wave function should nonetheless provide a good estimate for the *interaction energy*, provided the quasiparticle and the quasihole are not too close. This expectation is based on the observation that the density profile away from the core of an isolated quasiparticle or quasihole is obtained reasonably accurately by the unprojected theory, as is also the density profile in the overlap region of a state containing a quasiparticle and a quasihole (see, e.g., Ref. [16]), except when they are very close. This leads us to the hypothesis that $\Delta V + \Gamma$ should give a good approximation of the actual CM energy, except at small k, where the constant Γ corrects for the error in the creation part.

We first test this hypothesis in finite system calculations. The unprojected energies ΔV are computed using variational Monte Carlo techniques. The exact diagonalization energies, $\Delta V_{\rm ex}$, and $\Delta V + \Gamma$ (with a suitable choice of Γ) are shown in Fig. 2 for some of the biggest systems for which exact diagonalization has been performed. The unprojected theory does indeed capture the essential features of the true collective mode; in particular, it obtains correctly the minima and maxima. $\Delta V + \Gamma$ also provides a reasonably good quantitative approximation for the true collective mode energy. We have found that the agreement becomes better for larger systems; for small systems, the quasiparticle and quasihole are not sufficiently far separated, especially for $2/5$ and $3/7$ (whose quasiparticles are of larger extent).

The advantage of working with the unprojected CF wave functions is that a treatment of large systems becomes possible for all FQHE states. Figure 1 depicts $\Delta V + \Gamma$ for a 20-electron system for the $1/3$ FQHE state. A lack of any significant size dependence for systems with slightly larger N shows that these results are close to the thermodynamic limit. Let us first concentrate only on the range $kl_0 > 0.5$. Here, $\Delta V + \Gamma$ provides a good approximation for ΔV_p, and, in particular, obtains the additional minima. We note that there is no principle that rules out the existence of more than one minimum in the CM dispersion of the $1/3$ state; the structure in the CM dispersion arises simply from an interplay between the (several) maxima and minima in the density profiles of the quasiparticle and quasihole, as the distance between them is varied. The CM dispersions for $2/5$ and $3/7$ are shown in Fig. 3. The positions of the two deep minima for the $2/5$ collective mode in Fig. 3 agree well with those found in the exact diagonalization results of Refs. [3,18].

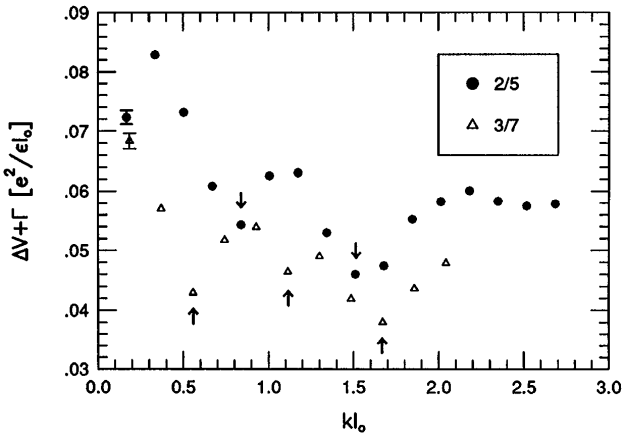

FIG. 3. $\Delta V + \Gamma$ is shown for for $2/5$, with $N = 30$ and $\Gamma = 0.037 e^2/\epsilon l_0$; and for $3/7$, with $N = 27$ and $\Gamma = 0.028 e^2/\epsilon l_0$. The values of Γ are chosen so as to provide a reasonable large-k limit. The typical error bar is shown on the first point in each case.

Additional weaker structure, analogous to the $1/3$ case, also appears. The relatively complicated structure in the dispersion clarifies why the small system calculations are unable to provide a coherent picture.

A curious feature of the CM dispersion in Fig. 3 is that ΔV bends downward at small wave vectors ($kl_0 < 0.5$ for $1/3$). The unprojected scheme is not trustworthy at small k, since the projection is known to alter the CM wave function significantly at small k. In fact, at $L = 1$, the CM states have a zero projection on the lowest LL [14]. Indeed, ΔV_p in Fig. 1 shows no such bending, and the $k \to 0$ value of ΔV_p is consistent with the SMA value of $0.15 e^2/\epsilon l_0$, as expected.

A comparison with the collective mode of the $\nu^* = n$ IQHE state [9] is illuminating. First of all, the interaction energy of the collective excitation decreases for small wave vectors, similar to that found above. In general, the number of minima or inflection points in the CM dispersion of the $\nu^* = n$ IQHE state is n, which correlates with the number of *strong* minima at $n/(2n + 1)$. In fact, even the positions of the latter can be understood by analogy to the IQHE: They occur in both cases at the same wave vectors (or, in spherical geometry, at the same L). The k values of the minima/inflection points in the CM dispersion of the corresponding IQHE states are shown in Figs. 1, 2, and 3 by vertical arrows. The weaker minima have no analog in the IQHE; they appear only after multiplication by the Jastrow factor, and refer to features beyond the mean-field theory.

Several effects left out in the above study must be incorporated before a comparison with experiment may be made. Modification in the Coulomb interaction because of the finite width of the quantum well, LL mixing, and disorder are all known to change the numerical values of the excitation energies. A good first approximation for the experimental CM dispersion should be obtained by $\Delta V + \Gamma$, with a choice of Γ that makes the large-k limit equal to the experimental transport gap.

In conclusion, we have used the CF wave functions to investigate the collective mode excitations of various FQHE states. This provides new qualitative information as well as better quantitative estimates than available previously.

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