

Nonthermal Phase Transitions after Inflation

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At the first stage of reheating after inflation, parametric resonance may rapidly transfer most of the energy of an inflaton field ϕ to the energy of other bosons. We show that quantum fluctuations of scalar and vector fields produced at this stage are much greater than they would be in a state of thermal equilibrium. This leads to cosmological phase transitions of a new type, which may result in a copious production of topological defects and in a secondary stage of inflation after reheating.

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The theory of reheating is one of the most important parts of inflationary cosmology. Elementary theory of this process was developed many years ago [1,2]. Some important steps toward a complete theory have been made in [3]. However, the real progress in the understanding of this process was achieved only recently when the new theory of reheating was developed. According to this theory [4], reheating typically consists of three different stages. At the first stage, a classical oscillating scalar field ϕ (the inflaton field) decays into massive bosons due to parametric resonance. In many models the resonance is very broad, and the process occurs extremely rapidly. To distinguish this stage of explosive reheating from the stage of particle decay and thermalization, we called it *preheating*. Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. The second stage is the decay of previously produced particles. This stage typically can be described by methods developed in [1]. However, these methods should be applied not to the decay of the original homogeneous inflaton field, but to the decay of particles produced at the stage of preheating. This changes many features of the process including the final value of the reheating temperature. The third stage of reheating is thermalization.

Different aspects of the theory of explosive reheating have been studied by many authors [5–8]. In our presentation we will follow the original approach of Ref. [4], where the theory of reheating was investigated with an account taken both of the expansion of the Universe and of the backreaction of created particles.

One should note that there exist such models where this first stage of reheating is absent; e.g., there is no parametric resonance in the theories where the field ϕ decays into fermions. However, in the theories where preheating is possible one may expect many unusual phenomena. One of the most interesting effects is the possibility of specific nonthermal postinflationary phase transitions which occur after preheating. As we will see, these phase transitions in certain cases can be much

more pronounced than the standard high-temperature cosmological phase transitions [9,10]. They may lead to copious production of topological defects and to a secondary stage of inflation after reheating.

Let us first remember the theory of phase transitions in theories with spontaneous symmetry breaking in the theory of scalar fields ϕ and χ with the effective potential

$$V(\phi, \chi) = \frac{\lambda}{4} (\phi^2 - \phi_0^2)^2 + \frac{1}{2} g^2 \phi^2 \chi^2. \quad (1)$$

Here $\lambda, g \ll 1$ are coupling constants. $V(\phi, \chi)$ has a minimum at $\phi = \phi_0, \chi = 0$ and a maximum at $\phi = \chi = 0$ with the curvature $V_{\phi\phi} = -m^2 = -\lambda\phi_0^2$. This effective potential acquires corrections due to quantum (or thermal) fluctuations of the scalar fields [9,10], $\Delta V = \frac{3}{2}\lambda\langle(\delta\phi)^2\rangle\phi^2 + g^2\langle(\delta\chi)^2\rangle\phi^2/2 + g^2\langle(\delta\phi)^2\rangle\chi^2/2 + \dots$, where the quantum field operator is decomposed as $\hat{\phi} = \phi + \delta\phi$ with $\phi \equiv \langle\hat{\phi}\rangle$, and we have written only leading terms depending on ϕ and $\chi \equiv \langle\hat{\chi}\rangle$. In the large temperature limit $\langle(\delta\phi)^2\rangle = \langle(\delta\chi)^2\rangle = T^2/12$. The effective mass squared of the field ϕ ,

$$m_{\phi,\text{eff}}^2 = -m^2 + 3\lambda\phi^2 + 3\lambda\langle(\delta\phi)^2\rangle + g^2\langle(\delta\chi)^2\rangle, \quad (2)$$

