Domain Coexistence in Two-Dimensional Optical Patterns

S. Residori, P. L. Ramazza, E. Pampaloni, S. Boccaletti,* and F. T. Arecchi*
Istituto Nazionale di Ottica, 50125 Firenze, Italy
(Received 7 August 1995)

We give evidence of coexisting transverse patterns of different symmetry in an optical beam circulating in a loop which contains a nonlinear medium. The symmetry of the patterns is controlled by the azimuthal rotation introduced in the feedback loop (nonlocality), while the competition is ruled by the input intensity which determines the distance from threshold (nonlinearity). Domains corresponding to patterns with different wave vectors (either different wavelength or different orientation) coexist, nucleating and moving. This gives rise to a complex spatiotemporal dynamics which is characterized by means of suitable collective indicators.

PACS numbers: 42.65.Sf, 42.79.Kr, 82.40.Ck

Preliminary experiments on pattern formation and competition in nonlinear optics have shown that patterns of different symmetry can alternate, either periodically or chaotically [1]. Alternation means that one pattern per time is mainly present, with a negligible amount of mixing with other configurations. This was explained in terms of heteroclinic cycles joining unstable fixed points corresponding to different configurations, with a long persistence time in the neighborhood of each fixed point and a fast transition from one fixed point to the other [2].

On the other hand, in the one-dimensional (1D) case, recent evidence has been presented of the coexistence of patterns of different symmetry in different regions of the available domain [3,4]. The patterns can have either a different wave number [3] or the same wave number but different phase [4]. In the former case the theory has to account for the formation of domain walls [5], in the latter case the domain walls will be phase defects [4].

In the 2D case, the first evidence of coexisting patterns of different symmetries was provided in an experiment of parametrically excited surface waves [6]. In 2D the different symmetries can be due either to selection of different wave vectors corresponding to the same wave number or to selection of different wave numbers. In the former case, there is a large body of experimental reports referring to bistable situations with the coexistence, e.g., of rolls and hexagons in Rayleigh-Bénard convection [7] or in optical patterning [8]. More recently, 2D domain coexistence of different patterns has been observed in large aspect ratio systems in parametrically excited surface waves [9].

Here we report evidence of the coexistence of domains of different wavelengths within the same 2D optical pattern [10]. The patterns we refer to are transverse patterns in an optical system consisting of a ring cavity where an impinging optical field, dephased after crossing a Kerr medium, modifies the properties of the same medium after a propagation in free space [11]. This is obtained via the use of a liquid crystal light valve (LCLV) [12] consisting of a thin layer of liquid crystal molecules sandwiched between two electrodes, together with a photoconductor. If the photoconductor is illuminated, most of the voltage drop is across the liquid crystal, thus providing an overall molecular alignment and hence a large Kerr effect. When the illumination is nonuniform, the pattern of the optical beam is transcribed into a dephasing pattern. The Kerr medium is thin compared to its diffusive length, hence the pattern formation is 2D, on a plane transverse to the direction of optical propagation.

The experimental setup consists of an LCLV with a front illumination via a collimated He-Ne laser beam. The backreflected light, Kerr dephased, undergoes diffraction and is then applied as a feedback signal on the back side (photoconductor) of the LCLV. A nonlocal feedback is provided by an image rotation introduced in the feedback loop through a fiber bundle rotation. For different settings of the rotation angle $\Delta = 2\pi/N$ (N integer), different types of pattern symmetries are excited, and at low intensity one succeeds in isolating the first unstable branch resulting from the interplay of diffraction in free space, diffusion in the Kerr medium and nonlocal feedback [13].

In the experiment reported here, we adjust the LCLV voltage at 12.3 rms and 3 kHz, the free propagation length at $L = 10$ cm, and the angle of rotation of the fiber at $\Delta = 2\pi/7$. Under these conditions, the linear stability analysis [13] predicts that, as the incident intensity I_0 overcomes I_{th}, the first unstable wave number is $q_2 = 2\pi\sqrt{3}/\sqrt{2\lambda L}$. This is indeed observed experimentally, as shown in Fig. 1(a). When, however, I_0 is increased well above I_{th}, the predictions of the linear stability analysis no longer hold.

