Measurements of Higher Order Photon Bunching of Light Beams

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A two-photon detection scheme is used to measure three- and four-photon correlations in a light beam and study their time dependence.

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Photon correlations of light beams are intimately connected to source dynamics. They are characteristic of sources that produce them [1]. Thus while photons from thermal sources exhibit bunching [2], those from nonthermal sources may exhibit antibunching [3]. Photon correlations are usually discussed in terms of the second order intensity correlation function $\langle I(t)I(t+\tau)\rangle$, where $I(t)$ is the intensity of light beam and angular brackets indicate averaging with respect to the state of the field. The second order correlation function is proportional to the probability of detecting two photons separated by an interval τ. It is clear that even for uncorrelated photons there is some finite probability, proportional to $\langle I(t)\rangle\langle I(t+\tau)\rangle = \langle I\rangle^2$, of detecting a pair of photons separated by an interval τ. A measure of intrinsic two-photon bunching is therefore provided by the correlation function $\kappa_2(\tau) = \langle \Delta I(t)\Delta I(t+\tau)\rangle/\langle I\rangle^2$, where $\Delta I(t) = I(t) - \langle I\rangle$ represents deviations of light intensity from the mean. Here and in what follows we assume statistically stationary light beams. Similar to $\kappa_2(\tau)$, we can introduce higher order correlation functions of light.

The third order intensity correlation function $\langle I(t)I(t+\tau_1)I(t+\tau_1+\tau_2)\rangle$ is proportional to the probability of detecting three photons at times t, $t+\tau_1$, and $t+\tau_1+\tau_2$, respectively. Coincidences of third order include effects of three-photon chance coincidences, proportional to $\langle I\rangle^3$, and two correlated photons in chance coincidence with a third one, proportional to $\langle I(t)I(t+\tau_1)\rangle\langle I(t+\tau_1+\tau_2)\rangle$, etc. After subtracting these contributions, we find the intrinsic third order correlations are given by $\kappa_3(\tau_1,\tau_2) = \langle \Delta I(t)\Delta I(t+\tau_1)\Delta I(t+\tau_1+\tau_2)\rangle/\langle I\rangle^3$. Similarly, the intrinsic fourth order correlations are given by the function $\kappa_4(\tau_1,\tau_2,\tau_3) = \langle \Delta I(t)\Delta I(t+\tau_1)\Delta I(t+\tau_1+\tau_2)\Delta I(t+\tau_1+\tau_2+\tau_3)\rangle/\langle I\rangle^4 - \kappa_2(\tau_1)\kappa_2(\tau_3) - \kappa_2(\tau_1+\tau_2)\kappa_2(\tau_2+\tau_3) - \kappa_2(\tau_1+\tau_2+\tau_3)\kappa_2(\tau_2)$ [4]. The zero-delay correlations $\kappa_3(0,0)$ and $\kappa_4(0,0,0)$ describe the degree of intrinsic three- and four-photon bunching, respectively. The zero-delay intensity correlations were measured in counting experiments based on single-photon detectors by Chang and co-workers [5]. The accuracy of these measurements of higher order correlations is limited by the detector and electronic dead-time effects. This limitation becomes especially significant for fields that exhibit large intensity fluctuations [5,6]. Ironically, it is precisely for this type of fields that measurements of higher order correlations provide information that cannot be inferred from knowledge of lower order correlations.

Counting experiments do not allow us to determine the time dependence of intensity correlations. The time dependence of intensity correlations is measured in delayed coincidence experiments [7–10]. Counting experiments also require large count rates for measuring higher order moments, whereas correlation measurements can be carried out with relatively low (at least by a factor of 10^{-2}) count rates. The measurements of the time dependence of the third order correlation function were studied by several workers [9,10] using single-photon detection schemes. These experiments clearly underscore the increasing difficulty of measuring the time dependence of intensity correlations higher than the second order by these techniques.

In this paper we wish to describe a scheme based on two-photon detection of light that allows us to measure third and fourth order intensity correlations. This scheme partly overcomes dead-time limitations and involves the measurements of autocorrelation and cross-correlation functions of the second harmonic and the fundamental field. We demonstrate our method by measuring the third and fourth order correlation functions of photons in a laser near threshold. Our experiments also yield the correlation times over which three- and four-photon correlations persist. The method is applicable to a wide range of experiments where higher order correlations of light play an important role.

