Evidence for a Superfluid Density in t-J Ladders

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Applying three independent techniques, we give numerical evidence for a finite superfluid density in isotropic hole-doped t-J ladders. We show the existence of anomalous flux quantization, emphasizing the contrasting behavior to that found in the "Luttinger liquid" regime stabilized at low electron densities. We consider the nature of the low-lying excitation modes, finding the 1D analog of the superconducting state; using a density matrix renormalization group approach, we find long range pairing correlations and exponentially decaying spin-spin correlations.

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The behavior of strongly correlated electrons confined to coupled chains is at present a topic undergoing much investigation; the reasons for this attention are numerous. Firstly, with the behavior of electrons under t-J or Hubbard type interactions in one-dimension now relatively well understood, the two-chain systems provide an interesting "first step" towards the challenge of two dimensions. Secondly, the unusual nature of the ground state of the undoped system, in particular the existence of a spin gap [1], leads to further interest with regards to "gapped" superconducting behavior. Furthermore, it is believed that compounds such as $(VO_2)P_2O_7$ [2] and $SrCu_2O_3$ [3] may be described by a lattice of coupled chains. While there is considerable literature concerning the possible phases in a t-J ladder, a complete picture is still far from being realized and our aim is to clarify the nature of the gapped state when the system is doped [4]. Various techniques have been applied previously in this hole-doped region [5], giving some indication of hole pairing and modified d-wave superconducting correlations.

The t-J Hamiltonian on the $2 \times L$ ladder is defined as

$$\begin{aligned}\mathcal{H} = J' &\sum_{j} (\mathbf{S}_{j,1} \cdot \mathbf{S}_{j,2} - \tfrac{1}{4} n_{j,1} n_{j,2}) \\ &+ J \sum_{\beta,j} (\mathbf{S}_{j,\beta} \cdot \mathbf{S}_{j+1,\beta} - \tfrac{1}{4} n_{j,\beta} n_{j+1,\beta}) \\ &- t \sum_{j,\beta,s} P_G (c^{\dagger}_{j,\beta;s} c_{j+1,\beta;s} + \text{H.c.}) P_G \\ &- t' \sum_{j,s} P_G (c^{\dagger}_{j,1;s} c_{j,2;s} + \text{H.c.}) P_G \,, \end{aligned} \tag{1}$$

where most notations are standard. β $(= 1, 2)$ labels the two legs of the ladder (oriented along the x axis), while j is a rung index ($j = 1, \ldots, L$). We shall concentrate on the isotropic case where the intraladder (along x) couplings J and t are equal to the interladder (along y) couplings J' and t'.

At half filling, the Hamiltonian reduces to the Heisenberg model and the behavior is generally relatively well understood [1]. A simple interpretation is given by considering the strong coupling limit ($J = 0$) in which the ground state consists of a singlet on each rung with a spin gap ($\sim J'$) which corresponds to forming a triplet on one of the rungs. With the introduction of interchain coupling J, the triplets can propagate and form a coherent band, thereby reducing the spin gap. In the isotropic case, the gap remains ($\sim 0.5J$) and it is the nature of the state formed on doping such a system that we shall concentrate on.

A possible phase diagram for the isotropic t-J ladder as a function of J/t and doping has been proposed recently [6]: Away from half filling, the spin-gapped region persists, exhibiting hole pairing and, as we will show, possible superconducting correlations. This behavior is observed up to $J/t >\sim 2.1$ [7] where the system phase separates. As the system is doped further, a Luttinger-like phase is stabilized exhibiting gapless spin and charge excitations. At very small electron densities, an electron-paired phase exists.

In this Letter we will describe three *independent* forms of evidence for a finite superfluid density in the spin-gapped region of the phase diagram (working specifically with an electron density $\langle n \rangle = 0.8$). The first set of results are based on the existence of anomalous flux quantization. Secondly, we consider the spin and charge excitation modes. Finally, we present direct calculations of correlation functions obtained using the density matrix renormalization group method.

