Phenomenological Transport Equation for the Cuprate Metals

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We observe that the appearance of two transport relaxation times in the various transport coefficients of cuprate metals may be understood in terms of scattering processes that discriminate between currents that are even or odd under the charge conjugation operator. We develop a transport equation that illustrates these ideas and discuss its experimental and theoretical consequences.

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The normal state of the cuprate superconductors exhibits the extraordinary feature of two transport relaxation time scales. In optimally doped compounds, conductivity and photoemission measurements indicate a scattering rate which grows linearly with temperature $\tau_{\rm tr}^{-1} = \eta T$, where for YBCO $\eta \approx 2$ [1]. By contrast, Hall constant and magnetoresistance measurements indicate that the cyclotron relaxation rate τ_H^{-1} has a qualitatively different *quadratic* temperature dependence:

$$\tau_H^{-1} = T^2/W_s + b_i \,. \tag{1}$$

Experimentally, τ_H^{-1} is inferred from the Hall angle $\theta_H = \omega_c \tau_H$, manifested in both the Hall conductance $\sigma_{xy} = \sigma_{xx}\theta_H$ and the magnetoconductance $\Delta\sigma_{xx} \approx -\sigma_{xx}(\theta_H)^2$. Experiments on YBCO demonstrate that b_i is proportional to the impurity concentration and W_s is estimated to be of the order of 800 K [2]. Thus in the relevant temperature range the ratio of the cyclotron and charge transport relaxation times $\tau_{\rm tr}/\tau_H \approx T/2W_s$ is small.

This is unprecedented behavior, for in conventional metals scattering at the Fermi surface does not discriminate between transverse and longitudinal currents. Anderson [3] has proposed that two relaxation rates are evidence for two distinct species of quasiparticle which independently relax the longitudinal and transverse currents [4]. Two alternative proposals, involving either strong momentum dependence of the electron self-energy [5–7] or singular skew scattering [8] as the origin of two relaxation time scales, require special conditions to be realized on the Fermi surface. The former requires that the weakly scattered parts of the Fermi surface do not short circuit the conductivity; the skew scattering model requires near-perfect particle-hole symmetry.

In this paper we reconsider the idea of two quasiparticle types. For its development, this radical idea requires an understanding of how longitudinal and transverse components of the electromagnetic current could couple selectively to two different quasiparticles. To this end, we link the discussion with the concept of charge conjugation symmetry [9,10]. Charge conjugation, the interconversion of electrons and holes, is an asymptotic low-energy symmetry of a Fermi surface. The parity $C = \pm 1$ under this symmetry operation delineates longitudinal electric currents, which are odd ($C = -1$), from transverse currents and a whole range of other neutral currents, which are even ($C = +1$). Scattering at the Fermi surface is normally "blind" to charge conjugation symmetry, leading to a single transport relaxation time. Making the tentative observation that in the cuprates odd parity currents relax at the fast rate $\tau_{\rm tr}^{-1}$, whereas other even parity currents relax at the slow rate τ_H^{-1}, we are led to hypothesize that new kinds of low-energy scattering processes are present in the cuprate metals which *depend on the charge conjugation symmetry of the quasiparticles.* By formulating this idea as a phenomenological transport equation we show that the fastest relaxation rate dominates the resistivity, but that the slowest relaxation rate *selectively* short circuits all other current relaxation processes. These results constrain a large class of in-plane thermal and electric transport coefficients, allowing the hypothesis to be tested.

Consider a Fermi surface described by the Hamiltonian

$$H_0 = \sum_{\vec p} \epsilon_{\vec p - e\vec A}\, \psi^\dagger_{\vec p\sigma}\psi_{\vec p\sigma} \,. \tag{2}$$

We define charge conjugation as

$$\psi_{\vec p\sigma} \to \sigma \psi^\dagger_{\vec p^* - \sigma}\,, \qquad \vec A \to -\vec A\,, \tag{3}$$

where $\vec p = \vec p_F + \delta p \hat n$ and $\vec p^* = \vec p_F - \delta p \hat n + O(\delta p^2)$ locate degenerate electron and hole states along the normal $\hat n$ from the Fermi surface. Physical operators $\hat O$ can be categorized according to their *conserved* parity under charge conjugation $\hat O \to C\hat O$ ($C = \pm 1$). For example, the electric current operator divides into independent "longitudinal" and "transverse" components $\vec j_e = \vec j_E + \vec j_H$,

$$\begin{aligned} \vec j_E &= e \sum_{\vec p} \vec v_{\vec p_F} \psi^\dagger_{\vec p\sigma}\psi_{\vec p\sigma} && (C = -1)\,, \\ \vec j_H &= e \sum_{\vec p} [\underline{m}_{\hat p}^{-1}(\delta p - e\vec A)] \psi^\dagger_{\vec p\sigma}\psi_{\vec p\sigma} && (C = +1)\,, \end{aligned} \tag{4}$$

with opposite charge conjugation parities. Here, $\underline{m}_{\hat p}$ is the effective mass tensor. The transverse current has the same $C = +1$ parity as the thermal current operator, and

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it is this term which gives rise to a Hall and thermoelectric response.

