

## Evidence for Two-Dimensional Thermal Fluctuations of the Vortex Structure in $\text{Bi}_{2.15}\text{Sr}_{1.85}\text{CaCu}_2\text{O}_{8+\delta}$ from Muon Spin Rotation Experiments

S. L. Lee,<sup>1</sup> M. Warden,<sup>1</sup> H. Keller,<sup>1</sup> J. W. Schneider,<sup>1</sup> D. Zech,<sup>1</sup> P. Zimmermann,<sup>1</sup> R. Cubitt,<sup>2</sup>  
E. M. Forgan,<sup>2</sup> M. T. Wylie,<sup>2</sup> P. H. Kes,<sup>3</sup> T. W. Li,<sup>3</sup> A. A. Menovsky,<sup>4</sup> and Z. Tarnawski<sup>4</sup>

<sup>1</sup>Physik-Institut der Universität Zürich, CH-8057 Zürich, Switzerland

<sup>2</sup>School of Physics and Space Research, University of Birmingham, Birmingham B15 2TT, United Kingdom

<sup>3</sup>Kamerlingh Onnes Laboratorium, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands

<sup>4</sup>Physics Department, University of Amsterdam, 1018 XE Amsterdam, The Netherlands

(Received 16 November 1994)

In the mixed state of  $\text{Bi}_{2.15}\text{Sr}_{1.85}\text{CaCu}_2\text{O}_{8+\delta}$  the temperature dependence of the muon spin rotation ( $\mu\text{SR}$ ) linewidth is found to have a strong *field* dependence, which is attributed to the effect on the line shapes resulting from thermal fluctuations of the vortex positions. This provides important information concerning the form of the fluctuations and moreover gives a measure of the anisotropy parameter  $\gamma$ . By combining this result with a  $\mu\text{SR}$  determination of the penetration depth  $\lambda(T)$  and the melting line  $B_m(T)$ , we are further able to obtain the Lindemann number  $c_L$ .

PACS numbers: 74.40.+k, 74.25.Dw, 76.75.+i, 74.72.Hs

In high- $T_c$  superconductors (HTS) thermal fluctuations of the flux vortices play a significant role in determining many of the properties in the mixed state [1]. Recent muon spin rotation ( $\mu\text{SR}$ ) [2] and small-angle neutron diffraction (SANS) [3] measurements clearly demonstrated that thermal fluctuations give rise to a vortex melting transition in the highly anisotropic material  $\text{Bi}_{2.15}\text{Sr}_{1.85}\text{CaCu}_2\text{O}_{8+\delta}$  (BSCCO). Techniques such as  $\mu\text{SR}$ , NMR, and SANS are highly sensitive to the internal field variation within the vortex structure and may therefore be sensitive to fluctuations of the vortex positions even at temperatures well below the melting transition  $T_m$  [4]. The amplitude of the fluctuations depends on the rigidity of the lattice, which is expected to be low in materials such as BSCCO with large penetration depths and large anisotropy [1]. The ratio of vortex-fluctuation amplitude to lattice parameter at which melting occurs (the Lindemann number  $c_L$ ) is also of interest [1,5].

In the layered HTS, the idea of vortex pancakes existing within the layers may be invoked to describe the quasi-2D nature of the vortex currents, and the concept of a vortex line may be recovered if the interaction between pancakes in adjacent layers is sufficient to align the vortex cores along the direction perpendicular to the planes. At low fields the coupling between the layers is sufficient for this to occur, but, as the field is increased, the energy required for short-range tilt deformations of flux lines becomes smaller than that for shear deformations within the planes. When this occurs, the layers of pancakes may move with respect to one another so that the core positions in nearby layers no longer coincide. For instance, pinning-induced deformations of this nature, occurring above a characteristic crossover field  $B_{cr}$ , are revealed by recent  $\mu\text{SR}$  [2] and neutron [3] measurements. A knowledge of the strength of the coupling between the layers is of theoretical interest, and is often expressed in terms of the anisotropy parameter  $\gamma = \lambda_c/\lambda_{ab}$ , where  $\lambda_c, \lambda_{ab}$  are

the magnetic penetration depths due to currents flowing perpendicular and parallel to the planes, respectively. Measuring  $\gamma$  in the highly anisotropic materials has proved to be difficult [6,7]: The usually reliable method of torque magnetometry has so far only been able to place a lower limit on  $\gamma$  in BSCCO, due to the failure of existing models to describe the data [6]. On the other hand, techniques relying on the shape of the measured irreversibility line make assumptions regarding the temperature dependence of  $\lambda$  and the value of  $c_L$  [7].