becomes positive and symmetry is restored (i.e., $\phi = 0$ becomes the stable equilibrium point) for $T > T_c$, where $T_c^2 = 12m^2/(3\lambda + g^2) \gg m^2$. At this temperature the energy density of the gas of ultrarelativistic particles is given by $\rho = N(T_c)\pi^2 T_c^4/30 = 24m^4 N(T_c)\pi^2/5(3\lambda + g^2)^2$. Here $N(T)$ is the effective number of degrees of freedom at large temperature, which in realistic situations may vary from 10^2 to 10^3 . Note that for $g^4 < 96N\pi^2\lambda/5$ this energy density is greater than the vacuum energy density $V(0) = m^4/4\lambda$. Meanwhile, for $g^4 \gtrsim \lambda$ radiative corrections are important, they lead to the creation of a local minimum of $V(\phi, \chi)$, and the phase transition occurs from a strongly supercooled state [9]. That is why the first models of inflation required supercooling at the moment of the phase transition.

An exception to this rule is given by supersymmetric theories, where one may have $g^4 \gg \lambda$ and still have a

potential which is flat near the origin due to cancellation of quantum corrections of bosons and fermions [11]. In such cases thermal energy becomes smaller than the vacuum energy at $T < T_0$, where $T_0^4 = 15m^2\phi_0^2/2N\pi^2$. Then one may even have a short stage of inflation which begins at $T \sim T_0$ and ends at $T = T_c$. During this time the Universe may inflate by the factor

$$\frac{a_c}{a_0} = \frac{T_0}{T_c} \sim 10^{-1} \left(\frac{g^4}{\lambda} \right)^{1/4} \approx 10^{-1} g \sqrt{\frac{\phi_0}{m}}. \quad (3)$$

In supersymmetric theories with flat directions Φ it may be more natural to consider potentials of the so-called "flaton" fields Φ without the term $\lambda\Phi^4/4$ [11],

$$V(\Phi, \chi) = -\frac{m^2\Phi^2}{2} + \frac{\lambda_1\Phi^6}{6M_P^2} + \frac{m^2\Phi_0^2}{3} + \frac{1}{2}g^2\Phi^2\chi^2, \quad (4)$$

where $\Phi_0 = \lambda_1^{-1/4}\sqrt{mM_P}$ corresponds to the minimum of this potential. The critical temperature in this theory for $\lambda_1\Phi_0^2 \ll g^2M_P^2$ is the same as in the theory (1) for $\lambda \ll g^2$, and expansion of the Universe during thermal inflation is given by $10^{-1}g\sqrt{\Phi_0/m}$, as in Eq. (3). This short additional stage of "thermal inflation" may be very useful; in particular, it may provide a solution to the Polonyi field problem [11].

The theory of cosmological phase transitions is an important part of the theory of the evolution of the Universe, and during the last 20 years it was investigated in a very detailed way. However, typically it was assumed that the phase transitions occur in the state of thermal equilibrium. Now we are going to show that similar phase transitions may occur even much more efficiently prior to thermalization, due to the anomalously large expectation values $\langle(\delta\phi)^2\rangle$ and $\langle(\delta\chi)^2\rangle$ produced during preheating.

We will first consider the model (1) without the scalar field χ and with the amplitude of spontaneous symmetry breaking $\phi_0 \ll M_P$. In this model inflation occurs during the slow rolling of the scalar field ϕ from its very large values until it becomes of the order M_P . Then it oscillates with the initial amplitude $\phi \sim 10^{-1}M_P$ and initial frequency $\sim 10^{-1}\sqrt{\lambda}M_P$. Within a few dozen oscillations it transfers most of its energy $\sim \frac{\lambda}{4}10^{-4}M_P^4$ to its long-wave fluctuations $\langle(\delta\phi)^2\rangle$ in the regime of broad parametric resonance [4].

