Determining the Cabibbo-Kobayashi-Maskawa Unitarity Triangle from B Decays to Charged Pions and Kaons

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Decay rates of $B^0(t) \to \pi^+\pi^-$, $B^0 \to \pi^- K^+$, $B^+ \to \pi^+ K^0$ ($K_S \to \pi^+\pi^-$) and of charge-conjugate processes are studied within flavor SU(3) symmetry and first-order SU(3) breaking. We show that these measurements can determine with a reasonable accuracy the two angles α and γ of the Cabibbo-Kobayashi-Maskawa unitarity triangle.

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B decays provide a variety of CP asymmetry measurements [1], which can test the currently favored hypothesis that phases in elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2] are the source of the observed *CP* violation in the neutral kaon system [3]. The time-dependent rate asymmetry between the process $B^0(t) \to \pi^+\pi^-$ and its *CP* conjugate measures one of these phases, the angle α of the CKM unitarity triangle. Penguin amplitudes [4] and higher order electroweak penguin contributions [5] complicate the situation somewhat. However, by measuring also the rates of $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$ and of their charge-conjugate counterparts one can isolate the amplitudes contributing to final states with isospin 0 and 2 and thereby determine α with a rather good accuracy [6,7]. The detection of the modes involving neutral pions poses an interesting challenge for future experiments.

A few alternative ways to learn the penguin effect in $B^0 \to \pi^+\pi^-$ were suggested recently. DeJongh and Sphicas [8] have studied in detail the dependence of the asymmetry in $B^0(t) \to \pi^+\pi^-$ on the (unknown) magnitude and relative phase of the tree and penguin amplitudes contributing to this process. Using flavor SU(3) symmetry, Silva and Wolfenstein [9] proposed to approximately estimate the penguin contribution by comparing the tree-dominated decay rate of $B^0 \to \pi^+\pi^-$ with that of $B^0 \to \pi^- K^+$ which has a large penguin term. Buras and Fleischer [10] suggested to isolate the penguin term in $B^0 \to \pi^+\pi^-$ from its [SU(3)-related] dominant effect in the time-dependent asymmetry of $B^0(t) \to K^0\overline{K}^0$.

In the present Letter we describe a method which determines simultaneously both the angle α and the angle γ of the unitarity triangle from the decay rates of $B^0(t) \to \pi^+\pi^-$, $B^0 \to \pi^- K^+$, $B^+ \to \pi^+ K^0$ (where $K^0 \to K_S \to \pi^+\pi^-$) and their charge conjugates. All these modes are detected by charged pions and kaons in the final state. Other ways to measure γ, based on charged B decays, were proposed in Ref. [11]. Our method employs flavor SU(3) symmetry [12–14], and neglects "annihilation" amplitudes in which the spectator quark (the light quark accompanying the b in the initial meson) enters into the decay Hamiltonian [15]. These amplitudes in B decays are expected to be suppressed by f_B/m_B, where $f_B \simeq 180$ MeV. In order to improve the precision of the method, we also include first-order SU(3) breaking terms [16]. Second-order corrections, which are expected to be at a level of a few percent, will be neglected.

In the SU(3) limit and neglecting annihilation terms all B decay amplitudes into $\pi\pi$, πK, and $K\overline{K}$ states can be decomposed in terms of three independent amplitudes [7,15]: a "tree" contribution $t(t')$, a "color-suppressed" contribution $c(c')$, and a "penguin" contribution $p(p')$. These amplitudes contain both the leading-order and electroweak penguin contributions:

$$\begin{aligned} t &\equiv T + (c_u - c_d)P_{\rm EW}^C, \\ c &\equiv C + (c_u - c_d)P_{\rm EW}, \\ p &\equiv P + c_d P_{\rm EW}^C . \end{aligned} \tag{1}$$

Here the capital letters denote the leading-order contributions defined in Ref. [15], and $P_{\rm EW}$ and $P_{\rm EW}^C$ are color-favored and color-suppressed electroweak penguin amplitudes defined in Ref. [7]. The values $c_u = \frac{2}{3}$ and $c_d = -\frac{1}{3}$ are those which would follow if the electroweak penguin coupled to quarks in a manner proportional to their charges. (Small corrections, which we shall ignore and which do not effect our analysis, arise from axial-vector Z couplings and from WW box diagrams.) The $\Delta S = 0$ amplitudes are denoted by unprimed quantities and the $\Delta S = 1$ processes by primed quantities.