The temperature dependence of the  $\mu\text{SR}$  linewidth is often used to extract  $\lambda(T)$ , which in turn is expected to reflect the symmetry and coupling strength of the superconducting-electron pairing state. The results are found to be widely different between different HTS [8–10]. In this Letter we report our measurements of the temperature dependence of the  $\mu\text{SR}$  linewidth in BSCCO, which is a strong function of applied field. We attribute this to the effect of thermal fluctuations of the vortex positions which, in this *highly anisotropic system*, have a significant effect on the linewidth even well below the melting line  $B_m(T)$ . Theoretical calculations [1] [see also Eqs. (3) and (4)] indicate that the ratio of the vortex-fluctuation amplitude to the flux-lattice parameter *increases* with field, as do pinning-induced distortions in this system [2,3]. Hence we believe that for this material it is the *low-field* behavior of the  $\mu\text{SR}$  signal which more accurately reflects the intrinsic behavior of  $\lambda(T)$ . At higher fields, where fluctuations are important, they appear to be quasi-2D; from the field dependence we obtain a measure of the anisotropy  $\gamma$ . By combining these results with the melting line  $B_m(T)$  obtained via  $\mu\text{SR}$ , we also deduce without further assumptions the Lindemann number  $c_L$  at the melting transition.

The sample consisted of a mosaic of high-quality single crystals of BSCCO with  $T_c \approx 84$  K [11]. After growth, they were annealed in air at 500 °C and then quenched in

order to give a definite value of oxygen concentration, on which both  $T_c$  and the anisotropy are known to depend [12]. The crystals were aligned with their  $c$  axes perpendicular to the sample plate and parallel to the applied field. The  $\mu$ SR experiments were performed on beam line  $\pi$ M3 at the Paul Scherrer Institute, Switzerland. Low momentum muons were incident on the sample with their momentum parallel to the magnetic field and with their spins polarized perpendicular to the field. The mosaic was backed by an  $\text{Fe}_2\text{O}_3$  plate in order to reduce background signal from muons not hitting the sample [13].

In a  $\mu$ SR experiment one observes the precession of the spin-polarized muons in the local internal fields of the mixed state. There is a spatial modulation  $B(r)$  of the internal field in the mixed state, and hence a range of precession frequencies in the  $\mu$ SR time spectrum. Thus the probability distribution of internal fields  $p(B)$  can be deduced [14]. The details of  $p(B)$  are closely related to  $B(r)$ , thus the  $\mu$ SR line shapes contain much information concerning the internal vortex arrangement (see, e.g., [2]).

In Fig. 1 are the measured temperature dependences of the square root of the second moment of the field distribution  $\langle \Delta B^2 \rangle^{1/2}$  obtained as in Ref. [2] for a series of fields initially applied at  $T > T_c$ . The second moment  $\langle \Delta B^2 \rangle$  is a good measure of the linewidth of the probability distribution  $p(B)$ . If one assumes an ideal 3D vortex lattice, then for  $\lambda \gg \xi$  (where  $\xi$  is the superconducting coherence length) the temperature dependence follows from that of the penetration depth  $\lambda(T)$  [14],

$$\langle \Delta B^2 \rangle(T) = B^2 \sum_{\tau \neq 0} \frac{1}{[1 + \lambda^2(T)\tau^2]^2}, \quad (1)$$

where  $\tau$  are the reciprocal lattice vectors,  $B$  is the average internal flux density,  $\lambda(T) = \lambda(0)/(1 - t^n)^{1/2}$ ,  $t = T/T_c$ , and  $n = 4$  for the two-fluid model. Evaluation of the sum in Eq. (1) for various fields shows that  $\langle \Delta B^2 \rangle$  should be independent of applied field provided that  $\mu_0 H_{c1} < B \ll \mu_0 H_{c2}$ . In this case the temperature dependence is determined by  $\lambda(T)$  and closely follows the form  $\langle \Delta B^2 \rangle(T) = \langle \Delta B^2 \rangle(0)(1 - t^n)^2$ .

