Comment on "Asymptotic Scaling in the Two-Dimensional O(3) σ Model at Correlation Length 10^5"

In their recent Letter [1] Caracciolo *et al.* claim to have determined the correlation length of the 2D O(3) σ model up to 10^5 and to find excellent (4%) agreement with the Hasenfratz-Maggiore-Niedermayer (HMN) formula [2]. Their results come from applying finite size scaling (FSS) to Monte Carlo (MC) data taken on lattices of linear size $L \leq 512$, 200 times smaller than the alleged correlation lengths. Although this fact alone casts doubt upon such claims, we would like to repeat here why such procedures cannot be relied upon to study asymptotic scaling in 2D O(N) models (see also [3,4]).

FSS is a statement about the limit $L \to \infty$ at $x \equiv \xi(L)/L$ fixed. So if asymptotic scaling would hold, L would have to be increased like $O(e^{2\pi\beta})$; if, as we believe, there is a critical point at a finite value of β, L has to increase with β even faster. There is no easy answer to the crucial question how large L should be chosen to keep the corrections to FSS smaller than a given percentage, but a criterion is provided by perturbation theory (PT). PT provides the correct asymptotic expansion at L fixed, $\beta \to \infty$ and suggests that if $L \ll O(e^{\pi\beta})$, any MC measured quantity will just reproduce PT. Moreover, at fixed L, the accuracy with which PT reproduces MC data increases with increased x. We have verified this explicitly in Ref. [4] and so have Caracciolo *et al.* (see their Fig. 2, where for $x > 0.7$, the PT prediction is indistinguishable from the MC data). So contrary to what they say, implicitly they do assume asymptotic scaling by working in the perturbative regime for the crucial large x values.

There is another, related, trouble with PT at fixed L: As we have shown explicitly [5], the two limits $L \to \infty$ and $\beta \to \infty$ cannot be interchanged. If by their procedure Caracciolo *et al.* did determine the true $\xi_\infty^{(2)}(\beta)$, the result should be independent of the boundary conditions (BC) used. But in [5] we showed that in the non-Abelian models O(N) $N \geq 3$, the termwise limits of the PT coefficients, and even of the so-called universal coefficients of the β function depend upon the BC.

The only safe way to avoid the pollution of the FSS predictions by the BC is to work on lattices with $L \gg O(e^{\pi\beta})$. The authors not only did not obey this criterion, but for $x > 0.7$ they reduced $L_{\rm min}$ from 128 to 64. They state that they needed a larger $L_{\rm min}$ for $x < 0.7$ to eliminate certain scaling violations. These are, in fact, systematic: In their Fig. 1, for $x < 0.6$ the data points taken at the same x but larger L (i.e., larger β) generally produce larger values for the scaling function F_ξ. These nonperturbative scaling violations shift to larger L values with increasing x and for $x > 0.7$ are no longer visible in the limited range of L values studied. This certainly cannot be taken as proof that the limit $L \to \infty$ has been reached; it would rather be worth some effort to study these violations in more detail.

Those FSS violations are also reflected in the extrapolated values of the correlation length $\xi_\infty^{(2)}$ produced by Caracciolo *et al.* as well as those of Kim, reported in their Table II: generally larger lattices lead to larger values of $\xi_\infty^{(2)}$.

Caracciolo *et al.* state that their work establishes FSS for $L_{\rm min} \leq L \leq 256$ and $1.65 \leq \beta \leq 3$. This statement is incorrect: As said above, all their results employing FSS data with $x > 0.7$ are perturbative, hence, in principle, polluted by BC effects and cannot be regarded as true determinations of $\xi_\infty^{(2)}(\beta)$. So contrary to their claim implicity in Table II, one does not know $\xi_\infty^{(2)}(\beta)$ for $\beta > 1.9$; it is unknown if it varies in agreement with asymptotic scaling or whether it diverges for $\beta < 3$.

The authors invoke as support for their claims the improved agreement between their $\xi_\infty^{(2)}(3.0)$ and the HMN formula. But if one accepts their premises, one can extend their procedure to arbitrarily large β and see if the agreement still improves: Instead of the nonexisting MC data at very large β one can for $x > 0.7$ use the PT values for $\xi_L^{(2)}$—which will be very good at fixed L and those large β values—and determine $\xi_\infty^{(2)}$ using the PT form of their FSS function [their Eq. (7)]. The agreement with the HMN prediction does not improve, so the good agreement found at $\beta = 3$ has to be considered as accidental.

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[1] S. Caracciolo, R.G. Edwards, A. Pelissetto, and A.D. Sokal, Phys. Rev. Lett. **75**, 1891 (1995).
[2] P. Hasenfratz, M. Maggiore, and F. Niedermayer, Phys. Lett. B **245**, 522 (1990).
[3] A. Patrascioiu and E. Seiler Report No. MPI-Ph/91-88 (to be published).
[4] A. Patrascioiu and E. Seiler, Phys. Rev. Lett. **73**, 3325 (1994).
[5] A. Patrascioiu and E. Seiler, Phys. Rev. Lett. **74**, 1920 (1995).

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