Let us define a reduced pump parameter $\epsilon = (I_0 - I_{\mathrm{th}})/I_{\mathrm{th}}$. Experimentally, a gradual increase of ϵ starting from $\epsilon = 0$ leads initially to an increase of the amplitude of the quasicrystalline patterns, without a scale change. A further increase in ϵ results in the destabilization of a second band at $q_1 = 2\pi/\sqrt{2\lambda L}$ [Fig. 1(b)]. In this situation the near field signal does not appear as a uniform superposition of patterns at the two different wavelengths, but rather as a collection of spatially separated domains,

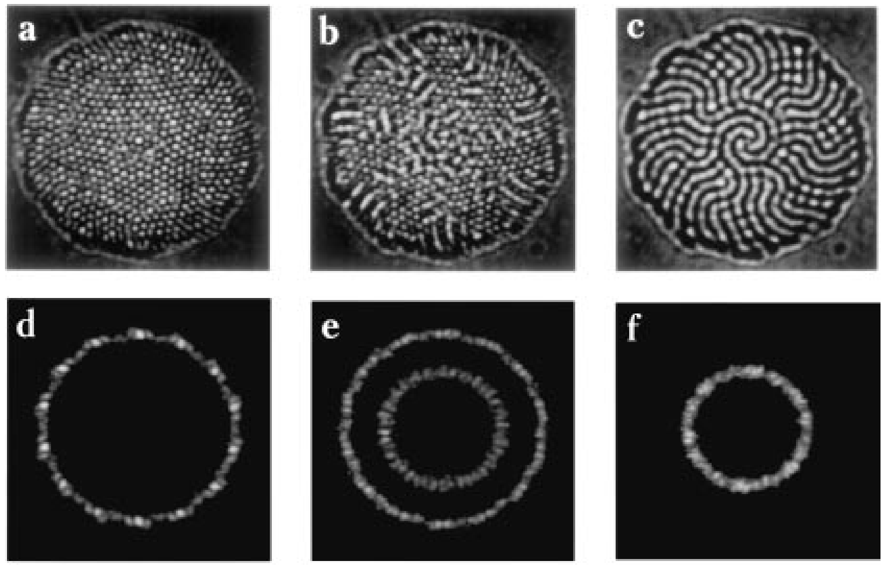

FIG. 1. Near field (upper) and far field (lower) patterns observed for $\epsilon = 0.5$ (a),(d), $\epsilon = 2$ (b),(e), and $\epsilon = 4.2$ (c),(f). Left (right) column corresponds to excitation of only the q_2 (q_1) band, in the middle column the two bands coexist. The single wave number cases (left and right) show coexistence of many sets of $2N = 14$ vectors.

each one containing patterns at only one of the two spatial scales. The average size of the domains with $q = q_1$ increases for increasing ϵ and eventually the whole wave front is made of domains at this wave number, while the domains at $q = q_2$ are completely suppressed [Fig. 1(c)].

For ϵ very small, a single q band is associated with a far field made of $2N$ spots (fixed orientation of the wave vectors), and hence the near field shows mainly a single domain (besides some boundary perturbations) [13]. On the contrary, here (rather larger ϵ) even a single band is a collection of wave vectors with different orientations, and hence even for a single wavelength we have a many-domain pattern, with grain boundaries separating different orientations. As ϵ is increased [Figs. 1(a) and 1(b)] domains with the smaller wave number q_1 emerge at the grain boundaries of the previous q_2 multiorientation patterns, thus showing that defects are sources that trigger the onset of the q_1 patterns [14].