Thermal and electric transport is normally described in terms of four fundamental transport tensors [11]

$$\vec{j}_e = \underline{\sigma}\vec{E} + \underline{\beta}\vec{\nabla}T\,, \qquad \vec{j}_t = \underline{\gamma}\vec{E} + \underline{\zeta}\vec{\nabla}T\,. \tag{5}$$

These tensors are directly linked to microscopic charge and thermal current fluctuations via Kubo formulas. Table I compares the leading temperature dependences of the various transport tensors measured in the optimally doped cuprates with a series of calculations we now describe. The thermoelectric conductivity $\underline{\beta}$, determined from the conductivity and Seebeck coefficients, $\underline{S}$, $\underline{\beta} = -\underline{\sigma S}$ has a particularly revealing temperature dependence. In a naive relaxation time treatment, the temperature dependence of β is directly related to the relevant quasiparticle relaxation rate τ_{TE}^{-1} according to [12]

$$\beta = -\left(\frac{\pi^2 k_B}{3e}\right)\left(\frac{k_B T}{\epsilon_F}\right)\frac{ne^2}{m}\,\tau_{TE}\,, \tag{6}$$

where ϵ_F is the Fermi energy. Combining this with the electrical conductivity, $\sigma = (ne^2/m)\tau_{\rm tr}$, the dimensionless thermopower is then

$$\tilde{S} = \frac{eS}{k_B} = \left(\frac{\tau_{TE}}{\tau_{\rm tr}}\right)\left(\frac{\pi^2}{3}\right)\left(\frac{k_B T}{\epsilon_F}\right). \tag{7}$$

In optimally doped compounds [13], the thermopower contains an unusual constant part, $\tilde{S} \approx \tilde{S}_0 - bT$ where $\tilde{S}_0 \sim 0.1$, which indicates that

$$\tau_{TE}^{-1} = T^2/W_{\rm th} \tag{8}$$

is a factor $T/\eta W_{\rm th}$ smaller than the transport relaxation rate, where $W_{\rm th} = (3\tilde{S}_0/\pi^2\eta)\epsilon_F \sim \epsilon_F/10$. The comparable size and temperature dependence of τ_{TE}^{-1} and τ_H^{-1} suggest that the same type of quasiparticle carries both the Hall current and the thermocurrent.

TABLE I. Leading temperature dependences of transport coefficients compared with proposed decomposition into two Majorana relaxation times ($\mathcal{L}_0$ is the Lorentz number $\pi^2 k_B^2/3e^2$).

Conductivity	Majorana fluid $\times(\frac{m}{ne^2})$	Leading T behavior $\Gamma_f \gg \Gamma_s$ $(T \gg T^2)$	Expt.	Ref.
σ_{xx}	$\frac{2}{\Gamma_f+\Gamma_s}$	T^{-1}	T^{-1}	
σ_{xy}	$\frac{\omega_c}{\Gamma_f\Gamma_s}$	T^{-3}	T^{-3}	
$\Delta\sigma_{xx}$	$-\frac{\sigma_{xx}}{2}(\frac{\omega_c^2}{\Gamma_s^2} + \frac{\omega_c^2}{\Gamma_f^2})$	T^{-5}	T^{-5}	
β_{xx}	$-\frac{eT\mathcal{L}_0}{2\epsilon_F}(\frac{1}{\Gamma_+} + \frac{\Gamma_+}{\Gamma_s\Gamma_f})$	T^{-1}	T^{-1}	[14]
β_{xy}	$\beta_{xx}\frac{\omega_c}{\Gamma_+}$	T^{-2}	T^{-3}(?)	[14]
ζ_{xx}	$-\frac{\mathcal{L}_0}{2}(\frac{T}{\Gamma_f} + \frac{T}{\Gamma_s})$	T^{-1}	(?)	[15]
ζ_{xy}	$\zeta_{xx}\frac{\omega_c}{\Gamma_+}$	T^{-2}	T^{-1}(?)	[15,16]

