Direct Observation of Transfer of Angular Momentum to Absorptive Particles from a Laser Beam with a Phase Singularity

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Black or reflective particles can be trapped in the dark central minimum of a doughnut laser beam produced using a high efficiency computer generated hologram. Such beams carry angular momentum due to the helical wave-front structure associated with the central phase singularity even when linearly polarized. Trapped absorptive particles spin due to absorption of this angular momentum transferred from the singularity beam. The direction of spin can be reversed by changing the sign of the singularity.

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It is well known that a circularly polarized beam carries angular momentum. Each photon of such a beam has an angular momentum of $\hbar$. The effects produced by the optical angular momentum are hard to observe in most circumstances as they represent extremely small quantities. The angular momentum flux carried by a circularly polarized 10 mW He-Ne laser beam is of the order of 10^{-18} mN. The first attempt to measure the torque produced by the optical angular momentum was made by Beth [1] 59 years ago. Beth reported that his results agreed with theory in sign and magnitude. More recently Santamato *et al.* [2] observed the light induced rotation of liquid crystal molecules, Chang and Lee [3] calculated the optical torque acting on a weakly absorbing optically levitated sphere, and Ashkin and Dziedzic [4] observed rotation of particles optically levitated in air.

A linearly polarized wave containing a central phase singularity also carries angular momentum associated with its helical structure. Figure 1 shows this structure for a TEM_{01}^* beam [5]. Each photon carries $l\hbar$ angular momentum where l is called the topological charge of the singularity. This contribution to the angular momentum is sometimes referred to as "orbital angular momentum" to distinguish it from "spin" angular momentum associated with circular polarization [6,7].

Allen *et al.* [6] have proposed to measure the angular momentum carried by a doughnut laser beam by measuring the torque acting on an optical device which reverses the chirality of the beam.

We have demonstrated in a previous paper [8] that black or reflective particles of sizes of 1–2 μm can be trapped optically in a liquid by higher order doughnuts produced by high efficiency computer generated holograms. We also mentioned that some bigger absorptive particles were set into rotation by slightly defocused doughnuts. In this paper we report on further experiments concerned with the transfer of angular momentum to absorptive particles and their subsequent rotation. The detection is performed using a video camera. A large number of small particles have been trapped and have been recorded. Our results clearly show that the rotation directions of all these trapped particles agree with the sign of the doughnut.

In this Letter we analyze the results of the rotating particles in terms of possible torques acting on a trapped absorptive particle.

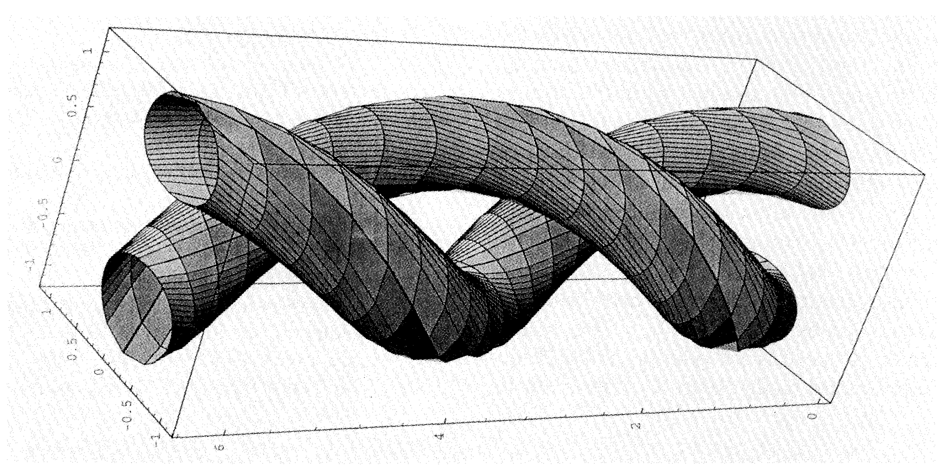

FIG. 1. Snapshot of the irradiance structure of a TEM_{01}^* $[(r/\omega)e^{-r^2/\omega^2}e^{i\theta}e^{ikz}]$ beam containing a first order phase singularity. The surface represented is that where the irradiance has half its peak value and we take $k = 1$, $\omega = 1$ for simplicity.

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The angular momentum carried by light can be understood in two ways. Classically, electromagnetic radiation carries momentum, which can be both linear and angular. Allen *et al.* [6] have shown that for a Laguerre-Gaussian beam the angular momentum density is given as

$$M_z = \frac{l}{\omega}|u|^2 + \frac{\delta_z r}{2\omega}\frac{\partial |u|^2}{\partial r}, \tag{1}$$

where u is the amplitude of the light field, l is the azimuthal index number of the Laguerre polynomial, $\delta_z = \mp 1$ for right-handed or left-handed circularly polarized light and $\delta_z = 0$ for linearly polarized light.

Quantum mechanically, one would say that each photon carries $(l \mp \delta_z)\hbar$ of angular momentum because of the well-known analogy between paraxial theory and quantum mechanics [9].

Although the above simple relationship is valid only within the paraxial approximation as shown recently by Barnett and Allen [7], for a linearly polarized light ($\delta_z = 0$), even when tightly focused as in an optical trapping experiment, the total angular momentum per second is still given by

$$\Gamma_z = \frac{P}{\omega} l, \tag{2}$$

where P is the laser power.