For fields of 10 mT and below we find a  $\langle \Delta B^2 \rangle^{1/2}(T)$  consistent with calculations using Eq. (1) and a two-fluid model variation of  $\lambda(T)$ , as illustrated in Fig. 1. In the same figure we also show the  $\langle \Delta B^2 \rangle^{1/2}(T)$  at fields of 20, 30, and 45 mT, in a region of the phase diagram where the flux structure is known from  $\mu$ SR [2] and SANS [3] experiments to be 3D. The melting of the flux lattice is evident from a sudden decrease in the value of  $\langle \Delta B^2 \rangle^{1/2}$  at the transition temperature  $T_m$ , in addition to the sharp change in the line shape as reported earlier [2] (see also Fig. 2). Moreover, at temperatures below  $T_m(B)$  a clear change in the temperature dependence  $\langle \Delta B^2 \rangle^{1/2}(T)$  is observable at larger fields, so that by 45 mT it falls well below the "two-fluid" line, which fits the results at low field. It is this change in the temperature dependence, within a region of the phase diagram where the vortex structure is 3D, that is one of the main results of this Letter.

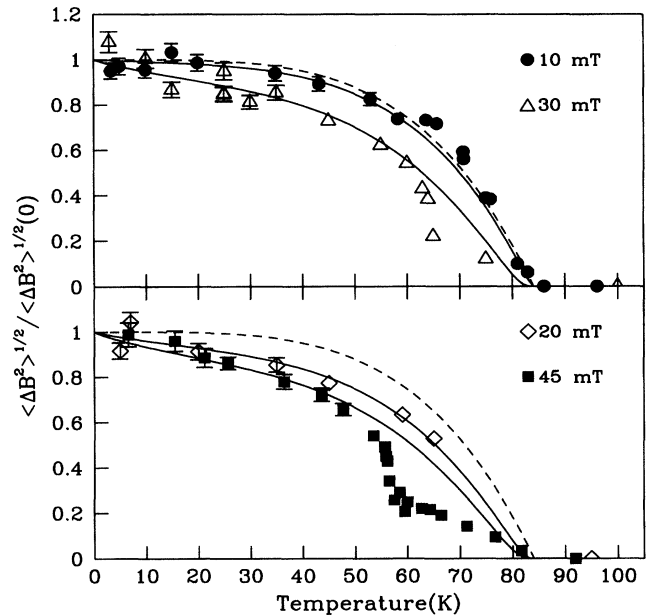


FIG. 1. The square root of the second moment of the field distribution  $\langle \Delta B^2 \rangle^{1/2}$  as a function of temperature at applied fields of 10 mT (circles), 20 mT (diamonds), 30 mT (triangles), and 45 mT (squares) (corrected for the small line broadening due to nuclear moments). The solid lines are calculations which combine Eqs. (2) and (4) in order to take thermal fluctuations of the vortex positions into account, which are expected to describe the data for temperatures below the melting temperature  $T_m(B)$ . For  $T > T_m$ ,  $\langle \Delta B^2 \rangle^{1/2}(T)$  shows a rapid drop, which may be clearly seen in the 30 and 45 mT data. For the calculated curves, values of  $\lambda(0) = 1800 \text{ \AA}$  and  $\gamma = 350$  (see text) have been used, and the data are normalized at zero temperature. The dashed lines are the expected curves if the effects of fluctuations are neglected. For all of the theoretical curves, a two-fluid temperature variation of the penetration depth is used, as suggested by the data at 10 mT, where the effects of fluctuations are small.

Such behavior can be accounted for by considering the thermal fluctuations of the vortices about their average positions [4,10]. Since the typical time scale for thermal flux motion  $\sim 10^{-10}$  s [4] is much shorter than that of the muon lifetime or precession time  $\sim 10^{-6} - 10^{-7}$  s, the muon detects the field averaged over vortex fluctuations. (The averaging is over time and also over a distance  $\sim \lambda_{ab}$  in the  $c$  direction [15].) This averaging effectively smears out the high fields that only occur very close to the vortex cores. The effect on the  $\mu$ SR line shape is to truncate the high-field "tail," which for an ideal static lattice would extend out to the core field. Such an effect can easily be seen in the experimental line shapes of Fig. 2(a), taken at an applied field of 45 mT. For increasing temperature, the increasing amplitude of the fluctuations averages the core field to lower values. In contrast, the line shapes taken at 10 mT [Fig. 2(b)] are much less strongly affected. For temperatures below  $T_m(B)$  time averaged fluctuations lead to a Debye-Waller