Our first set of results then concern the existence of anomalous flux quantization. The calculation involves threading the double chain ring with a flux Φ and studying the functional form of the ground state energy with respect to the threaded flux, namely $E_0(\Phi)$ (we will measure the flux Φ in units of the flux quantum $\Phi_0 = hc/e$). In general, $E_0(\Phi)$ consists of an envelope of a series of parabola, corresponding to the curves of individual many-body states $E_n(\Phi)$, exhibiting a periodicity of 1. Byers and Yang [8] have shown that in the thermodynamic limit, $E_0(\Phi)$ exhibits local minima at quantized values of Φ, the separation of which is $1/n$ where n is the sum of charges in the basic group; these local minima

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in $E_0(\Phi)$ must be separated by a finite energy barrier. Hence, for a superconducting state, we would expect minima in $E_0(\Phi)$ at intervals of 1/2; these minima are related to the existence of supercurrents that are trapped in the metastable states corresponding to the flux minima and are thus unable to decay away [9]. It should be mentioned that this phenomenon, known as anomalous flux quantization, is an indication of pairing and is not sufficient in itself to imply a superconducting state.

Detailed studies of the attractive Hubbard model on two-dimensional lattices [10] have indicated the presence of anomalous flux quantization, confirming the existence of superconducting correlations in the ground state. In contrast, the repulsive Hubbard model exhibits no anomalous flux quantization.

In addition to the existence of flux quantization, the function $E(\Phi)$ also gives a quantitative value of the superfluid density, defined in one dimension by $D_s = (\partial^2/\partial\Phi^2)\,(\lim_{L\mapsto\infty}[LE_0(\Phi)])$. A distinction should be made between D_s (the superfluid density) and D (the Drude weight) [11], which are in general different: The superfluid density corresponds to the curvature of the envelope of the individual many-body states as a function of flux, while the Drude weight is obtained from the curvature of a single ground-state many-body energy level. In one dimension, however, there are only a finite number of energy level crossings in the thermodynamic limit [11], and D and D_s are equal. In the thermodynamic limit no particular applied flux is preferred when calculating D_s [12], and hence we consider the curvature of the whole $LE_0(\Phi)$ curve. Note that the existence of a superfluid would require both anomalous flux quantization and a finite D_s.

Numerically, the application of a flux through a double chain ring is achieved by modifying the kinetic term of the Hamiltonian such that $c^\dagger_{j,\beta;s}c_{j+1,\beta;s} \mapsto c^\dagger_{j,\beta;s}c_{j+1,\beta;s}e^{i2\pi\Phi/L}$, where Φ is the flux through the ring. Hence, the application of a flux is numerically equivalent to a change in the boundary conditions of the problem: $\Phi = 0$ representing periodic and $\Phi = \frac{1}{2}$ representing antiperiodic boundary conditions.

The technique we have employed is exact diagonalization of finite systems, specifically 2×5 and 2×10 double chain rings with electron densities $\langle n \rangle = 0.8$ and $\langle n \rangle = 0.4$ corresponding to the regions of the phase diagram where we expect spin-gapped or Luttinger liquid behavior, respectively. The modes of the system are characterized first by their spin: singlet and triplet excitations correspond to charge and spin modes, respectively. It is also useful to consider the parity of the states of the system under a reflection in the symmetry axis of the ladder along the direction of the chains. Even ($R_x = 1$) or odd ($R_x = -1$) excitations correspond to bonding (B) or antibonding (A) modes, respectively. Finally, it is necessary to consider the momentum, $k_x = 2\pi n/L$, in order to determine the dispersion relation of each mode. Implementation of these quantum numbers and symmetries is straightforward using exact diagonalization methods.

Concentrating initially on $J/t = 0.5$, $\langle n \rangle = 0.8$, we show in Fig. 1 all possible spin and charge modes of the 2×10 system, for all possible momenta, as a function of applied flux. We show the full spectrum only for $\Phi < 0.25$ in order to simplify the diagram. The minimum energy function $E_0(\Phi)$ is formed by charge (spin zero) bonding modes; the excited modes with different quantum numbers move further from the ground state as the system size increases (a result we have checked by finite size scaling techniques) and hence will not interfere with $E_0(\Phi)$. The existence of minima at intervals of half a flux quantum (i.e., anomalous flux quantization) clearly indicates the existence of pairing.