In Fig. 2 we report the local intensities at one point of the near field for the three cases described above. When the wave number q_2 is excited ($\epsilon = 0.2$) we have relatively slow drifts of the domain boundaries. When only q_1 is excited ($\epsilon = 4.5$) the corresponding eigenvalue λ is complex [13], and thus we obtain rotating patterns. The rotation gives rise to a high frequency as observed in Fig. 2(c). Finally, in Fig. 2(b) ($\epsilon = 1.9$) the two wave numbers coexist, and at a given pixel we have an alternation between the two regimes.

Further information about the observed phenomena can be gained from the spatial power spectra of the signal, corresponding to the far field. Typical examples of these spectra are shown in Figs. 1(d)–1(f). In order to obtain some global information about the temporal behavior of the signal, we define the quantity $\eta(t) = S_1(t)/[S_1(t) + S_2(t)]$ as the fraction of the total power that instantaneously belongs to the first band. Here $S_j(t)$ ($j = 1, 2$) is the instantaneous power radially integrated in the Fourier space over a circular corona of radius q_j. A plot of $\eta(t)$ for three different values of ϵ is shown in Fig. 3. It is seen here that, when the system is dominated by one of the two competing bands, the time fluctuations of $\eta(t)$ are very small. On the contrary, the range of ϵ for which

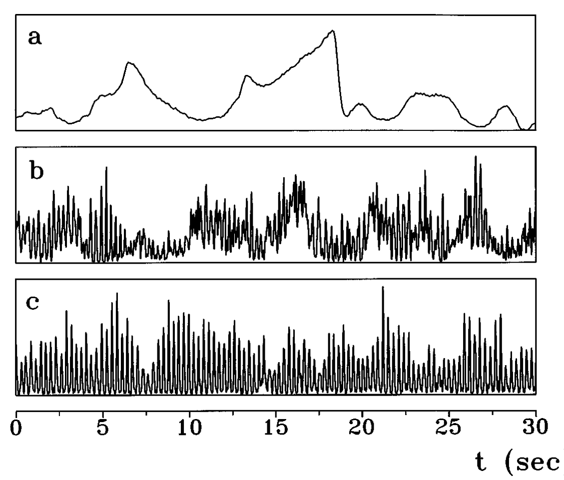

FIG. 2. Near field local intensity (arbitrary units) vs time. In (a) ($\epsilon = 0.2$, q_2 band) the fluctuations are due only to domain dynamics; in (c) ($\epsilon = 4.5$, q_1 band) there is also a fast oscillation due to the imaginary part of the eigenvalue; (b) ($\epsilon = 1.9$, both q_1 and q_2 bands) is a superposition of the other two cases.

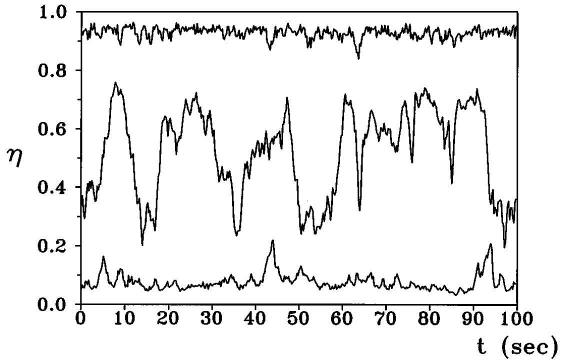

FIG. 3. Temporal evolution of the normalized spectral power η on the first ring. $\epsilon = 1$, q_2 band (lower curve), $\epsilon = 4.1$, q_1 band (upper curve), and $\epsilon = 2.1$, both q_1 and q_2 bands (middle curve).

the two bands show coexistence corresponds to regions of high fluctuation for $\eta(t)$, meaning that there neither the coexistence of the two bands nor the domination of one band over the other are stable phenomena.