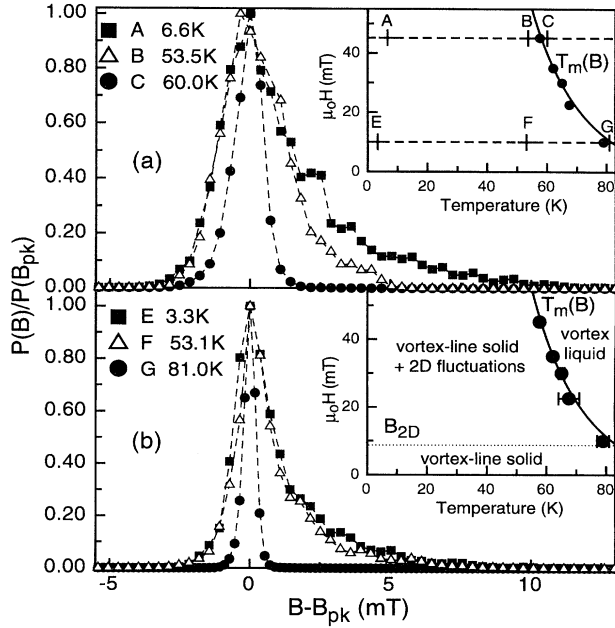


FIG. 2. (a) The probability density of the internal field  $p(B)$  derived from  $\mu$ SR data, shown for various temperatures and an applied field of 45 mT. For clarity of presentation the data have been normalized to the peak of the spectra at  $B_{pk}$ , and the dashed lines are guides to the eye. Note the truncation of the high-field tail at 53.5 K due to the large thermal fluctuations, and also the change of line shape above  $T_m(B)$  (spectrum C). The inset shows the positions of the spectra in the phase diagram. Also included are the experimental melting line, obtained from the change of  $\mu$ SR line shape (circles), and the theoretical curve Eq. (5) using  $\gamma = 350$  and  $c_L = 0.137$  (solid line). (b) The  $p(B)$  for an applied field of 10 mT. Note that the tails are much less strongly truncated than those at 45 mT, due to the reduced effectiveness of thermal fluctuations at the lower field. The inset shows the melting line as in (a), together with the proposed phase diagram (see text).

factor in Eq. (1), which then becomes

$$\langle \Delta B^2 \rangle (T) = B^2 \sum_{\tau \neq 0} \frac{e^{-\tau^2 \langle u^2 \rangle / 2}}{[1 + \lambda^2(T) \tau^2]^2}, \quad (2)$$

where  $\langle u^2 \rangle$  is the mean square displacement of vortices from their equilibrium positions [4,15]. It is of interest to note that this correction to Eq. (1) is of the same form as that due to finite coherence length  $\xi$ : thus the effective field distribution near the cores is smeared out as if  $\xi_{\text{eff}} = \sqrt{2} \langle u^2 \rangle^{1/2}$ . The form of  $\langle u^2 \rangle (B, T)$  depends on the relation between the applied field and that field at which the vortex interactions become quasi-2D  $B_{2D} \sim \phi_0 / s^2 \gamma^2$  ( $s$  is the interlayer spacing  $\sim 15 \text{ \AA}$  and  $\phi_0$  is the flux quantum) [1]. For  $B \ll B_{2D}$  the system is expected to be dominated by fluctuations of 3D vortex-line structures which will have the form

$$\langle u^2 \rangle_{3D}(B, T) \approx \frac{4\pi^{1/2} \mu_0 k_B \gamma \lambda_{ab}^2(T) T}{\ln^{1/2}(k_{\text{max}} a) \phi_0^{3/2} B^{1/2}}, \quad (3)$$

where  $a$  is the vortex lattice parameter and  $k_{\text{max}}$  is as defined in Eq. (2a) of Ref. [1]. At fields above  $B_{2D}$  the

interactions between pancake vortices within a layer are greater than the interactions between vortices belonging to the same ‘‘vortex line,’’ which leads to a quasi-2D fluctuation behavior which may be described by

$$\langle u^2 \rangle_{2D}(B, T) = \frac{4\mu_0 k_B \lambda_{ab}^2(T) T}{s \phi_0 B} \ln \left[ \gamma s \left( \frac{\sqrt{3} B}{2 \phi_0} \right)^{1/2} \right]. \quad (4)$$

Evidence for the dimensional nature of the fluctuations can be gained by considering the detailed way in which the temperature dependence varies with field. Numerical calculations performed using  $\langle u^2 \rangle$  from Eqs. (3) and (4) in Eq. (2) may be compared to the data. The strong field dependence of  $\langle \Delta B^2 \rangle (T)$  predicted by Eq. (4) gives good agreement with the measurements across the *entire range of fields* considered, whereas Eq. (3) gives a weaker field dependence that fails to describe the data adequately. This suggests that the fluctuation behavior in this regime is dominated by ‘‘2D’’ fluctuations of pancake vortices about their average positions, rather than by fluctuations of the vortex lines themselves. This result is not entirely unexpected, since it reflects the known extreme ‘‘softness’’ of the vortex lines over distances of the order of the interlayer spacing [2,3].