Consider the generation of second harmonic (SH) light from a fundamental beam. The intensity of the SH beam $I_2(t)$ is related to the intensity $I(t)$ of the fundamental beam by

$$I_2(t) = \text{const} \times I^2(t)\,. \tag{1}$$

Using this relation in the cross-correlation function $C(\tau) = \langle I_2(t)I(t+\tau)\rangle/\langle I_2\rangle\langle I\rangle$ of the second harmonic and fundamental light intensity, we immediately see that

$$C(\tau) = \frac{\langle I^2(t)I(t+\tau)\rangle}{\langle I^2\rangle\langle I\rangle} \tag{2}$$

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measures three-photon correlations of the fundamental beam. Writing $I(t) = \langle I \rangle + \Delta I(t)$ in Eq. (2), and using the definitions of intrinsic bunching of various orders, we find

$$c(\tau) \equiv C(\tau) - 1 = \frac{2\kappa_2(\tau) + \kappa_3(0,\tau)}{1 + \kappa_2}, \qquad (3)$$

where $\kappa_2 \equiv \kappa_2(0)$. This equation shows that a measurement of the SH and fundamental cross-correlation function $c(\tau)$ yields the intrinsic three-photon correlation function $\kappa_3(0,\tau)$. Similarly we find that the normalized second order intensity correlation function of second harmonic light $S(\tau) = \langle I_2(t) I_2(t+\tau)\rangle / \langle I_2 \rangle^2$ measures four-photon correlations of the fundamental beam. In terms of intrinsic correlations it can be written as

$$\begin{aligned} s(\tau) &\equiv S(\tau) - 1 \\ &= \frac{4\kappa_2(\tau) + 2[\kappa_2(\tau)]^2 + 4\kappa_3(0,\tau) + \kappa_4(\tau)}{(1+\kappa_2)^2}, \end{aligned} \qquad (4)$$

where we have used the symmetry property $\kappa_3(0,\tau) = \kappa_3(\tau,0)$ and $\kappa_4(\tau) \equiv \kappa_4(0,\tau,0)$. Eq. (4) shows that a measurement of $s(\tau)$ together with the knowledge of lower order correlations yields degree of intrinsic four-photon bunching $\kappa_4(0,0,0)$.

An outline of the experimental setup is shown in Fig. 1. It consists of a folded standing wave single-mode He:Ne laser cavity [11]. A noncritically temperature phase-matched rubidium dihydrogen phosphate (RDP) crystal placed at the beam waist inside the cavity generates SH light. Cavity mirrors are highly reflecting (>99%) at the fundamental wavelength (632.8 nm) and highly transmitting (>85%) at the second harmonic (316.4 nm). The fundamental and its SH leaving the cavity are orthogonally polarized. They are separated by a polarizing beam splitter. The SH and the fundamental beams pass through line filters and fall on two separate high gain fast photomultiplier tubes. The SH light is passed through a monochromator to further reduce background light. The output pulses from the photomultiplier tubes are fed to the two inputs, labeled A and B, of a digital correlator. For the measurements of the cross-correlation function $c(\tau)$ we feed second harmonic pulses to input A and fundamental pulses to input B. For measuring the second order intensity autocorrelation function $s(\tau)$ or $\kappa_2(\tau)$ only input A is used. If input A is SH we get $s(\tau)$ and if it is fundamental we get $\kappa_2(\tau)$.

The correlator divides time into intervals of equal duration $\Delta\tau$. This time is referred to as sample time. In our experiments it was of the order of 2 μs. The number of pulses occurring during each sample time is counted by a shift register counter. At the end of each counting interval the counter shifts its contents into the first location of a 128-channel store-and-shift register. At the end of the next counting interval the number in the first location is moved to the second location and the number in the counter is moved to the first location. This procedure is continued until all 128 channels are filled. From this stage on the same procedure is continued with the number in the last channel being discarded. Building of the correlation function now begins. At the receipt of each pulse at input A (when in autocorrelation mode) or input B (when in cross-correlation mode) the contents of each location in the store-shift register are added to the contents of corresponding memory location in the correlator memory. Thus at the end of the ith sample time in which n_i counts were recorded contents of memory location j will increase by $n_i n_{i-j}$. After N_s such samples the number in the jth channel will be $N(j\Delta\tau) = \sum_{i=1}^{N_s} n_i n_{i-j}$. In practice, background light also contributes to counts in channels A and B. Let θ_A and θ_B denote the fractional signal count rates in the two channels. If the sample time is short compared to the correlation time of the light and N_s is large, the number $N(j\Delta\tau)$ in channel j is related to the correlation function of light by

$$N(j\Delta\tau) = N_s R_A R_B (\Delta\tau)^2 [1 + \theta_A \theta_B \lambda(\tau)], \qquad (5)$$

