Global Phase Diagram for the Quantum Hall Effect: An Experimental Picture

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We study the behavior of the extended states of a two-dimensional electron system in silicon in a magnetic field, B. Our results show that the extended states, corresponding to the centers of different Landau levels, *merge* with the lowest extended state as $B \to 0$. Using our data, we construct an experimental-based "disorder vs filling factor" phase diagram for the integer quantum Hall effect (QHE). Generalizing this diagram to the case of the fractional QHE, we show that it is consistent with the recently observed direct transitions between insulator and fractional QHE at $\nu = 2/5, 2/7$, and $2/9$.

It is generally accepted that in a two-dimensional electron system (2DES), placed in a strong perpendicular magnetic field B there exist extended states at the center of each Landau level [1]. At the same time, in zero magnetic field, a 2DES is believed to be insulating [2], and, therefore, there should be no extended states below the Fermi level. Khmelnitskii [3] and Laughlin [4] resolved this discrepancy by arguing that as the magnetic field decreases, the extended states neither vanish nor merge but float up in energy and finally leave the Fermi sea leading to an insulating state at $B = 0$ [5,6]. In spite of the fundamental importance of this prediction, there were no attempts to test it experimentally for almost 10 years. The interest in this and related problems was recently revived when magnetic-field-induced transitions between insulating and quantum Hall states were observed almost simultaneously in GaAs/AlGaAs heterostructures [7–9] and Si metal-oxide-semiconductor field-effect transistors (MOSFET's) [10–12]. In addition, theoretical global phase diagrams for the quantum Hall effect (QHE), recently proposed by Kivelson, Lee, and Zhang (KLZ) [13] and by Halperin, Lee, and Read (HLR) [14], have fueled experimental studies of the behavior of extended states in disordered two-dimensional systems.

Shashkin, Kravchenko, and Dolgopolov (SKD), studying a 2DES in silicon, reported the floating up of extended states with respect to their expected positions at half-integer filling factors $\nu \equiv n_s hc/eB = i + \frac{1}{2}$ as $B \to 0$ [15,16] (here n_s is the electron density, and i is the level index). However, contrary to the theoretical predictions of Khmelnitskii [3], Laughlin [4], and KLZ, the extended states did not float up indefinitely; instead, they coalesced at some finite energy as $B \to 0$. To map positions of the extended states, SKD used two methods. For the first method, the system was considered metallic if the diagonal conductivity σ_{xx} was higher than $e^2/20h$ and insulating otherwise. For the second, the metal/insulator

boundary was determined using the criterion of vanishing activation energy [12]. Both methods gave similar results.

Later, Glozman, Johnson, and Jiang (GJJ) [17] studied a low-mobility gated GaAs/AlGaAs structure and reported floating up of extended states *without merging* in agreement with theoretical predictions. They mapped the positions of extended states by identifying them as peaks in σ_{xx}. The principle discrepancy of their results with those of Ref. [15] was attributed to some arbitrariness in the first of two criteria used by SKD.

To resolve this inconsistency, we have performed a systematic study of the position of extended states in very high-quality silicon MOSFET's using the method suggested by GJJ. We have found that the extended states belonging to different Landau levels *do merge* with the lowest as the magnetic field vanishes, while the levels remain well resolved to the point of merging. Using the obtained map of the extended states, we construct an experimental-based "disorder vs filling factor" phase diagram for the integer QHE. Applying correspondence principles articulated by KLZ, we construct a global phase diagram, which appears to be consistent with direct transitions between an insulating state and the fractional QHE (FQHE) at $\nu = 2/5, 2/7$, and $2/9$, recently observed in high-quality GaAs/AlGaAs heterostructures [18–21].

The samples used were silicon MOSFET's with maximum mobility μ around 4×10^4 cm^2/V s similar to those used in Refs. [10–12,15,16]. The resistance was measured with a four-terminal dc technique using amplifiers with a very high input impedance ($>10^{14}$ Ω).