The crucial observation is the following. Suppose that the initial energy density of oscillations $\sim \frac{\lambda}{4}10^{-4}M_P^4$ were instantaneously transferred to thermal energy density $\sim 10^2T^4$. This would give the reheating temperature $T_r \sim 2 \times 10^{-2}\lambda^{1/4}M_P$, and the scalar field fluctuations $\langle(\delta\phi)^2\rangle \sim T_r^2/12 \sim 3 \times 10^{-5}\sqrt{\lambda}M_P^2$. Meanwhile particles created during preheating have much smaller energy $\sim 10^{-1}\sqrt{\lambda}M_P$. Therefore if the same energy density $\frac{\lambda}{4}10^{-4}M_P^4$ is instantaneously transferred to low-energy particles created during preheating, their number, and, correspondingly, the amplitude of fluctuations, will be much greater, $\langle(\delta\phi)^2\rangle \sim C^2M_P^2$, where $C^2 \sim 10^{-2}-10^{-3}$ [4]. Thermal fluctuations would lead to symmetry restora-

tion in our model only for $\phi_0 \sim T_r \sim 10^{-2}\lambda^{1/4}M_P \sim 10^{14}$ GeV for the realistic value $\lambda \sim 10^{-13}$ [10]. Meanwhile, according to Eq. (2), the nonthermalized fluctuations $\langle(\delta\phi)^2\rangle \sim M_P^2$ may lead to symmetry restoration even if the symmetry breaking parameter ϕ_0 is as large as $10^{-1}M_P$. Thus, the nonthermal symmetry restoration may occur even in those theories where the symmetry restoration due to high-temperature effects would be impossible.

In reality thermalization takes a very long time, which is inversely proportional to coupling constants. This dilutes the energy density, and the reheating temperature becomes many orders of magnitude smaller than 10^{14} GeV [10]. Therefore postinflationary thermal effects typically cannot restore symmetry on the grand unified theory (GUT) scale. Preheating is not instantaneous as well, and therefore the fluctuations produced at that stage are smaller than $C^2M_P^2$, but only logarithmically, $\langle(\delta\phi)^2\rangle \sim C^2M_P^2 \ln^{-2} 1/\lambda$ [4]. For $\lambda \sim 10^{-13}$ this means that nonthermal perturbations produced at reheating may restore symmetry on the scale up to $\phi_0 \sim 10^{16}$ GeV.

Later $\langle(\delta\phi)^2\rangle$ decreases as $a^{-2}(t)$ because of the expansion of the Universe. This leads to the phase transition with symmetry breaking at the moment $t = t_c \sim \sqrt{\lambda}M_P m^{-2}$ when $m_{\phi, \text{eff}} = 0$, $\langle(\delta\phi)^2\rangle = \phi_0^2/3$, $E_\phi \sim m$. Note that the homogeneous component $\phi(t)$ at this moment is significantly less than $\sqrt{\langle(\delta\phi)^2\rangle}$ due to its decay in the regime of the narrow parametric resonance after preheating [4]: $\overline{\phi^2} \propto t^{-7/6} \propto t^{-1/6}\langle(\delta\phi)^2\rangle$; an overbar means averaging over oscillations.

The mechanism of symmetry restoration described above is very general; in particular, it explains a surprising behavior of oscillations of the scalar field found numerically in the $O(N)$ -symmetric model of Ref. [6]. Thus in the interval between preheating and thermalization the Universe could experience a series of phase transitions which we did not anticipate before. For example, cosmic strings and textures, which could be an additional source for the formation of the large scale structure of the Universe, should have $\phi_0 \sim 10^{16}$ GeV [12]. It is hard to produce them by thermal phase transitions after inflation [13]. Meanwhile, as we see now, fluctuations produced at preheating may be quite sufficient to restore the symmetry. Then the topological defects are produced in the standard way when the symmetry breaks down again. In other words, production of superheavy topological defects can be easily compatible with inflation.

On the other hand, the topological defect production can be quite dangerous. For example, the model (1) of a one-component real scalar field ϕ has a discrete symmetry $\phi \rightarrow -\phi$. As a result, after the phase transition induced by fluctuations $\langle(\delta\phi)^2\rangle$ the Universe may become filled with domain walls separating phases $\phi = +\phi_0$ and $\phi = -\phi_0$. This is expected to lead to a cosmological disaster.