Various material-dependent parameters  $\lambda_{ab}$ ,  $s$ , and  $\gamma$  appear in Eq. (4), which together with Eq. (2) describe the measured linewidths. At low field, where fluctuations are small,  $\lambda_{ab}(0)$  is extracted using the difference between the average internal field and the peak field in the  $\mu$ SR line shape  $B - B_{pk} \propto 1/\lambda^2$  [4,16]. We have previously calculated a value of  $\lambda_{ab}(0) \approx 1800 \text{ \AA}$  using the  $\mu$ SR  $p(B)$  at 5 mT, and with  $B$  obtained from magnetization measurements [2]. Given  $\lambda_{ab}(0)$  and  $s$ , the temperature dependence of  $\langle \Delta B^2 \rangle$  is given by Eq. (2) provided that a suitable choice is made for  $\gamma$  in the expression for  $\langle u^2 \rangle_{2D}(B, T)$  [Eq. (4)]. A value of  $\gamma$  has thus been chosen that gives calculated temperature dependences  $\langle \Delta B^2 \rangle (T)$ , which are most consistent with our observations for  $T < T_m$  across the range of fields measured. This gives a value  $\gamma \sim 350(50)$ . In the analysis it was found necessary to divide the value of  $\langle \Delta B^2 \rangle^{1/2}$  obtained via Eq. (2) by an additional scaling factor at each field, in order to account for influences on the linewidth that arise from sources other than  $\lambda(T)$  and  $\langle u^2 \rangle_{2D}(B, T)$ . These include distortions of the flux lattice due to pinning [15], macroscopic field gradients arising from the sample geometry [17], and experimental difficulty in recovering the complete line shape for close-to-ideal distributions at low field [4,16]. The latter effect, which is dominant, arises because in any realizable experiment the signal at fields which correspond to a very low  $p(B)$  will be comparable to the statistical noise, so that the true linewidth will always be underestimated. The corresponding scaling factors are temperature independent and almost field independent (of value 1.8 at 10 mT, and 1.6 at 20, 30, and 45 mT). The value obtained for  $\gamma \sim 350(50)$  is somewhat larger than our earlier reported value of  $\gamma \sim 150$  [2] which was derived from the shape of the melting line, but

the difference arises largely because in that paper 3D fluctuations were assumed. However, we demonstrate below that this larger value of  $\gamma$ , together with the form of  $\lambda(T)$  obtained from this work, may nevertheless be used to describe the experimental melting curve for  $B < B_{cr}(T)$  in terms of 2D fluctuations. Using  $\gamma \sim 350$  the corresponding Josephson length  $\lambda_J = \gamma s \sim 5000 \text{ \AA} \gg \lambda_{ab}(0) \sim 1800 \text{ \AA}$ , which suggests that our sample is approaching the limit in which Josephson coupling between the layers may be neglected [1]. In the limit of vanishing Josephson coupling, pancake vortices in neighboring layers are coupled only via magnetic interactions, so the presence of quasi-2D fluctuations becomes more likely. From  $\gamma$  we may also estimate  $B_{2D} \sim \phi_0/s^2\gamma^2 \sim 8 \text{ mT}$  [1]. We note that the temperature dependence of the linewidth begins to show clearly the influence of quasi-2D thermal fluctuations as the field increases above 10 mT, confirming our estimate of  $B_{2D}$ . We may compare this value with that of  $B_{cr} \sim 65 \text{ mT}$ , which is determined by the field at which the pinning energy for a pancake vortex is comparable to the energetic cost of deforming the vortex-line lattice [2,3]. Clearly this pinning crossover at  $B_{cr}$  cannot occur unless the short-range flux vortex interactions are effectively 2D, so  $B_{cr} \gtrsim B_{2D}$ .