In Fig. 1(a) we show the diagonal (ρ_{xx}) and Hall (ρ_{xy}) resistivities obtained at very low n_s as functions of $1/\nu \propto B$. Despite low n_s, $\rho_{xy}(B)$ looks conventional, with wide plateaus at $\nu = 2$ and 1. Diagonal resistivity exhibits minima at $\nu = 6, 2$, and 1, which are also characteristic of the QHE. However, in contrast to the conventional QHE, the ρ_{xx} peaks between some of these minima pierce
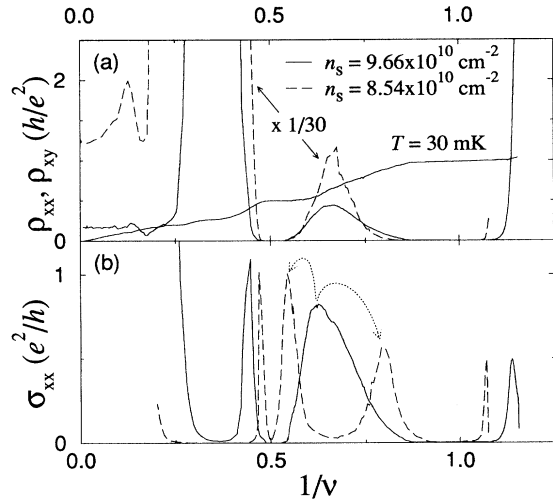

FIG. 1 ρ_{xx} and ρ_{xy} (a) and σ_{xx} (b) vs $1/\nu$ for two electron densities. To avoid complications not addressed in discussion, we show σ_{xx} only for $1/\nu > 0.2$.

FIG. 2.   The map of extended states. Numbers show σ_{xy} in units of e^2/h. The inset shows details at low B and n_s. Note that the lowest two triangles (corresponding to extended states between QHE with $s_{xy} = 6$ and 4) almost coincide with the QHE/I boundary. Merging with the QHE/I boundary is unambiguous.

through $\rho_{xy}(B)$ and exceed ρ_{xy} by orders of magnitude. The peaks with $\rho_{xx} \gg \rho_{xy}$ correspond to an insulating state with $\sigma_{xy} \ll \sigma_{xx} \ll e^2/h$. Note that lower-field ρ_{xx} peaks develop directly from the QHE at $\nu = 6$ [22].

Recalculating σ_{xx} from the data for ρ_{xx} and ρ_{xy} [the result is shown in Fig. 1(b)], it is possible to map the positions of extended states on the (B, n_s) plane [23]. When the Fermi energy E_F crosses an extended state, σ_{xx} exhibits a maximum. Most of the data was obtained by sweeping gate voltage (and therefore n_s) at $B = \text{const}$. We used data obtained by sweeping B at constant n_s only at very low n_s, because the phase boundary has regions almost independent of B for some ranges of n_s.

Figure 2 shows the resulting positions of the extended states on the (B, n_s) plane. Each set of symbols reflects the behavior of one σ_{xx} peak as a function of n_s and B, and the dotted lines represent the expected positions of the extended states at half-filled Landau levels, $\nu = i + \frac{1}{2}$. The regions between extended states correspond to the QHE with $\sigma_{xx} = 0$ and $s_{xy} \equiv \sigma_{xy}h/e^2$ shown by the numbers obtained from the data for σ_{xy}. The region with $s_{xy} = 0$, separated from regions with $s_{xy} \neq 0$ by the lowest extended state, is insulating. We will call this boundary, shown by the solid line, the "QHE/insulator (QHE/I) boundary."

The two most important features of Fig. 2 are (i) as the magnetic field is lowered, the higher n_s extended states, separating QHE with different values of s_{xy}, merge with the QHE/I boundary, and (ii) the QHE/I boundary oscillates as a function of magnetic field for $B \lesssim 3.5$ T, reaching maxima where the higher n_s extended states join. Both features are in agreement with the phase diagram obtained by SKD. They are not observed in the experimental results reported by GJJ. We note that
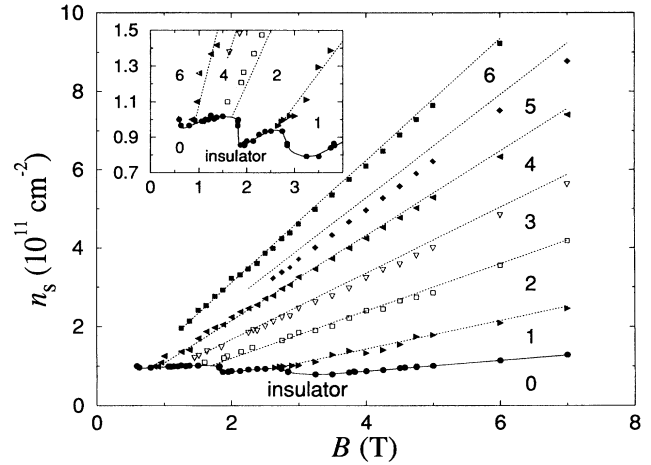
merging of extended states with a boundary between QHE and zero-field metallic state was suggested by HLR.

