Comment on "Life at the Edge of Chaos"

In a recent Letter Ray and Jan [1] have conjectured that the model of evolution proposed by Bak and Sneppen (BS) [2] may belong to the universality class of directed percolation (DP). The conjecture is based upon mean-field calculations and comparison of Monte Carlo data with some *static* exponents of DP. Simultaneously, Paczuski, Maslov, and Bak [3] have also made the same conjecture based upon theoretical field arguments and numerical simulations. Since a small error in the critical barrier x_c, such as for $x \geq x_c$ the BS model exhibits self-organized critical (SOC) behavior, may induce large errors in critical exponents of DP, a precise determination of x_c and the subsequent evaluation of critical exponents is desirable.

Following Grassberger and De La Torre [4], we have performed a spreading study of the BS model in one and two dimensions. Initially, all sites of the lattice are filled with random numbers within the interval $x^* \leq x \leq 1$. So, x^* introduces a control parameter which corresponds to the gap of the BS model. After that, the central site of the lattice is replaced by a random number $x < x^*$ at $t = 0$. Always taking the site with the smallest random number to be the next site to update, the following quantities are measured for sites with $x < x^*$: the survival probability $P(t)$, the average number of sites $N(t)$, and the mean square distance $R^2(t)$ over which the sites have been spread. Results are averaged over 2×10^5 samples. Lattices are taken large enough to prevent sites with $x < x^*$ from reaching the edges, so the results are free of finite size effects. At criticality, the following scaling behavior holds: $P(t) \propto t^{-\delta}$, $N(t) \propto t^{\eta}$, and $R^2(t) \propto t^z$ [4], where δ, η, and z are *dynamic* exponents.

Results for one dimensional lattices are shown in Fig. 1 as log-log plots of N vs t. For $x^* = 0.676$ the downward deviation of N indicates subcritical behavior while for $x^* = 0.678$ the upward curvature of N indicates that this value lies within the SOC state. Last, for $x^* = 0.677$ one has three decades of straight line behavior which is a signature of criticality. So, the critical gap becomes $x_c \approx 0.677(1)$, which may be compared with $x_c \approx 0.67(1)$ [2] and $x_c \approx 0.667$ [3]. Also we have $\eta \approx 0.2383(4)$ (statistical error). The error in x_c causes an error of about ± 0.008 in η. An additional small error may also be due to scaling corrections. In contrast, for DP in $(1 + 1)$ dimensions, one has $\eta \approx 0.308(9)$. Also, for the BS model $z \approx 1.343(2)$, in contrast to $z \approx 1.263(8)$ for DP. The log-log plot of $P(t)$ vs t exhibits pronounced curvature which hinders the precise evaluation of δ, however, it is expected that a smaller value than that of DP is given by $\delta \approx 0.162(4)$.

In two dimensions (see inset of Fig. 1) we have $x_c \approx 0.3289(1)$, in full agreement with [5]. Also, $\eta \approx 0.055(1)$, $\delta \approx 0.3333(1)$, and $z \approx 0.955(1)$. All these exponents are far from the DP values given by $\eta \approx 0.214$, $\delta \approx 0.460$, and $z \approx 1.134$. The small value of η is compatible with a logarithmic correction with $\eta \approx 0.065(1)$.

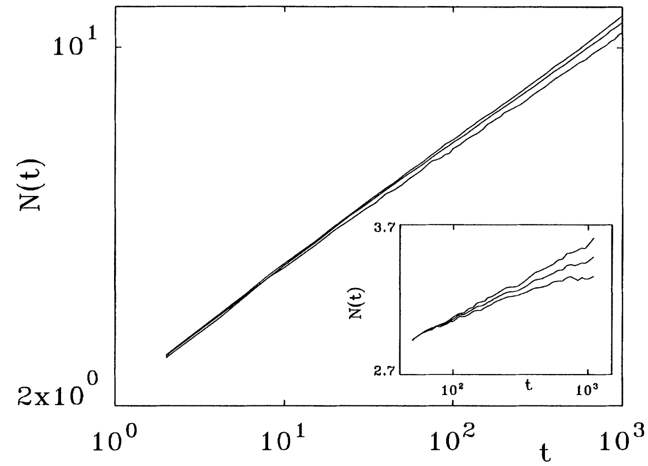

FIG. 1. Log-log plots of N vs t (one dimension) for $x^* = 0.676$ (lower curve), $x^* = 0.677$ (medium curve), and $x^* = 0.678$ (upper curve). Inset: Results of two dimensions for $x^* = 0.3288$ (lower curve), $x^* = 0.3289$ (medium curve), and $x^* = 0.3290$ (upper curve).

Since for DP *dynamic* and *static* critical exponents are related by well established scaling relations [4], our results suggest that these scaling relations are no longer valid in the case of the BS model. In fact, very recently it has been shown that, unlike DP, the BS model has only two independent exponents [5].

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