This question requires a more detailed analysis. Even though the point $\phi = 0$ after preheating becomes a minimum of the effective potential, the field ϕ continues oscillating around this minimum. Therefore, at the moment

t_c it may happen to be either to the right of the maximum of $V(\phi)$ or to the left of it everywhere in the Universe. In this case the symmetry breaking will occur in one preferable direction, and no domain walls will be produced. A similar mechanism may suppress production of other topological defects.

However, this would be correct only if the magnitude of fluctuations $(\delta\phi)^2$ were smaller than the average amplitude of the oscillations $\overline{\phi^2}$. In our case fluctuations $(\delta\phi)^2$ are greater than $\overline{\phi^2}$ [4], and they can have considerable local deviations from their average value $\langle(\delta\phi)^2\rangle$. Investigation of this question shows that in the theory (1) with $\phi_0 \ll 10^{16}$ GeV fluctuations destroy the coherent distribution of the oscillating field ϕ and divide the Universe into an equal number of domains with $\phi = \pm\phi_0$, which leads to the domain wall problem. This means that in consistent inflationary models of the type of (1) one should have either $\phi_0 = 0$ (no symmetry breaking), or $\phi_0 \geq 10^{16}$ GeV.

Now we will consider models where the symmetry breaking occurs for fields other than the inflaton field ϕ . The simplest model has the following potential [13,14]:

$$V(\phi, \chi) = \frac{\lambda}{4} \phi^4 + \frac{\alpha}{4} \left(\chi^2 - \frac{M^2}{\alpha} \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2. \quad (5)$$

We will assume here that $\lambda \ll \alpha, g^2$, so that at large ϕ the curvature of the potential in the χ direction is much greater than in the ϕ direction. In this case at large ϕ the field χ rapidly rolls toward $\chi = 0$. An interesting feature of such models is the symmetry restoration for the field χ for $\phi > \phi_c = M/g$, and symmetry breaking when the inflaton field ϕ becomes smaller than ϕ_c . As emphasized in [13], such phase transitions may lead to formation of topological defects without any need for high-temperature effects.

Now we would like to point out some other specific features of such models. If the phase transition discussed above happens during inflation [13] (i.e., if $\phi_c > M_p$ in our model), then no new phase transitions occur in this model after reheating. However, for $\phi_c \ll M_p$ the situation is much more complicated. First of all, in this case the field ϕ oscillates with the initial amplitude $\sim M_p$ (if $M^4 < \alpha \lambda M_p^4$). This means that each time the absolute value of the field becomes smaller than ϕ_c , the phase transition with symmetry breaking occurs and topological defects are produced. Then the absolute value of the oscillating field ϕ again becomes greater than ϕ_c , and symmetry restores again. However, this regime does not continue for too long a time. Within a few dozen oscillations, quantum fluctuations of the field χ will be generated with the dispersion $\langle(\delta\chi)^2\rangle \sim C^2 g^{-1} \sqrt{\lambda} M_p^2 \ln^{-2} 1/g^2$ [4]. For $M^2 < C^2 g^{-1} \sqrt{\lambda} \alpha M_p^2 \ln^{-2} 1/g^2$, these fluctuations will keep the symmetry restored. Note that for $g^2 \gg \lambda$ preheating is more efficient, and the coefficient C may become much greater than 10^{-2} . The symmetry

breaking finally completes when $\langle(\delta\chi)^2\rangle$ becomes small enough.

One may imagine an even more complicated scenario when oscillations of the scalar field ϕ create large fluctuations of the field χ , which in their turn interact with the scalar fields Φ breaking symmetry in GUTs. Then we would have phase transitions in GUTs induced by the fluctuations of the field χ . This means that no longer can the absence of primordial monopoles be considered as an automatic consequence of inflation. To avoid the monopole production one should use the theories where quantum fluctuations produced during preheating are small or decoupled from the GUT sector. This condition imposes additional constraints on realistic inflationary models. On the other hand, preheating may remove some previously existing constraints on inflationary theory. For example, in the models of GUT baryogenesis it was assumed that the GUT symmetry was restored by high-temperature effects, since otherwise the density of X , Y , and superheavy Higgs bosons would be very small. This condition is hardly compatible with inflation. It was also required that the products of decay of these particles should stay out of thermal equilibrium, which is a very restrictive condition. In our case the superheavy particles responsible for baryogenesis can be abundantly produced by parametric resonance, and the products of their decay will not be in a state of thermal equilibrium until the end of reheating.