A quantitative measurement of the transition from the band q_2 to the band q_1 dominated regime is given by the behavior of the time average $\overline{\eta} \equiv \langle \eta(t) \rangle_t$ and the standard deviation $\sigma \equiv [\langle \overline{\eta}^2 - \eta(t)^2 \rangle_t]^{1/2}$ of the quantity $\eta(t)$ versus the pump parameter ϵ. Plots of the results of these measurements are shown in Fig. 4 (left). These plots give a quantitative confirmation of the enhancement of fluctuations in the signal that accompanies the regimes of competition coexistence between the two bands.

The experimental results can be described in terms of a model that, though being oversimplified, retains the fundamental mechanisms of the process under consideration. At each point of real space, the local field $E(r,t)$ is expressed in terms of its Fourier expansion, which forms a discrete set

$$E(r,t) = \int d\mathbf{q}\, a_{\mathbf{q}}(\tilde{r}) e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{1}$$

where

$$a_{\mathbf{q}}(\tilde{r}) = \sum_{n=1}^{2N} a_n \delta_{\mathbf{q}-\mathbf{q}_n(\tilde{r})}\,. \tag{2}$$

This means that at each r position we have a fast space dependence due to the phase factor, plus a slow dependence due to the selection of a set of $2N$ vectors $[\mathbf{q}_n(\tilde{r})]$ specific of that domain, and which belong to either the q_1 or q_2 rings of Fig. 1. Via a Galerkin expansion, truncated to those modes whose wave numbers lie on the rings of radii q_1 and q_2, the two partial differential equations ruling the interaction of the field with the Kerr medium [9,15] are replaced by a set of ordinary equations describing the evolution of $a_{\mathbf{q}}(t)$, with linear terms and leading nonlinearities due to quadratic and cubic mode-mode coupling [16,17].

The mode coupling within one ring (at constant q modulus) was treated in [16], and for $N \neq 3l$ (l being a positive integer) the quadratic nonlinearity disappears by closure considerations, thus leaving a cubic mode coupling of the type considered in laser theory for population coupling in the absence of phase coupling [18]. This applies to our case since we have selected $N = 7$.

Thus far, however, no treatment has been provided for the competition between rings in q space. The data of Fig. 1 show that a situation of almost isotropic amplitude distribution on the two rings is easily reached. Even though the far field displays this isotropy, the closure relations in building the quadratic convolution term for the evolution equation of $a_q(\tilde{r},t)$ must be built with a unique set of $2N$ vectors. This rules out the possibility of having $\mathbf{q}_i^{\mathrm{I}} + \mathbf{q}_i^{\mathrm{II}} = \mathbf{q}_j$ ($i \neq j, i$ and $j = 1,2$), since with $N = 7$ ($2N = 14$ points regularly spaced over each ring) and with the ratio $|q_2|/|q_1| = \sqrt{3}$, the above relations are never satisfied. Thus also the inter-ring competitions are ruled only by cubic nonlinearities.

We find it convenient to follow the evolution of the corresponding integrated spectral powers $S_i = 2\pi q_i |a_{qi}|^2$ ($i = 1,2$). The equations for S_1 and S_2 are

$$\begin{aligned} \dot{S}_1 &= \mu_1 S_1 - \beta_1 S_1^2 - \gamma_1 S_1 S_2\,, \\ \dot{S}_2 &= \mu_2 S_2 - \beta_2 S_2^2 - \gamma_2 S_1 S_2\,. \end{aligned} \tag{3}$$

We have thus arrived at general equations analogous to those ruling the dynamics of competing populations [19] and already used in laser dynamics for two mode operation [18].

Because of the saturating characteristics of the LCLV [20], the linear growth rates μ_i depend on the input intensity I_0. The function $\mu_i(I_0)$ is increasing for moderate I_0 and decreasing for high I_0, where saturation of LCLV characteristic is effective. We choose as a functional form for $\mu_i(I_0)$ a parabola, that is, $\mu_i = \alpha_i I_0 - \rho_i I_0^2$, $i = 1,2$.