The amplitudes of the two processes $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ are expressed as

$$\begin{aligned} A_{\pi\pi} &\equiv A(B^0 \to \pi^+\pi^-) = -t - p \\ &= -T - P - \tfrac{2}{3} P_{\rm EW}^C , \\ A_{\pi K} &\equiv A(B^0 \to \pi^- K^+) = -t' - p' \\ &= -T' - P' - \tfrac{2}{3} P_{\rm EW}'^C , \end{aligned} \tag{2}$$

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while that for $B^+ \to \pi^+ K^0$ will be approximated by

$$A_+ \equiv A(B^+ \to \pi^+ K^0) = p' = P' - \tfrac{1}{3} P'^C_{\rm EW} \approx P' + \tfrac{2}{3} P'^C_{\rm EW}\,, \qquad (3)$$

neglecting a color-suppressed electroweak penguin effect of order $|P'^C_{\rm EW}/P'| = O((\frac{1}{5})^2)$ [7]. With this approximation, A_+ contains the same combination of electroweak and gluonic penguins as in the expression for $A_{\pi K}$.

The terms on the right-hand sides of (2) and (3) carry well-defined weak phases. The weak phase of T is $\mathrm{Arg}(V_{ud}V^*_{ub}) = \gamma$, and that of $P + \frac{2}{3}P^C_{\rm EW}$ is approximately $\mathrm{Arg}(V_{td}V^*_{tb}) = -\beta$, where we neglect corrections due to quarks other than the top quark. The effects of the u and c quarks become appreciable [17] when V_{td} obtains its currently allowed smallest values. This corresponds to a small deviation of the CP asymmetry in $B^0(t) \to \pi^+\pi^-$ from $\sin(2\alpha)\sin(\Delta m t)$ (where Δm is the neutral B mass difference). For large values of V_{td}, where the deviation due to the penguin amplitude becomes significant [18], the u and c contributions become vary small. T' also carries the phase γ, while the weak phase of $P' + \frac{2}{3}P'^C_{\rm EW}$ is $\mathrm{Arg}(V_{ts}V^*_{tb}) = \pi$. The ratio of $\Delta S = 1$ to $\Delta S = 0$ tree and penguin amplitudes are given by the corresponding ratios of CKM factors, $|T'/T| = |V_{us}/V_{ud}| \equiv r_u = 0.23$, $|P'/P| = |V_{ts}/V_{td}| \equiv r_t$.

Denoting $\mathcal{T} \equiv |T|$, $\mathcal{P} \equiv |P + \frac{2}{3}P^C_{\rm EW}|$ and assigning SU(3)-symmetric strong phases δ_T, δ_P to terms with specific weak phases, (2) and (3) may be transcribed as

$$\begin{aligned} A_{\pi\pi} &= \mathcal{T}\, e^{i\delta_T} e^{i\gamma} + \mathcal{P} e^{i\delta_P} e^{-i\beta}, \\ A_{\pi K} &= r_u \mathcal{T}\, e^{i\delta_T} e^{i\gamma} - r_t \mathcal{P} e^{i\delta_P}, \\ A_+ &= r_t \mathcal{P} e^{i\delta_P}. \end{aligned} \qquad (4)$$

To introduce first-order SU(3) breaking corrections, we note that in the T' amplitude the W turns into an $\bar{s}$ quark instead of a $\bar{d}$ in T. This SU(3) breaking term was denoted by T'_1 in Ref. [16]. Assuming factorization for T, which is supported by experiments [19,20] and justified for $B \to \pi\pi$ and $K\pi$ by the high momentum with which the two color-singlet mesons separate from one another, SU(3) breaking is given by the K/π ratio of decay constants

$$\frac{\mathcal{T}'}{\mathcal{T}} = \frac{|V_{us}|f_K}{|V_{ud}|f_\pi} \equiv \tilde{r}_u\,. \qquad (5)$$

Apart from small electroweak penguin terms, all amplitudes we consider are free of color-suppressed contributions, for which factorization might be more questionable. The situation would be very different were we to consider the amplitude for $B^0 \to \pi^0\pi^0$, where the color-suppressed contribution could be dominant.

