Comment on "Lagrange Equilibrium Points in Celestial Mechanics and Nonspreading Wave Packets for Strongly Driven Rydberg Electrons"

Reference [1] proposes that an electronic wave packet launched from a global equilibrium point in the hydrogen atom in a circularly polarized microwave field—an analog of the Lagrange points L_4 and L_5 [2]—will be nondispersive. Yet the packet presented in Ref. [1] spreads considerably after only a few Kepler orbits. We argue that the highly anharmonic nature of the effective potential in the vicinity of the equilibrium point will cause any such putative coherent state to disperse unless the diameter of the atom is ca. 0.1–1 cm; however, stable coherent states might be prepared with the addition of a magnetic field.

In the rotating frame and atomic units the Hamiltonian is

$$H = H_0 - \omega(xp_y - yp_x) - \mathcal{E}x\,, \qquad (1)$$

where $H_0 = (p_x^2 + p_y^2)/2 - 1/r$. The zero-velocity surface (ZVS) [2] is given by $V = -1/r - \omega^2(x^2 + y^2)/2 - \mathcal{E}x$, which, for the field strength $\mathcal{E} \neq 0$, has a maximum L_m (at $x = x_0$) and a saddle point that also lies along the x axis. The stability of a Gaussian wave packet centered on x_0 depends largely on the quality of a harmonic approximation at L_m. If $\mathcal{E} = 0$, the equilibria all lie along a flat (i.e., nonharmonic) circular rim in the ZVS. As the field is applied, an isolated equilibrium at x_0 emerges around which the ZVS becomes progressively more harmonic with increasing $\mathcal{E}$. However, a transition to instability [1,2] occurs at L_m when $\epsilon_c = \mathcal{E}_c/\omega^{4/3} \approx -0.1156$, and this limits the range of linear motion. Figure 1(a) shows a wave packet defined as in Ref. [1] ($q = 0.9562$, $\epsilon = -0.0444$). Much of the packet spills out of the harmonic regime thereby exposing these parts to the danger of dispersion. Simulations reveal that a sea of classical chaos surrounds the tiny regular island at L_m, which can further enhance dispersion of the packet. In fact, it is not until $x_0 \gtrsim 10^7$ a.u. that most of the coherent state can nest comfortably atop the harmonic part of the maximum—see Fig. 1(b).

The addition of a magnetic field (**B**) perpendicular to the plane of polarization [2] modifies Eq. (1) to give $\mathcal{H} = H + \omega_c^2(x^2 + y^2)/8$ with ω replaced by $\Omega = (\omega \pm \omega_c/2)$, where ω_c is the cyclotron frequency and the direction of the **B** field is chosen to select the + sign (selection of the − sign can actually produce an outer *minimum* in the ZVS [2]). Stability analysis and simulations [2] show that the maximum can indeed be stabilized by the **B** field, allowing the preparation of genuinely coherent states: e.g., if $\omega = 1$ and $\omega_c = 0.9$, L_m is linearly stable until $\mathcal{E} \approx -0.58$.

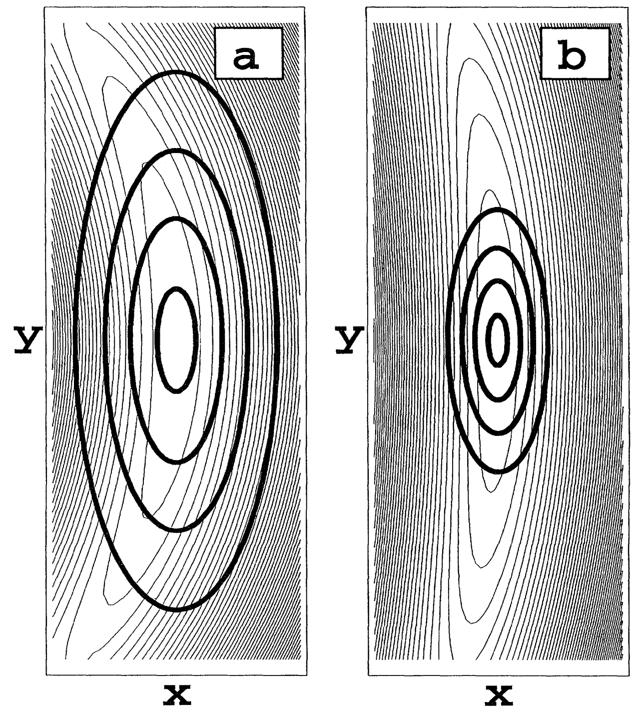

FIG. 1. Level curves of the ZVS. Thick lines are contours (at 0.25, 0.5, 0.75, and 0.95) of the Gaussian probability density centered at L_m (in a.u.), (a) $x_0 = 10^4$; (b) $x_0 = 10^7$.

David Farrelly
Department of Chemistry
Princeton University
Princeton, New Jersey 08544-1009

Ernestine Lee
Department of Chemistry and Biochemistry
Utah State University
Logan, Utah 84322-0300

T. Uzer
School of Physics
Georgia Institute of Technology
Atlanta, Georgia 30332-0430

Received 10 March 1995
PACS numbers: 32.80.Rm, 42.50.Hz, 95.10.Ce

[1] I. Bialynicki-Birula, M. Kaliński, and J. H. Eberly, Phys. Rev. Lett. **73**, 1777 (1994).
[2] D. Farelly and T. Uzev, Phys. Rev. Lett. **74**, 1720 (1995); D. Farrelly, E. Lee, and A. Brunello (to be published).

 0031-9007/95/75(5)/972(1)$06.00