By taking linear combinations of degenerate electron and hole states,

$$a_{\vec{p}\sigma} = \tfrac{1}{\sqrt{2}}[\psi_{\vec{p}\sigma} + \sigma\psi^\dagger_{\vec{p}^*-\sigma}] \quad (C = +1)\,, \qquad b_{\vec{p}\sigma} = \tfrac{1}{i\sqrt{2}}[\psi_{\vec{p}\sigma} - \sigma\psi^\dagger_{\vec{p}^*-\sigma}] \quad (C = -1)\,, \tag{9}$$

the low-energy excitations of a Fermi surface described by (1) may always be rewritten as eigenstates of the charge-conjugation operator [9],

$$H_0 = \sum_{|\vec{p}|>|\vec{p}_F|,\sigma} \Psi^\dagger_{\vec{p}\sigma}\epsilon_{\vec{p}-e\vec{A}\tau_y}\Psi_{\vec{p}\sigma}\,, \tag{10}$$

where $\Psi^\dagger_{\vec{p}\sigma} = (a^\dagger_{\vec{p}\sigma}, b^\dagger_{\vec{p}\sigma})$, and τ_y is the second Pauli matrix. Despite the superficial resemblance with Bogoliubov quasiparticles, this is merely an alternative, if unfamiliar representation of the unpaired electron gas in terms of eigenstates of charge conjugation, rather than eigenstates of charge. Note that from (10) photon absorption flips the charge conjugation parity of the excitation.

In this new basis the Boltzmann f function is a matrix

$$\underline{f}(\vec{p}\sigma, \vec{R}, t) = \begin{bmatrix} \langle a^\dagger{}_{\vec{p}\sigma}a_{\vec{p}\sigma}\rangle & \langle b^\dagger{}_{\vec{p}\sigma}a_{\vec{p}\sigma}\rangle \\ \langle a^\dagger{}_{\vec{p}\sigma}b_{\vec{p}\sigma}\rangle & \langle b^\dagger{}_{\vec{p}\sigma}b_{\vec{p}\sigma}\rangle \end{bmatrix}_{\vec{R},t}\,, \tag{11}$$

where $\langle\,\rangle$ represents an appropriate coarse grained average of the microscopic Green function in the vicinity of $\vec{R}$ [17]. Electric and thermal currents are given by taking the trace of $\underline{f}(\vec{p})$ with the current operators

$$j_e(\vec{p}) = e\vec{\mathcal{V}}_{\vec{p}}\underline{\tau}_y\,, \qquad \vec{j}_t(\vec{p}) = \epsilon_{\vec{p}}\vec{\mathcal{V}}_{\vec{p}}\,, \tag{12}$$

where $\vec{\mathcal{V}}_{\vec{p}} = \vec{v}_F\underline{1} + \vec{u}_{\vec{p}}\underline{\tau}_y$ is the velocity operator and $\vec{u}_{\vec{p}} = \underline{m}_{\vec{p}}^{-1}\delta\vec{p}$. The "transverse" current $\vec{j}_H = e\vec{u}_p$ is diagonal in this basis, whereas the "longitudinal" current $\vec{j}_E = e\vec{v}_F\tau_y$ is off-diagonal. In this representation the Boltzmann equation becomes

$$\dot{f} + \tfrac{1}{2}\{\vec{\mathcal{V}}_{\vec{p}}, \vec{\nabla}_R f\} + \tfrac{1}{2}e\{(\vec{E} + \vec{\mathcal{V}}_{\vec{p}} \times \vec{B})\tau_y, \vec{\nabla}_p f\} = I[g]\,, \tag{13}$$

where $I[g]$ is the collision functional, $g = f - f^{(0)}$ is the departure from equilibrium. Here the curly brackets represent anticommutators, which appear when making the gradient expansion of matrix Green functions. In this phenomenological discussion we shall use the relaxation time approximation to the collision integral, which is

$$I[g] = -\tfrac{1}{2}\{\Gamma, g\}\,, \tag{14}$$

where Γ is the relaxation matrix. For a conventional metal, where scattering is charge-conjugation invariant, $\underline{\Gamma} = \Gamma\underline{1}$.