In order to probe the behavior of $E_0(\Phi)$ further, we consider the quantity $L[E_0(\Phi) - E_0(\Phi = 0)]$ as a function of Φ for various values of J/t and $\langle n \rangle$. Note that the curvature of this function in the thermodynamic limit gives D_s. Figure 2(a) shows the contrasting behavior obtained when keeping J/t fixed at 1.0 and varying the electron filling, specifically $\langle n \rangle = 0.4$ and $\langle n \rangle = 0.8$ (both the 2×5 and 2×10 results are shown). This plot clearly shows the existence of anomalous flux quantization for a filling of $\langle n \rangle = 0.8$ and its absence for $\langle n \rangle = 0.4$. The occurrence of the absolute minima at different values of flux ($\Phi = 0$ and $\Phi = 1/2$ for $\langle n \rangle = 0.8$ and $\langle n \rangle = 0.4$, respectively) can be explained by considering the noninteracting Fermi sea for the two fillings; a lower energy state is formed by choosing the flux to give a closed shell. Figure 2(b) shows an equivalent plot but in this case keeping $\langle n \rangle$ constant at 0.8

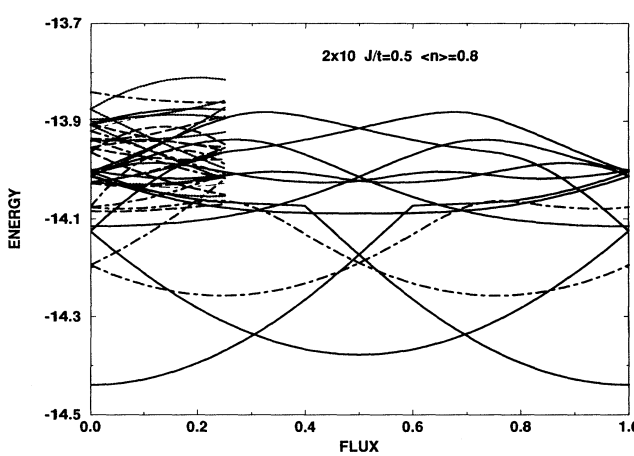

FIG. 1. Energy as a function of flux (in units of $\Phi_0 = hc/e$) for $J/t = 0.5, \langle n \rangle = 0.8$ for a system size of 2×10. We show all possible momenta for various quantum numbers. For the charge modes the solid lines correspond to bonding and the dotted lines to antibonding, while for the spin modes the dashed lines correspond to bonding and the dot-dashed lines to antibonding. We give only the charge bonding modes and the lowest lying spin antibonding mode in full to simplify the diagram.