Using these experimentally obtained positions of the extended states, we reconstruct a disorder vs filling factor phase diagram for the QHE to compare with the theoretical results of KLZ and HLR. To characterize the disorder quantitatively, we used the $B = 0$ electron scattering time extracted from $\mu(n_s)$ at $T = 1.8$ K. The resulting phase diagram is shown in Fig. 3. The level structure in silicon is complex, with both spin and valley splittings associated with each Landau level. Thus QHE states with $s_{xy} = 1$ to 4 all belong to Landau level zero, and the merging of different extended states belonging to this level is
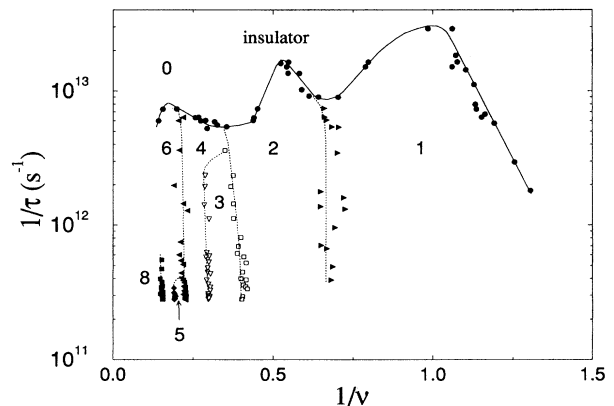

FIG. 3.   Disorder vs filling factor phase diagram recalculated from the data shown in Fig. 2. The dotted lines show the approximate behavior of the extended states, and the solid line is the QHE/I boundary. Numbers between the lines give σ_{xy} in units of e^2/h.

trivial. However, the QHE state with $s_{xy} = 6$ belongs to Landau level 1. Therefore the experimentally obtained phase diagram principally differs from the theoretical one because *it allows a direct transition from insulator to the QHE with $s_{xy} = 6$, associated with the next Landau level.* The QHE/I boundary (the upper solid line) oscillates reaching maxima near $\nu = 1$, 2, and 6, and minima at the points where it merges with extended states separating neighboring QHE regions [10,15,16]. Note there is no maximum near $\nu = 4$.

We believe that these oscillations of the QHE/I boundary may be caused by many-body (MB) enhancements of energy gaps between levels which occur at integer filling factors [24]. Indeed, suppose that the "starting" phase diagram (i.e., the diagram which does not include any MB enhancement) looks like the schematic diagram in the inset in Fig. 4. The QHE/I boundary is monotonic for $1/\nu < 0.5$, and extended states, which separate QHE with different s_{xy}, merge with it (for simplicity, we consider spinless electrons and ignore the valley splitting; however, we assume that all gaps are subject to MB enhancement). Then, we "turn on" the MB enhancement of gaps between the neighboring levels when E_F is at integer ν. This enhancement results in the penetration of the QHE, associated with that ν, deeper into insulating region, as shown by vertical arrows. At the same time, the points where the extended states merge with the QHE/I boundary remain on the "starting" boundary because the enhancement is unim-

FIG. 4. A suggested phase diagram for the quantum Hall effect. Shaded regions correspond to the QHE with $\sigma_{xx} = 0$ and values of s_{xy} given by numbers. Thick solid lines separating regions of QHE with different s_{xy} designate the extended states. The white region corresponds to the insulator with $s_{xy} = \sigma_{xx} = 0$, and the darker shaded regions correspond to the "metallic" states near $B = 0$ [6] and half-integer filling factors. The horizontal lines trace out transitions between QHE and insulating phases consistent with the experimental references with which they are labeled. Inset illustrates a possible mechanism leading to oscillations of the QHE/I boundary, see text.

portant when E_F coincides with an extended state. The result is an oscillating QHE/I boundary [25]. This picture leads to a reconciliation of the results of GJJ and SKD. For the case of pure Landau level gaps at even ν in GaAs systems, like the case for GJJ, there is little MB enhancement of the gaps. Therefore the QHE/I boundary is monotonic and a multireentrant insulating state is not seen.

The question may arise why the QHE/I boundary shows maxima at $\nu = 1$, 2, and 6, and not at $\nu = 4$. We believe this is due to the lack of MB enhancement at $\nu = 4$ and to complex level structure in Si MOSFET's. Splittings at $\nu = 4$ and 8 vanish at $B \lesssim 1$ T, as shown by direct measurements of the chemical potential [26].