So far, we have merely reformulated conventional transport theory. Our central phenomenological hypothesis is that in the cuprate metals the relaxation times of the different Majorana modes at the Fermi surface are no longer equal. We assign "fast" (Γ_f) and "slow" (Γ_s) scattering rates to quasiparticles of opposite parity,

$$\underline{\Gamma} = \mathrm{diag}[\Gamma_f(T), \Gamma_s(T)], \qquad (15)$$

or $\underline{\Gamma} = \Gamma_+ + \Gamma_- \tau_z$, where $\Gamma_\pm = \frac{1}{2}[\Gamma_f \pm \Gamma_s]$. Under this assumption, an electron is a linear combination of fast and slow eigenstates of $\hat{C}$. Since $\Gamma_f \gg \Gamma_s$, an electron will decay rapidly in time $\Delta t \sim \Gamma_f^{-1}$ into a quantum admixture of electron and hole,

$$e^- \xrightarrow{\Delta t \sim \Gamma_f^{-1}} (e^- - h^+)/\sqrt{2}. \qquad (16)$$

In this way, charged currents rapidly decay, leaving behind a "neutral" component which carries the slowly relaxing Hall, spin, thermal, and thermoelectric currents. This is an analog of neutral kaon decay [10].

Let us now follow these effects in the transport equations. Writing $g = g_0 + \vec{g} \cdot \vec{\tau}$ and resolving the components of the transport equation, we obtain

$$(a + b)\begin{pmatrix} g_0 \\ g_y \\ g_z \end{pmatrix} = -f' \begin{pmatrix} \Phi_{\vec{p}} \\ e\vec{E} \cdot \vec{v}_F \\ 0 \end{pmatrix}, \qquad (17)$$

where $f' \equiv \partial_\epsilon f|_{\epsilon = \epsilon_0(\vec{p})}$, $\Phi_{\vec{p}} = e\vec{E} \cdot \vec{u}_{\vec{p}} - \epsilon_0(\vec{p}) T^{-1} \times \vec{\nabla} T \cdot \vec{v}_F$, is the neutral current driving term and

$$a = \partial_t + \Gamma_+ \underline{1} + \Gamma_- \underline{\tau}_z,$$
$$b = [e(\vec{E} + \mathcal{V}_{\vec{p}} \times \vec{B})\underline{\tau}_y \cdot \vec{\nabla}_{\vec{p}} + \vec{\mathcal{V}}_{\vec{p}} \cdot \vec{\nabla} T \partial_T] \qquad (18)$$

are the collision and gradient terms, in which we have implicitly made the transformation

$$\underline{\tau}_y \to \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \underline{\tau}_z \to \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \qquad (19)$$

The equation for g_x decouples and has been omitted.

To solve the transport equations, we adopt the standard Zener-Jones multipole expansion, inverting (17) and expanding order by order in powers of b/a, $g = g^{(1)} + g^{(2)} + \cdots$, where $g^{(n)} = (-a^{-1}b)^{n-1} g^{(1)}$. By expanding the leading contributions to the electrical and thermal currents

$$\vec{j}_e = e \sum [\vec{v}_F g_y(\vec{p}) + \vec{u}_{\vec{p}} g_0(\vec{p})],$$
$$\vec{j}_t = \sum \epsilon(\vec{p}) [\vec{v}_F g_0(\vec{p}) + \vec{u}_{\vec{p}} g_y(\vec{p})], \qquad (20)$$

we obtain the transport coefficients. The results for a simplified parabolic band are summarized in Table I.

A simple physical picture of the effect of an electric field is provided in Fig. 1. When an electric field is applied, it produces an admixture of $C = +1$ and $C = -1$ quasiparticles whose joint relaxation rate $\Gamma_{\mathrm{tr}} = \frac{1}{2}[\Gamma_s + \Gamma_f] \approx \frac{1}{2}\Gamma_f$ is dominated by the rapidly relaxing quasiparticles. Magnetic fields couple diagonally to the Majorana quasiparticles, deflecting each component through a Hall angle $\theta_{s,f} = \omega_c/\Gamma_{s,f}$. Since $\theta_s \gg \theta_f$, the Hall current is dominated by the slow-relaxation quasiparticles.

A thermal gradient couples diagonally to the quasiparticles, so thermal and thermoelectric conductivities are determined by the slow relaxation rate. The difference in the relaxation times of the electrical and thermoelectric currents then gives rise to the unique temperature independent component in the Seebeck coefficient $S = -\rho\beta \propto T\Gamma_f/\Gamma_s$. The off-diagonal field-dependent part of the thermal conductivity is of interest because it is free from phonon contributions. The field-dependent part of thermal current is even under the charge conjugation operator, so the thermal Hall angle is determined by the fast relaxation rate, $\theta_T \sim \omega_c/\Gamma_+$, giving $\zeta_{xy} \sim 1/T^2$. Provisional measurements of the Hall conductivity [16] show that it grows as the temperature is lowered, but suggest $\zeta_{xy} \sim 1/T$, a result which, if sustained, would refute our approach.