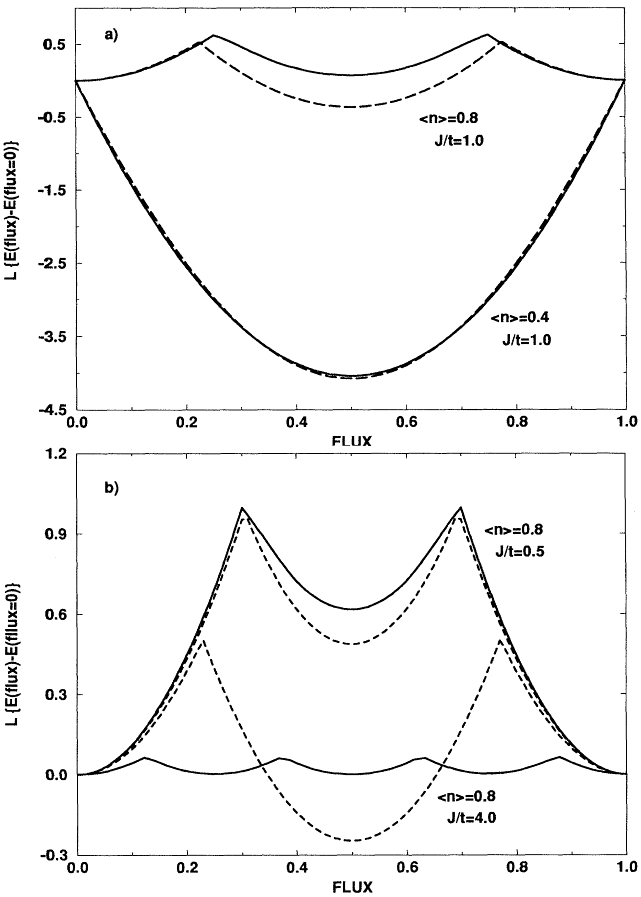

FIG. 2. $L[E_0(\Phi) - E_0(\Phi = 0)]$ where L is the length of the ladder and $E_0(\Phi)$ is the ground-state energy with an applied flux Φ. The dashed lines correspond to 2×5, the solid lines to 2×10. (a) The results for $\langle n \rangle = 0.4$ and $\langle n \rangle = 0.8$ both with $J/t = 1.0$, while (b) shows the results for $J/t = 0.5$ and $J/t = 4.0$ both with $\langle n \rangle = 0.8$.

and varying J/t from 0.5 to 4.0. In this case anomalous flux quantization is exhibited for $J/t = 0.5$, while for $J/t = 4.0$ $L[E_0(\Phi) - E_0(\Phi = 0)]$ appears to scale to a flat function, consistent with the existence of a phase separated region ($D_s = 0$).

Except for the region believed to be phase separated, the form of the curve $L[E_0(\Phi) - E_0(\Phi = 0)]$ may be easily extrapolated to the thermodynamic limit, thereby allowing an accurate determination of D_s. In a future publication [13] we analyze the specific values in more detail but for this Letter we emphasize that D_s scales to a finite value in the regions which are not phase separated.

The second form of evidence for a superfluid density lies in the behavior of the low energy spin and charge modes which can be used to characterize the possible phases of a particular model. The one-dimensional analog of a superconductor has one gapless charge mode at zero momentum, a gap to all spin excitations and dominant pairing. The phase diagram for the various possible phases for a chain with a Hubbard (rather than t-J) Hamiltonian has recently been found [14] and interestingly on doping away from half filling, this spin-gapped phase with one gapless charge mode is stabilized, exactly the situation we will suggest for the t-J case. In addition, both this reference and recent papers by Nagaosa and Schulz [15] argue that in this gapped phase, a four-fermion operator associated with the superfluid density would exhibit $4k_f = 2(k_f^A + k_f^B)$ charge-density oscillations which decay as $l^{-1/\theta}$ when the pair-field correlations decay as $l^{-\theta}$; in contrast to a Luttinger liquid, the power law term in the density-density correlation function at $2k_f$ is missing.

In Fig. 3 we show the dispersion of the spin and charge modes for the 2×10 ladder with $\langle n \rangle = 0.8$, $J/t = 1.0$, corresponding to the region of the phase diagram where we believe the superfluid to exist. There are several obvious features: Firstly, there is a finite gap to spin excitations (a result that finite size scaling calculations confirm) [6]. Secondly, there is (at least) one vanishing charge mode (bonding) as $k_x \mapsto 0$ [16]. Thirdly, there is no sign of $2k_f$ charge gapless modes. All of these features are consistent with a spin liquid state with dominant superconducting correlations. We should note the existence of a dip in the bonding charge mode in Fig. 3 which may be related to the fluctuations in pair density described above (more detailed descriptions of these results are given in [6]).

The final results we present are direct numerical calculations of various correlation functions obtained using the density matrix renormalization group approach [17];

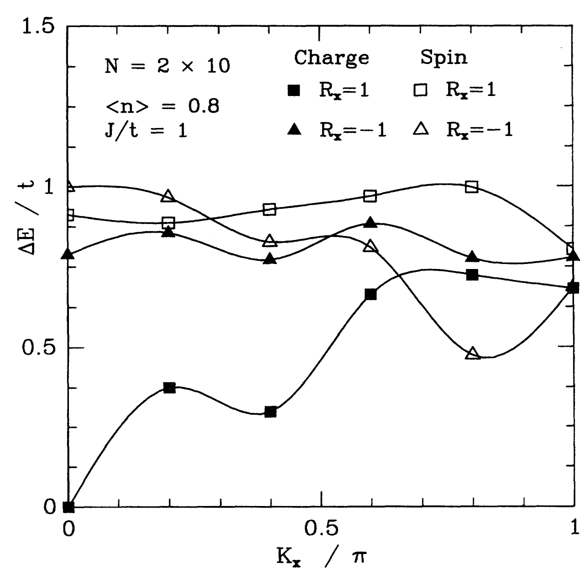

FIG. 3. Spin and charge excitation modes of a 2×10 ladder versus momentum k_x (in units of π); $\langle n \rangle = 0.8$ and $J/t = 1.0$. The quantum numbers associated with the various symbols are shown on the plots. The continuous lines are included as a guide to the eye.