We have also performed detailed measurements to determine the transition from the vortex-solid phase to the vortex-fluid phase by observing changes in the  $\mu$ SR line shape (for a discussion see Ref. [2]). In order to describe this in terms of a transition driven by 2D fluctuations, an approximate expression for the melting line may be derived from the Lindemann criterion  $\langle u^2 \rangle_m = c_L^2 a^2$ , where  $c_L$  is a quantity  $\sim 0.1$  [1]. Combining this criterion with Eq. (4) yields the following expression for the melting line, valid for  $B > B_{2D}$ :

$$B_m^{2D}(T) = \frac{2\phi_0}{\sqrt{3}\gamma^2 s^2} \exp\left[\frac{c_L^2 \phi_0^2 s}{\sqrt{3}\mu_0 k_B \lambda^2(T) T}\right]. \quad (5)$$

By comparing the function to the data, and using  $\gamma = 350$  as found above, a value of  $c_L$  may be obtained. For example, at 35 mT this gives  $c_L \approx 0.137$ . The agreement of the resulting curve with the data is excellent, as shown in the insets of Fig. 2 for  $B < B_{cr}$ . We note, however, that  $c_L$  may itself be a function of field, with early Monte Carlo simulations predicting  $0.12 < c_L < 0.17$  for  $10 < B < 50 \text{ mT}$  [5]. Analysis of the present data over the same region gives  $0.09(2) < c_L < 0.14(1)$ , in very reasonable agreement.

In conclusion, we have shown that the anomalous temperature dependence of the  $\mu$ SR linewidth, in a region of the phase diagram where the vortex structure is a

3D solid, can be described by the effects of thermal fluctuations of the vortex positions. The observed field dependence in our field range strongly suggests the dominance of 2D fluctuations, and from this we obtain a value of the anisotropy parameter  $\gamma \sim 350(50)$ . This further implies that when melting occurs in this field regime it is driven by 2D thermal fluctuations, involving a transition from a weakly coupled 3D vortex lattice to a quasi-2D vortex liquid. The shape of the melting line  $B_m(T)$  as measured by  $\mu$ SR is in good agreement with that expected from such a transition, and from this we obtain a typical Lindemann number  $c_L \sim 0.14$ . We note that these conclusions are consistent with results of SANS [3], and magneto-optical and forced torsion pendulum measurements [17], where a transition to a 2D liquid is implied. We further note that in our very low-field data, where the effects of thermal fluctuations on the  $\mu$ SR linewidth are becoming negligible, we observe a temperature dependence which is not inconsistent with a two-fluid variation of  $\lambda_{ab}(T)$ .

We are very grateful to B. Ivlev for useful discussions and to D. Herlach for technical assistance. This work was supported by the Swiss National Science Foundation (NFP30 No. 4030-32785), the Engineering and Physical Sciences Research Council of the United Kingdom, and by a special grant from the British-Swiss Joint Research Program.

- 
- [1] L. I. Glazman and A. E. Koshelev, Phys. Rev. B **43**, 2835 (1991).
  - [2] S. L. Lee *et al.*, Phys. Rev. Lett. **71**, 3862 (1993).
  - [3] R. Cubitt *et al.*, Nature (London) **365**, 407 (1993).
  - [4] Y.-Q. Song *et al.*, Phys. Rev. Lett. **70**, 3127 (1993); Y.-Q. Song *et al.*, Physica (Amsterdam) **241C**, 187 (1995).
  - [5] R. S. Ryu *et al.*, Phys. Rev. Lett. **68**, 710 (1992).
  - [6] J. C. Martinez *et al.*, Phys. Rev. Lett. **69**, 2276 (1992).
  - [7] A. Schilling *et al.*, Phys. Rev. Lett. **71**, 1899 (1993).
  - [8] H. Keller *et al.*, Physica (Amsterdam) **185-189C**, 1089 (1991).
  - [9] J. E. Sonier *et al.*, Phys. Rev. Lett. **72**, 744 (1994).
  - [10] D. R. Harshman *et al.*, Phys. Rev. Lett. **61**, 3152 (1991).
  - [11] T. W. Li *et al.*, J. Cryst. Growth **135**, 481 (1994).
  - [12] R. Kleiner and P. Müller, Phys. Rev. B **49**, 1327 (1994).
  - [13] R. Cubitt *et al.*, Physica (Amsterdam) **213C**, 126 (1993).
  - [14] E. H. Brandt, Phys. Rev. B **37**, 2349 (1988).
  - [15] E. H. Brandt, Phys. Rev. Lett. **66**, 3213 (1991).
  - [16] A. D. Sidorenko *et al.*, Physica (Amsterdam) **166C**, 167 (1990).
  - [17] M. V. Indenbom *et al.*, Physica (Amsterdam) **222C**, 203 (1994).