In the penguin amplitudes (including electroweak penguin) of both $B^0 \to \pi^- K^+$ and $B^+ \to \pi^+ K^0$ the $\bar{b}$ quark turns into an $\bar{s}$ quark instead of a $\bar{d}$ in $B^0 \to \pi^+\pi^-$. This SU(3) breaking term was denoted by P'_1 in Ref. [16]. Here we will denote the magnitude of the $\Delta S = 1$ penguin amplitude by $r_t\tilde{\mathcal{P}}$, to allow for SU(3) breaking. Since factorization is questionable for penguin amplitudes, one generally expects $\tilde{P} \neq (f_K/f_\pi)\mathcal{P}$. We will assume that the phase δ_P is unaffected by SU(3) breaking. Since this phase is likely to be small [21], this assumption is not expected to introduce a significant uncertainty in the determination of the weak phases.

Thus, including first-order SU(3) breaking, Eqs. (4) are modified to become

$$\begin{aligned} A_{\pi\pi} &= \mathcal{T}\, e^{i\delta_T} e^{i\gamma} + \mathcal{P} e^{i\delta_P} e^{-i\beta}, \\ A_{\pi K} &= \tilde{r}_u \mathcal{T}\, e^{i\delta_T} e^{i\gamma} - r_t \tilde{\mathcal{P}} e^{i\delta_P}, \\ A_+ &= r_t \tilde{\mathcal{P}} e^{i\delta_P}. \end{aligned} \qquad (6)$$

It will be shown that the numerous *a priori* unknown parameters in (6), including the two weak phases $\alpha \equiv \pi - B - \gamma$ and γ, can be determined from the rate measurements of the above three processes and their charge conjugates.

First, we note that the amplitudes for the corresponding charge-conjugate decay processes are simply obtained by changing the signs of the weak phases γ and β. We denote the charge-conjugate amplitudes corresponding to (6) by $\overline{A}_{\pi\pi}$, $\overline{A}_{\pi K}$, A_-, respectively.

The time-dependent tagged B^0 and $\overline{B}^0$ decay rates to $\pi^+\pi^-$ are given by

$$\begin{aligned} \Gamma[B^0(t) \to \pi^+\pi^-] &= e^{-\Gamma t}\left[|A_{\pi\pi}|^2 \cos^2\!\left(\frac{\Delta m}{2}t\right) + |\overline{A}_{\pi\pi}|^2 \sin^2\!\left(\frac{\Delta m}{2}t\right) + \mathrm{Im}(e^{2i\beta}A_{\pi\pi}\overline{A}^*_{\pi\pi})\sin(\Delta m t)\right], \\ \Gamma[\overline{B}^0(t) \to \pi^+\pi^-] &= e^{-\Gamma t}\left[|A_{\pi\pi}|^2 \sin^2\!\left(\frac{\Delta m}{2}t\right) + |\overline{A}_{\pi\pi}|^2 \cos^2\!\left(\frac{\Delta m}{2}t\right) - \mathrm{Im}(e^{2i\beta}A_{\pi\pi}\overline{A}^*_{\pi\pi})\sin(\Delta m t)\right]. \end{aligned} \qquad (7)$$

Measurement of these rates determines $|A_{\pi\pi}|^2$, $|\overline{A}_{\pi\pi}|^2$, and $\mathrm{Im}(e^{2i\beta}A_{\pi\pi}\overline{A}^*_{\pi\pi})$:

$$\begin{aligned} |A_{\pi\pi}|^2 &= \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos(\delta - \alpha)\,, \\ |\overline{A}_{\pi\pi}|^2 &= \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos(\delta + \alpha)\,, \\ \mathrm{Im}(e^{2i\beta}A_{\pi\pi}\overline{A}^*_{\pi\pi}) &= -\mathcal{T}^2\sin(2\alpha) \\ &\quad + 2\mathcal{T}\mathcal{P}\cos\delta\sin\alpha\,, \end{aligned} \qquad (8)$$

where we used $\beta + \gamma = \pi - \alpha$ and where we defined $\delta \equiv \delta_T - \delta_P$. The rates of the self-tagging modes $\pi^- K^+$, $\pi^+ K^-$, and $\pi^+ K^0$ determine $|A_{\pi K}|^2$, $|\overline{A}_{\pi K}|^2$, and $|A_+|^2$, respectively:

