

Bialynicki-Birula, Kalinski, and Eberly Reply: The authors of the Comment [1] based all their estimates of the Trojan wave packet stability on purely classical considerations (sea of classical chaos, bifurcations, zero velocity surfaces, etc.). In this way they arrived at an extremely pessimistic estimate of the size of the orbits (0.1–1 cm) ($l \sim 10\,000$) that can support stable, coherent wave functions. The quantum effects, however, are not negligible in this problem, and they radically change the situation for the better. All that is needed to achieve a better stability is to improve the shape of the wave packet by going beyond the simplest Gaussian approximation.

In order to show how quantum effects may contribute additional stabilization as compared to classical theory, let us consider a quantum mechanical particle in one dimension moving in a potential field $V(x)$. The motion of various pieces of the wave packet is influenced not only by the classical potential $V(x)$ but also by the additional “quantum potential” $-(\hbar^2/2m)R''(x)/R(x)$ (cf., for example, [2]), where $R(x)$ is the (real) amplitude of the wave function. For a Gaussian wave function [3] this additional potential is always repulsive (i.e., the force is acting away from the maximum), and the stability will not be improved. However, even small departures from the Gaussian shape may lead to attraction, thus changing

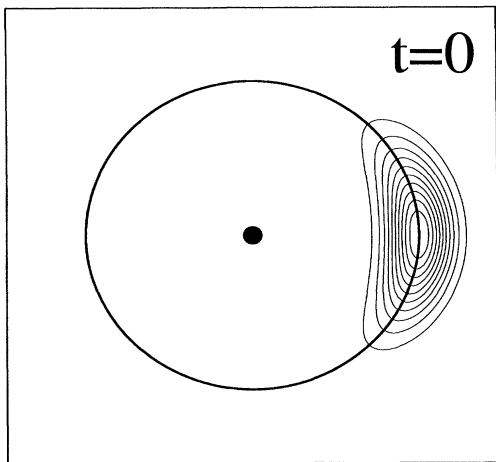


FIG. 1. Contour plot of the probability distribution of a Trojan wave packet calculated analytically [4] in polar coordinates. The circular path of the packet's orbit is also shown.

the estimates presented in the Comment. As a simple example, let us consider $R(x) = \exp(-x^2/2 - x^4/10)$. The quantum potential associated with this wave function is attractive for $|x| < 0.347$ and will act against the dispersion of the wave packet.

These qualitative considerations are consistent with our recent numerical experiments [4]. We observed a clear improvement in the stability of the solutions of the time-dependent Schrödinger equation for Trojan wave packets for $l = 60$ ($r \sim 0.0001$ cm) when our initial wave packet [3] was simply rewritten as a Gaussian function of cylindrical rather than Cartesian coordinates, similar to the classical form of Klar [5].

In Fig. 1 we show the Trojan wave packet obtained by using polar coordinates. This shape is well preserved over very many periods. We have estimated from detailed numerical calculations [4] the packet dissipation rate to be less than 0.07% per period.

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