The merging of extended states separating two different QHE states with the QHE/I boundary can be seen from the bare data for ρ_{xx} and ρ_{xy}. Consider a trivial situation between QHE minima at, e.g., $\nu = 1$ and 2 [Fig. 1 (a)]. As long as the ρ_{xx} peak lies below ρ_{xy} (lower disorder, the solid curve), its position approximately coincides with the maximum of σ_{xx} [see Fig. 1(b)] and, therefore, with the position of the extended state separating QHE with $s_{xy} = 1$ and 2. However, as soon as the peak of ρ_{xx} pierces through ρ_{xy} (higher disorder, the dashed curve), the single peak of σ_{xx} splits into two peaks separated by an insulator, as shown in Fig. 1(b) by arrows. In other words, the extended state between QHE with $s_{xy} = 1$ and 2 becomes a part of the QHE/I boundary. A similar merging happens every time a peak of ρ_{xx} overtakes ρ_{xy}, e.g., between QHE minima with $s_{xy} = 2$ and 6 and possibly 6 and 10 [22]. More interestingly, *this happens also between some of neighboring FQHE minima*, e.g., between $\nu = 2/5$ and $1/3$ [19,21]. This ρ_{xx} peak grows by an order of magnitude as a result of decreasing hole density (see Fig. 1 in Ref. [19]) and overtakes $\rho_{xy} \approx \frac{5}{2}h/e^2$ [21]. Therefore the extended state between fractions $2/5$ and $1/3$ merges with the QHE/I boundary in a way similar to the extended state between $s_{xy} = 1$ and 2 considered above.

The similarity between the integer QHE (IQHE) at $\nu = 1, 2, 3, \ldots$ and FQHE with ν equal, e.g., to $1/3$, $2/5$, $3/7, \ldots$ follows from the composite fermion approach of Jain [27] and HLR. It can also be seen in the global phase diagram of KLZ. This similarity prompts us to generalize the experimentally obtained IQHE phase diagram to FQHE using correspondence principles presented by KLZ. Of course, this can be possible only if the nature of the insulating phases is essentially the same in Si MOSFET's and in GaAs heterostructures. There is a recent experimental indication [28] that this is the case. For simplicity, we ignored spin and valley splittings. In this case, the regions with $s_{xy} = 1, \ldots, 4$ collapse into a single region with $s_{xy} = 1$, the regions with $s_{xy} = 5, \ldots, 8$—into a single region with $s_{xy} = 2$, etc. We used two main features of the experimental phase diagram shown in Fig. 3: (i) merging of the upper extended
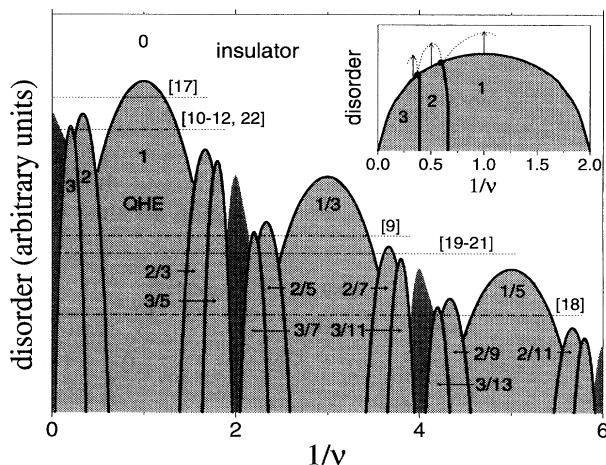
states with the QHE/I boundary, and (ii) the oscillatory QHE/I boundary. The suggested resulting phase diagram is shown schematically in Fig. 4. The main feature of this phase diagram is that it allows direct transitions between insulating states and QHE with $s_{xy} = 2, 3, 2/3,$ $2/5, 2/7, 2/9,$ and $2/11$, which are forbidden according to the global phase diagram of KLZ and HLR. However, recently there have been a number of reports on experimental observation of direct transitions between insulator and QHE with $s_{xy} = 6$ and possibly 10 [22], $s_{xy} = 2/5$ [19,21], $s_{xy} = 2/7$ [2021], and $s_{xy} = 2/9$ [18], which are consistent with the proposed phase diagram. Horizontal dotted lines show schematically the transitions realized in some of these experiments. It can be seen that, in principle, there should exist a direct transition between insulator and the QHE with $s_{xy} = 2/11$.

In summary, we report an experimentally determined map of extended states in a magnetic field. We show that the higher-n_s extended states can merge with the lowest one, in contrast to theoretical predictions. Using our data, we construct a disorder vs filling factor phase diagram for the IQHE and show that the generalization of our results to the FQHE explains recently observed transitions between insulating states and the FQHE states with $s_{xy} = 2/5, 2/7,$ and $2/9$.

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