$$\begin{aligned} |A_{\pi K}|^2 &= \tilde{r}_u^2\mathcal{T}^2 + r_t^2\tilde{\mathcal{P}}^2 - 2\tilde{r}_u r_t \mathcal{T}\tilde{\mathcal{P}}\cos(\delta + \gamma)\,, \\ |\overline{A}_{\pi K}|^2 &= \tilde{r}_u^2\mathcal{T}^2 + r_t^2\tilde{\mathcal{P}}^2 - 2\tilde{r}_u r_t \mathcal{T}\tilde{\mathcal{P}}\cos(\delta - \gamma)\,, \\ |A_+|^2 &= |A_-|^2 = r_t^2\tilde{\mathcal{P}}^2. \end{aligned} \qquad (9)$$

Measurement of the six quantities in (8)–(9) suffices to determine all six parameters α, γ, $\mathcal{T}$, $\mathcal{P}$, $\tilde{\mathcal{P}}$, and δ up to discrete ambiguities. The CKM parameter $r_t \equiv |V_{ts}/V_{td}|$, which is still largely unknown, is obtained from the unitarity triangle in terms of α and γ:

$$r_u r_t = \frac{\sin\alpha}{\sin\gamma}. \tag{10}$$

We note immediately that

$$|A_{\pi K}|^2 - |\overline{A}_{\pi K}|^2 = -\left(\frac{f_K}{f_\pi}\right)\left(\frac{\tilde{\mathcal{P}}}{\mathcal{P}}\right)(|A_{\pi\pi}|^2 - |\overline{A}_{\pi\pi}|^2), \tag{11}$$

which determines the magnitude of SU(3) breaking in the penguin amplitude $\tilde{\mathcal{P}}/\mathcal{P}$. The relation (11) between the particle-antiparticle rate differences in $B \to \pi K$ and in $B \to \pi\pi$ was recently derived [22] in the SU(3) limit $f_K/f_\pi \to 1$, $\tilde{\mathcal{P}}/\mathcal{P} \to 1$. The authors assumed for SU(3) breaking a value $\tilde{\mathcal{P}}/\mathcal{P} = f_K/f_\pi$ (based on factorization of penguin amplitudes) which is questionable. In our approach, this ratio is a free parameter to be determined by experiment. We expect it to differ from 1 by up to 30%.

Both sides of Eq. (11) are proportional to $\sin\delta$, and thus would vanish in the absence of a strong phase difference. In that case, one would have to assume a relation between $\tilde{\mathcal{P}}$ and $\mathcal{P}$ in order to obtain a solution. If, on the other hand, $\delta \neq 0$, leading to a rate asymmetry between the self-tagging decays $B^0 \to \pi^- K^+$ and $\overline{B}^0 \to \pi^+ K^-$, the present method permits one to interpret that rate asymmetry in a manner independent of δ.

A combined sample of the decays $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ has already been observed [23] with a joint branching ratio of about 2×10^{-5}. Equal mixtures of the two modes are likely, although confirmation of this estimate awaits a better π/K separation. A similar branching ratio is expected for $B^+ \to \pi^+ K^0$, where the efficiency of observing a K^0 by a K_S decay to two charged pions is 1/3. Samples of hundreds of events in each of these modes (combining $B^+ \to \pi^+ K^0$ and $B^- \to \pi^- \overline{K}^0$) are expected to be obtained in future e^+e^- colliders operating at the $\Upsilon(4S)$ resonance. The resulting statistical accuracy of determining the weak phases α and γ using the above method thus is expected to be at a level of 10%. The theoretical uncertainty of the method is at a similar level, involving the following corrections all of which are of order a few percent: a correction from an electroweak penguin amplitude in $B^+ \to \pi^+ K^0$, corrections due to u and c quarks in the $B^0 \to \pi^+\pi^-$ penguin amplitude, second-order SU(3) breaking in the magnitudes of weak amplitudes, first-order SU(3) breaking in the (small) strong phase of the penguin amplitude, and $O(f_B)$ annihilation amplitudes.

To summarize, we have shown that measurements of the rates for B decays to modes involving charged pions and kaons in the final states can determine the shape of the unitarity triangle. The accuracy of this method of determining the angles α and γ in future e^+e^- B factories is roughly estimated to be at a level of 10%. More detailed studies of the precision of this method are worthwhile.

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