Now let us return to the theory (1) including the field χ for $g^2 \gg \lambda$. In this case the main fraction of the potential energy density $\sim \lambda M_p^4$ of the field ϕ predominantly transfers to the energy of fluctuations of the field χ due to the explosive χ particles creation in the broad parametric resonance. The dispersion of fluctuations after preheating is $\langle(\delta\chi)^2\rangle \sim C^2 g^{-1} \sqrt{\lambda} M_p^2 \ln^{-2} 1/g^2$. These fluctuations lead to the symmetry restoration in the theory (1) with $\phi_0 \ll C(g^2/\lambda)^{1/4} M_p \ln^{-1} 1/g^2$, which may be much greater than 10^{16} GeV for $g^2 \gg \lambda$.

Later the process of decay of the field ϕ in this model continues, but less efficiently, and because of the expansion of the Universe the fluctuations $\langle(\delta\chi)^2\rangle$ decrease approximately as $g^{-1} \sqrt{\lambda} M_p^2 [a_i/a(t)]^2$, whereas their energy density ρ decreases as the energy density of ultra-relativistic matter, $\rho(t) \sim \lambda M_p^4 [a_i/a(t)]^4$, where a_i is the scale factor at the end of inflation. This energy density becomes equal to the vacuum energy density $m^4/4\lambda$ at $a_0 \sim a_i \sqrt{\lambda} M_p/m$, $t \sim \sqrt{\lambda} M_p m^{-2}$. Since that time and until the time of the phase transition with symmetry breaking the vacuum energy dominates, and the Universe enters the secondary stage of inflation.

The phase transition with spontaneous symmetry breaking occurs when $m_{\phi, \text{eff}} = 0$, $\langle(\delta\chi)^2\rangle = g^{-2} m^2$. This happens at $a_c = a_i \lambda^{1/4} g^{1/2} M_p/m$. Thus, during this additional period of "nonthermal" inflation the Universe expands $a_c/a_0 \sim \sqrt{g} \sqrt{\phi_0/m} = (g^2/\lambda)^{1/4}$ times. This is greater than expansion during "thermal" inflation (3) by the factor $O(g^{-1/2})$, and in our case inflation occurs even if $g^4 \ll \lambda$.

In this example we considered the second stage of inflation driven by the inflaton field ϕ . However, the same effect can occur in theories where other scalar fields are coupled to the field χ . For example, in the theories of the type of (4) fluctuations $\langle(\delta\chi)^2\rangle$ produced at the first stage of reheating by the oscillating inflaton field ϕ lead to a secondary “nonthermal” inflation driven by the potential energy of the “flaton” field Φ . During this stage the Universe expands $\sim\sqrt{g}\sqrt{\Phi_0/m}$ times. To have a long enough inflation one may consider, e.g., supersymmetric theories with $m\sim 10^2$ GeV and $\Phi_0\sim 10^{12}$ [11]. This gives a relatively long stage of inflation with $a_c/a_0\sim\sqrt{g}10^5$, which may be enough to solve the Polonyi field problem if the constant g is not too small.

If the coupling constant g is sufficiently large, fluctuations of the field χ will thermalize during this inflationary stage. Then the end of this stage will be determined by the standard theory of high-temperature phase transition, and the degree of expansion during this stage will be given by $10^{-1}g\sqrt{\Phi_0/m}$; see Eq. (3). It is important, however, that the inflationary stage may begin even if the field χ has not been thermalized at that time.