this technique allows much larger systems to be considered than is possible with exact diagonalization techniques. We present three types of correlations: Firstly, the equal-time rung-rung pair-field correlation function $\langle \Delta_i \Delta_j^\dagger \rangle$ where $\Delta_j^\dagger = (c_{j,1;\uparrow}^\dagger c_{j,2;\downarrow}^\dagger - c_{j,1;\downarrow}^\dagger c_{j,2;\uparrow}^\dagger)$, i and j are rung indices and 1 and 2 indicate the chain; this correlation function then creates a singlet pair on rung j and removes a singlet pair from rung i, a direct measure of the motion of hole pairs in the spin-gapped state. The second correlation function that we consider is the spin-spin correlation function $\mathbf{S}_i^z \cdot \mathbf{S}_j^z$ along one of the chains. Finally we measure the density-density correlations along one of the chains defined by $\langle \rho_{i,1} \rho_{j,1} \rangle$ where $\rho_{i,1} = \sum_\sigma c_{i,1;\sigma}^\dagger c_{i,1;\sigma}$.

In Fig. 4 we show the results of a direct calculation of these correlation functions for a 2×30 ladder with open boundary conditions and $\langle n \rangle = 0.8$ and $J/t = 1.0$, i.e., corresponding to the region where we expect superfluid behavior. The most immediate feature of the results is the fact that of the two-fermion operator correlation functions, the pairing correlations are longer range than the others, decaying slower than $|i - j|^{-1}$. Note that the $|i - j|^{-2}$ decay of the charge density-density correlations is just the usual leading behavior; the "$2k_f = k_f^A + k_f^B$" charge and spin correlations (indicated by the oscillations in the correlation functions) appear to decay exponentially. The fact that the pair-field correlations decay more slowly than $|i - j|^{-1}$ implies that $\theta < 1$, and therefore the pairing correlations decay more slowly than the "$4k_f$ charge-density-wave" correlations. These results are then further evidence for dominant superconducting correlations with a spin singlet (gapped) wave function.

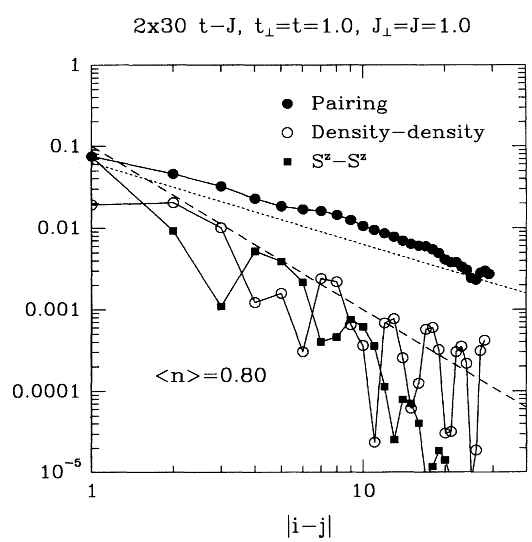

FIG. 4. Log-log plot of various correlation functions versus $|i - j|$ (real space separation) for a 2×30 open chain with $\langle n \rangle = 0.8$ and $J/t = 1.0$. The dashed line has a slope -2 and the dotted line -1. The correlation functions are explained in the main text.

In summary, we have presented three independent forms of evidence for a superfluid density in hole-doped t-J ladders. Firstly, we have shown the existence of anomalous flux quantization and a well-converged and finite D_s. Secondly, we have studied the low lying modes, finding a spin-gapped state with a gapless charge mode and no gapless $2k_f$ excitations. Finally, we have presented direct calculations of correlation functions, showing long range pairing correlations and exponentially decaying spin correlations.

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