The system admits the following four fixed points: $O = (0,0)$, $F1 = (\mu_1/\beta_1, 0)$, $F2 = (0, \mu_2/\beta_2)$, and $C = ((\mu_1\beta_2 - \gamma_1\mu_2)/(\beta_1\beta_2 - \gamma_1\gamma_2), (\beta_1\mu_2 - \gamma_2\mu_1)/(\beta_1\beta_2 - \gamma_1\gamma_2))$. The spatial interaction neglected in Eqs. (3) permits the birth of coherent $F1$ or $F2$ domain structures, nucleating from local defects. Indeed, when a single family locally displays a defect, this becomes a nucleation center for the other family. Hence, the observed sharing process on the near field can be interpreted as a continuous nucleation and competition of the two coherent domains, and it can be modeled by adding $\mu_2\xi(t)$ and $\mu_1\xi(t)$ to the first and second of Eqs. (3), respectively, where $\xi(t)$ is a wideband stochastic process with zero average. The noise contribution in the S_1 equation has been multiplied for μ_2 to account for the fact that the perturbation to S_1 arises from S_2 domains nucleating from local defects, hence, it is proportional to the growth rate of the second family. Similar considerations hold for the S_2 equation.

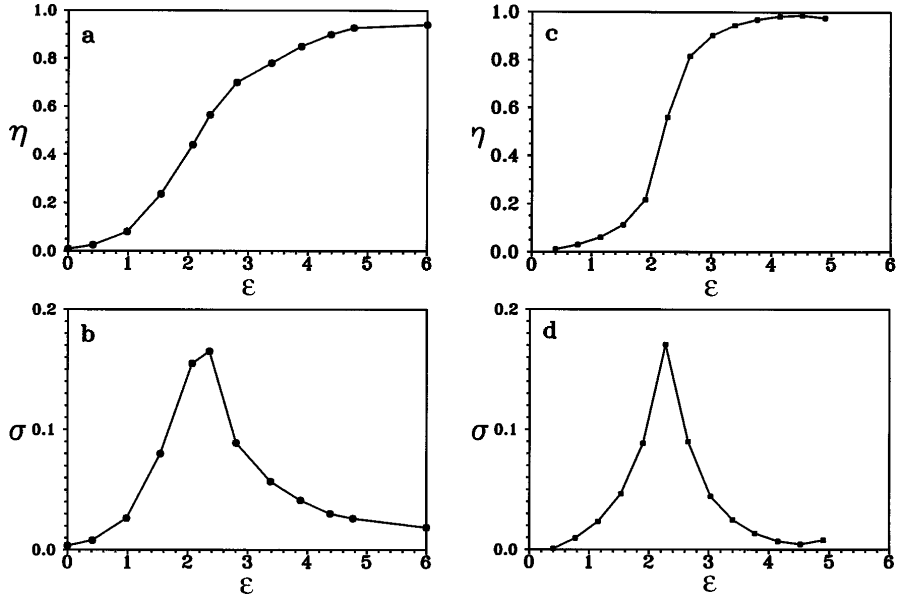

FIG. 4. Experimental (left column) and theoretical (right column) plots of $\eta(\epsilon)$ (a),(c) and $\sigma(\epsilon)$ (b),(d). Experimental error bars are within the size of the black circles. Theoretical points (black squares) are obtained from numerical integration of Eqs. (3) with $\mu_1 = 1 - (I_0 - 5)^2$, $\mu_2 = 1 - (I_0 - 5.5)^2$, $\beta_1 = \beta_2 = 1.5$, $\gamma_1 = \gamma_2 = 2.4$, and the noise addition. The x axis has been normalized to the reduced pump parameter ϵ. In all cases, lines are just a guide connecting points.

In Fig. 4 (right) we report the plots of $\eta(\epsilon)$ and of $\sigma(\epsilon)$ extracted from the numerical solutions of Eqs. (3) with the noise addition. For a suitable choice of parameters, they are in good qualitative agreement with the experiment.