The stage of inflation described above occurs in the theory with a potential which is not particularly flat near the origin. But what happens in the models which have flat potentials, like the original new inflation model in the Coleman-Weinberg theory [15]? One of the main problems of inflation in such models was to understand why the scalar field ϕ should jump onto the top of its effective potential, since this field in realistic inflationary models is extremely weakly interacting and, therefore, it could not be in the state of thermal equilibrium in the very early Universe. Thus, it is much more natural for inflation in the Coleman-Weinberg theory to begin at very large ϕ , as in the simplest version of chaotic inflation in the theory $\lambda\phi^4$. However, during the first few oscillations of the scalar field ϕ at the end of inflation in this model, it produces large nonthermal perturbations of vector fields $\langle(\delta A_\mu)^2\rangle\sim C^2g^{-1}\sqrt{\lambda}M_P^2\ln^{-2}/g^2$. This leads to symmetry restoration and initiates the second stage of inflation beginning at $\phi=0$. It suggests that in many models inflation most naturally begins at large ϕ as in the simplest version of the chaotic inflation scenario [16]. But then, after the stage of preheating, the second stage of inflation may begin like in the new inflationary scenario. In other words, the new theory of reheating after chaotic inflation may rejuvenate the new inflation scenario.

The main conclusion of this paper is the following. In addition to the standard high-temperature phase transition, there exists a new class of phase transitions which may occur at the intermediate stage between the end of inflation and the establishing of thermal equilibrium. These phase transitions take place even if the scale of symmetry breaking is very large and the reheating temperature is very small. Therefore, phase transitions of the new type

may have dramatic consequences for inflationary models and the theory of physical processes in the very early Universe.

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- [1] A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982); L. F. Abbott, E. Fahri, and M. Wise, Phys. Lett. **117B**, 29 (1982).
 - [2] A. A. Starobinsky, in *Proceedings of the Second Seminar on Quantum Theory of Gravity, Moscow, 1981*, edited by M. A. Markov and P. C. West (Plenum, New York, 1984), p. 103.
 - [3] J. Traschen and R. Brandenberger, Phys. Rev. D **42**, 2491 (1990); A. D. Dolgov and D. P. Kirilova, Sov. Nucl. Phys. **51**, 273 (1990).
 - [4] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); L. A. Kofman, A. D. Linde, and A. A. Starobinsky (to be published).
 - [5] Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51**, 5438 (1995).
 - [6] D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, Phys. Rev. D **51**, 4419 (1995); D. Boyanovsky, M. D’Attanasio, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, Report No. PITT-17/95, hep-ph/9507414, 1995.
 - [7] M. Yoshimura, Report No. TU/95/484, hep-th/9506176, 1995; H. Fujisaki, K. Kumekawa, M. Yamaguchi, and M. Yoshimura, Report No. TU/95/488, hep-ph/9508378, 1995.
 - [8] D. Kaiser, Harvard University Report No. HUTP-95/A027, astro-ph/9507108, 1995.
 - [9] D. A. Kirzhnits, JETP Lett. **15**, 529 (1972); D. A. Kirzhnits and A. D. Linde, Phys. Lett. **42B**, 471 (1972); Sov. Phys. JETP **40**, 628 (1974); Ann. Phys. (N.Y.) **101**, 195 (1976); S. Weinberg, Phys. Rev. D **9**, 3320 (1974); L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3357 (1974).
 - [10] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
 - [11] D. H. Lyth and E. D. Stewart, Phys. Rev. Lett. **75**, 201 (1995); Report No. LANCASTER-TH/9505, hep-ph/9510204, 1995.
 - [12] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge University Press, Cambridge, 1994).
 - [13] L. A. Kofman and A. D. Linde, Nucl. Phys. **B282**, 555 (1987).
 - [14] A. D. Linde, Phys. Lett. B **259**, 38 (1991); Phys. Rev. D **49**, 748 (1994).
 - [15] A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
 - [16] A. D. Linde, Phys. Lett. **129B**, 177 (1983).