$\lambda(\tau)$ represents the appropriate correlation function—$\kappa_2(\tau)$, $c(\tau)$, or $s(\tau)$. The correlator also records numbers

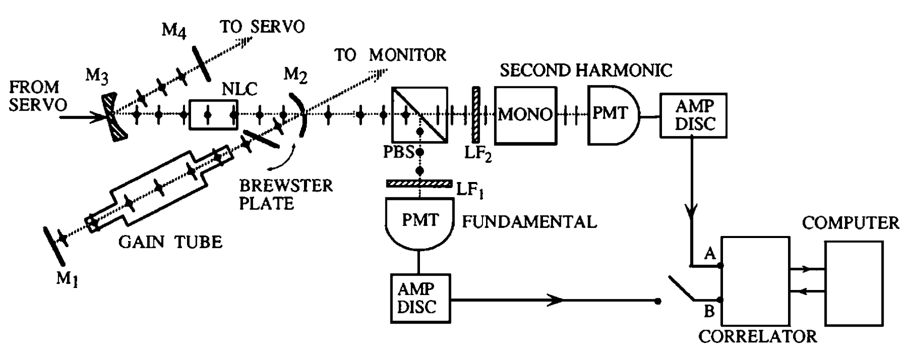

FIG. 1. An outline of the experimental setup. NLC is the nonlinear crystal, PBS is a polarizing beam splitter to separate second harmonic from the fundamental, LF_1 is a line filter centered at 632.8 nm, LF_2 is a uv line filter centered at the second harmonic wavelength 316.4 nm, MONO is a monochromator, PMT is a photomultiplier tube, AMP-DISC represents the amplifier-discriminator combination. Not shown are the electronic servo to stabilize the laser intensity, temperature control electronics, and the electronics for monitoring the fundamental intensity.

in 16 channels corresponding to delays of the order of $1028\,\Delta\tau$. Since all correlations have decayed down to zero, the average number in these channels will simply be

$$N_\infty = N_s R_A R_B (\Delta\tau)^2. \tag{6}$$

With the help of Eqs. (4) and (5) we can extract $c(\tau)$, $s(\tau)$, and $\kappa_2(\tau)$ from the experimental measurements. Count rates R_A and R_B were kept below 100 kHz. With such low count rates dead-time corrections were found to be negligible. From the measured correlation function $\lambda(\tau)$ we can introduce a correlation time

$$T = \int_0^\infty \frac{\lambda(\tau)}{\lambda(0)}\, d\tau\,, \tag{7}$$

where $\lambda(\tau) = \kappa_2(\tau)$, $c(\tau)$, and $s(\tau)$ define the correlation times T_2, T_3, and T_4, respectively, for two-, three-, and four-photon bunching. These times become a measure of the temporal extent of photon bunching of various orders.

The measured correlation functions were fitted by one or two exponentials. By extrapolating these fitted functions to zero delay we immediately obtain $\kappa_2(0)$, $c(0)$, and $s(0)$. The measured values of $\kappa_2(0)$ were used to determine the operating point of the laser characterized by the dimensionless pump parameter a [6,11]. As a check the operating point of the laser was also determined, independently, by carrying out photon counting measurements of the fundamental. These two determinations of the operating point yielded consistent results.

In comparing our measurements with the theoretical predictions we have chosen to plot $c(0)$ and $s(0)$ which are related to $\kappa_3(0,0)$ and $\kappa_4(0,0,0)$ by Eqs. (3) and (4), respectively. Both quantities are readily computed theoretically. From the normalization it is clear that $c(0)$ and $s(0)$ measure excess three- and four-photon bunching relative to the two-photon bunching. The experimental data are compared with theoretical calculations in Fig. 2. We see that both three- and four-photon bunching are prominent below threshold but decrease rapidly as the operating point of the laser increases and passes threshold. Above threshold, both three- and four-photon bunching is almost entirely due to pure chance coincidences. Thus intrinsic bunching rapidly decreases above threshold as the laser approaches a coherent state.