Various experiments can be used to both test and contrast our picture with alternative theories. Most importantly, we predict that the fast relaxation rate will only appear in charge-conjugation-odd currents; all other currents will be short circuited by the slowly relaxing quasiparticles. ac Hall conductivity $\sigma_{xy}(\omega)$ is another discriminatory probe. Provided $\Gamma_s \ll \Gamma_f$, our model predicts

$$\cot\theta_H(\omega) = \frac{\sigma_{xx}(\omega)}{\sigma_{xy}(\omega)} = \frac{-i\omega + \Gamma_s(T)}{\omega_c}. \qquad (21)$$

In the skew-scattering model [8], $\omega_c \to \omega_c^*(T) \propto 1/T$ is renormalized and there is only one relaxation rate $\Gamma_s = \Gamma_f$, so $\mathrm{Im}[\cot\theta(\omega)] \propto \omega T$ is proportional to temperature. By contrast, in the two-relaxation-time scenario, this quantity is temperature independent. The extension of existing ac Hall measurements on YBCO [18] to a variety of temperatures can thus delineate these two scenarios.

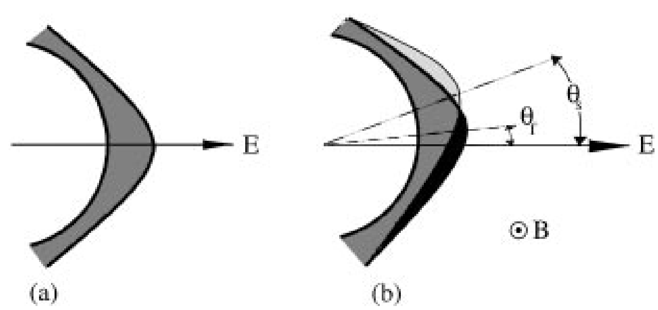

FIG. 1. (a) Application of field creates a mixture of slowly and rapidly relaxing quasiparticles. (b) Slow and fast components of the Majorana fluid precess in a field, equilibrating at large and small Hall angles, respectively.

Symmetry arguments have led us to suggest that if Hall and electric currents decay at qualitatively different rates, the imaginary part of the electron self-energy has the form

$$\Gamma = \Gamma_+ + \Gamma_- \hat{C}, \tag{22}$$

where $\hat{C}$ is the charge conjugation operator. We now discuss the interpretation of this hypothesis. Microscopically, terms proportional to $\hat{C}$ represent the inelastic interconversion of electrons and holes. This is superficially similar to "charge imbalance relaxation" in superconductors, where inelastic collisions give rise to a relaxation of quasiparticle charge into the condensate at a rate Γ_Q [19,20]. Evidently we have no condensate, but for consistency we do need a coherent charge reservoir to which charge is transferred by anomalous scattering events; its coherence length has to be *finite*, but it must also be long compared with the quasiparticle mean-free path

$$\xi(T) \gtrsim v_F / \Gamma_s(T). \tag{23}$$

We are thus tempted to interpret $\Gamma_- \sim 2T$ as a charge relaxation rate which survives in the normal state by virtue of these large coherent patches. It may be possible to directly test this idea, for once the cuprate metals become superconducting, the charge relaxation rate can be measured using a normal-superconducting-normal tunnel junction [19] or phase slip centers in narrow wires. In conventional superconductors as $T \rightarrow T_c$, $\Gamma_Q \rightarrow 0$ [19,20] in the cuprates we expect the charge imbalance decay rate to remain finite as $T \rightarrow T_c$.

These lines of reasoning suggest the presence of a mutual inelastic decay channel for electrons and holes $h^+ \rightleftharpoons$ neutral state $\rightleftharpoons e^-$ which involves the emission or absorption of a charge $+1e$ object into the charge reservoir. Perhaps there is a loose link here with the "holon" excitation in Anderson's Luttinger liquid scenario [3]? We also note that recent theoretical work on non-Fermi-liquid behavior in impurity models indicates one origin of scale-invariant marginal Fermi liquid behavior [21] is the formation of fermionic three-body bound states with *definite* charge-conjugation symmetry [22,23]. The formation of such objects around the Fermi surface might provide a microscopic basis for our phenomenological hypothesis.

In summary, we have proposed that the appearance of two transport relaxation times in the cuprate metals is a consequence of scattering effects that are sensitive to the charge-conjugation symmetry of the quasiparticles. We have formulated this hypothesis in a transport equation where one eigenstate of charge-conjugation symmetry decays much more rapidly than its partner. The decay of electric current is dominated by the rapid relaxation rate while "neutral" transport currents, including the Hall and thermoelectric currents, are governed by the slower decay rate.

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