In summary, we have shown that 2D patterns of different symmetries can coexist over different domains even when they belong to different wavelengths, we have introduced global indicators characterizing this coexistence, and we have built a simple model which describes the main experimental features.

Work partly supported by EEC Contract No. CHRX-Ct93-0107.

*Also at Physics Department, University of Florence, Florence, Italy.

[1] F.T. Arecchi, G. Giacomelli, P.L. Ramazza, and S. Residori, Phys. Rev. Lett. **65**, 2531 (1990).
[2] F.T. Arecchi, S. Boccaletti, G.B. Mindlin, and C. Perez Garcia, Phys. Rev. Lett. **69**, 3723 (1992).
[3] J. Hegseth, J.M. Vince, M. Dubois, and P. Berge', Europhys. Lett. **17**, 413 (1992).
[4] G. Giacomelli, R. Meucci, A. Politi, and F.T. Arecchi, Phys. Rev. Lett. **73**, 1099 (1994).
[5] D. Raitt and H. Riecke, Physica (Amsterdam) **82D**, 79 (1995).
[6] S. Ciliberto and J.P. Gollub, Phys. Rev. Lett. **52**, 922 (1984).
[7] S. Ciliberto, E. Pampaloni, and C. Perez-Garcia, Phys. Rev. Lett. **61**, 1198 (1988); E. Pampaloni, C. Perez Garcia, L. Albavetti, and S. Ciliberto, J. Fluid Mech. **234**, 393 (1992).
[8] E. Pampaloni, S. Residori, and F.T. Arecchi, Europhys. Lett. **24**, 647 (1993).
[9] D.P. Vallette, W.S. Edwards, and J.P. Gollub, Phys. Rev. E **49**, R4783 (1994); B.J. Gluckman, C.B. Arnold, and J.P. Gollub, Phys. Rev. E **51**, 1128 (1995); K. Kumar and K.M.S. Bajaj, Phys. Rev. E **52**, 4606 (1995); F. Melo, P.B. Umbanhowar, and H.L. Swinney, Phys. Rev. Lett. **75**, 3838 (1995).
[10] We have been recently informed of an experiment of competition between two different wave numbers [B. Tuering and T. Tschudi, Physica D (to be published)]. However, due to the different experimental realization, that experiment does not show domain coexistence.
[11] W.J. Firth, J. Mod. Opt. **37**, 151 (1990); G. D'Alessandro and W.J. Firth, Phys. Rev. Lett. **66**, 2597 (1991).
[12] S.A. Akhmanov, M.A. Vorontsov, and V.Yu. Ivanov, JETP Lett. **47**, 707 (1988); S.A. Akhmanov, M.A. Vorontsov, V.Yu. Ivanov, A.V. Larichev, and N.I. Zheleznykh, J. Opt. Soc. Am. B **9**, 78 (1992).
[13] E. Pampaloni, P.L. Ramazza, S. Residori, and F.T. Arecchi, Phys. Rev. Lett. **74**, 258 (1995).
[14] S. Ciliberto, P. Coullet, J. Lega, E. Pampaloni, and C. Perez-Garcia, Phys. Rev. Lett. **65**, 2370 (1990).
[15] F.T. Arecchi, Nuovo Cimento Soc. Ital. Fis. A **107**, 1111 (1994).
[16] B.A. Malomed, A.A. Nepomnyaschiĭ, and M.I. Tribelskiĭ, Sov. Phys. JETP **69**, 388 (1989).
[17] H. Haken, *Synergetics* (Springer-Verlag, Berlin, 1977).
[18] W.E. Lamb, Jr., Phys. Rev. **134A**, 1429 (1964).
[19] J.D. Murray, *Mathematical Biology* (Springer-Verlag, Berlin Heidelberg, 1989).
[20] M.A. Vorontsov, M.E. Kirakosyan, and A.V. Larichev, Sov. J. Quantum Electron. **21**, 105 (1991).