A measure of how long these correlations persist in time is provided by the correlation times introduced in Eq. (6). To avoid large uncertainties due to multiple subtractions that are required in order to obtain intrinsic correlation times, we have chosen to compare theoretical predictions with the correlation times extracted from the data by using the fitted functions in Eq. (7). Theoretical curves were obtained by solving the time-dependent Fokker-Planck equation [12]. To compare the experimental correlation times with the dimensionless times predicted by the theory a time scaling parameter was needed. This scaling parameter was determined according to the procedure explained

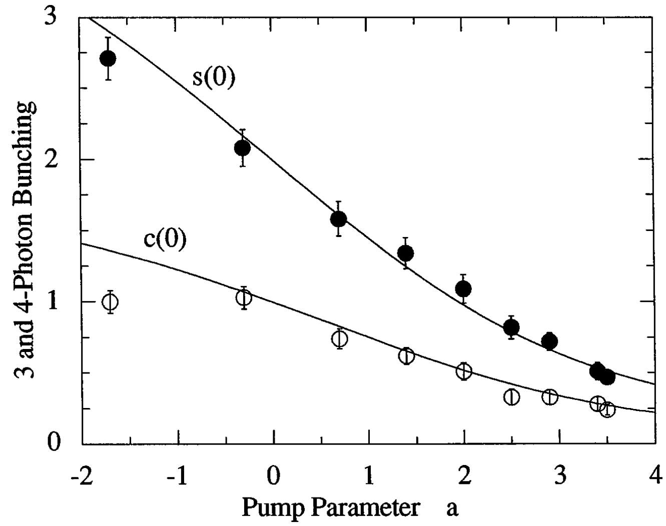

FIG. 2. Variation of the degree of three- and four-photon bunching $c(0)$ and $s(0)$, respectively, relative to the two-photon bunching as a function of the laser pump parameter a. The points represent the experimental data and the curves are the theoretical predictions.

in Refs. [6,7,9]. Figure 3 shows the variation of the correlation times for two-, three-, and four-photon bunching as a function of the operating point of the laser. In all cases correlations persist longest near threshold. This behavior is typical of all physical systems undergoing a phase transition. Figure 2 also shows that as the order of photon bunching increases the corresponding correlation time is shortened.

To summarize, we have measured the degrees of higher order photon bunching and their time dependence for an optical field using a two-photon detector. Several variations of the technique described in the paper are worth

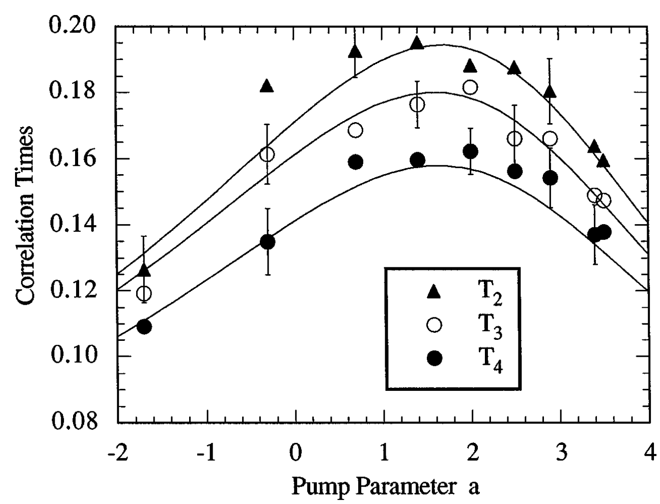

FIG. 3. Variation of the correlation times for two-, three-, and four-photon bunching as a function of the laser pump parameter a. The points represent the experimental data and the curves are the theoretical predictions.

mentioning. For example, using frequency mixing we can measure the correlations of weak signals by superimposing them with strong coherent fields. By introducing time delays between signals we can measure correlation functions with multiple delays. The agreement between our measurements and the theoretical predictions clearly demonstrates the efficacy of this scheme for studying higher order correlations of light from other sources.

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