A linearly polarized charge 3 singularity beam corresponds to $l = 3, \delta_z = 0$.

This suggests that an absorptive particle illuminated by such a singularity beam should be set into rotation in the same sense as the helical beam. It should reverse its rotation direction when the direction of rotation of the helical wave is reversed.

The experimental setup for the optical trap used in this work is similar to the one used in [10]. A linearly polarized TEM_{00} laser beam from a 15 mW He-Ne laser (632.8 nm) illuminates a blazed charge 3 phase hologram. The hologram then produces a phase singular laser doughnut beam equivalent to a linearly polarized Gauss-Laguerre LG_{03} mode with a power of approximately 7 mW. As the hologram used here is blazed, the sign of the doughnut can be simply reversed by turning the hologram around. We can also switch the diffracted beam between doughnut mode and Gaussian mode by moving the hologram sideways. The laser beam is now introduced into a microscope (Olympus CHT) through the aperture for the vertical illuminator. A dichroic mirror reflects the beam to fill the back aperture of an oil-immersion, high numerical aperture ($NA = 1.30$), 100× objective. The tightly focused doughnut beam had a diffraction limited beam waist within micrometers of the object plane of the objective.

The particles were illuminated from below by a lamp and green filters. A video camera placed vertically on the top of the microscope was used to record the motion of the trapped particles. We normally used black high-T_c superconductor ceramic powder as absorptive particles, dispersed in kerosene. Sizes of the particles that can be trapped are around 1–2 μm. We have performed similar experiments using CuO particles dispersed in water.

Using the charge 3 doughnut, we can trap a particle in the dark central spot. It can then be moved around relative to its surroundings by moving the microscope slide. The maximum measured trapping speed is around 5 μm/s. Particles adhering to the slide can often be set free by switching from the doughnut to the Gaussian mode and "kicking" them with the strong repulsive force.

Using a video camera, we clearly see a trapped particle rotating always in one direction determined by the helicity of the beam. The particle also keeps rotating in the same direction while being moved relative to its surroundings. The rotation of a trapped particle can be easily maintained for long periods of time. The particle can be set free and trapped again with the same rotation direction. In a session, more than 30 particles have been trapped, moved, set free, and trapped again. They all rotate in the same direction as the helicity of the beam. A few particles with very irregular shapes tumble wildly and occasionally rotate in the opposite direction for one or two turns but much more slowly than they rotate the other way. However, most of the time they rotate in the same direction as other particles. Those particles with close to spherical shapes always rotate in the same direction with high constant speed.

Turning the hologram back to front, that is reversing the charge of the singularity, we repeat the above procedures and we observe the same effects except that the rotation direction of all particles is reversed.

The trapped particle rotation speed varies from 1 to 10 Hz depending on shape and size.

Figure 2 shows six successive frames of a video recording. A fairly asymmetric particle, about 2 μm across, tumbling and rotating at about 1 Hz, has been chosen for illustration. It lies near the top of the frame and over a period of 0.24 s rotates through an angle of a little over $\pi/2$. Surrounding objects are stationary. More rapidly rotating symmetrical particles show up poorly in individual frames. However, the rotation is clear in continuous playback, in spite of motional blurring, very limited depth of field, and the fact that the microscope is operating near its limit of resolution.

When trapping CuO particles in water, if a little detergent is added, all the particles move very freely, and when one is trapped, almost immediately surrounding particles begin to move in radially and join a general circulation about it, in the same direction as the rotation of the trapped one.

Taking losses into consideration, the power incident on the focal plane is around 4 mW. According to Eq. (2), the angular momentum flux of such a doughnut beam produced by a charge 3 blazed hologram illuminated by a 633 nm HeNe laser is about 4×10^{-18} mN.

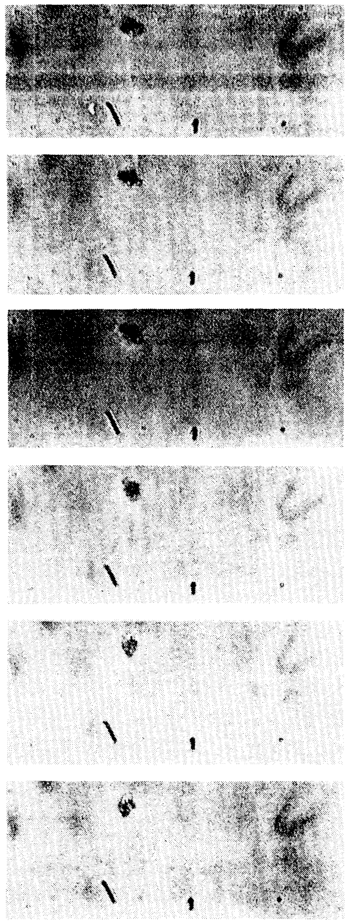

FIG. 2. Six successive frames of a video recording of a particle of black ceramic trapped in a charge three doughnut beam. The particle is near the top; other objects in the field are stationary. A rotation of just over $\pi/2$ occurs in the period shown.

Assume a particle absorbs 25% of the beam and hence 25% of the angular momentum. The rotation speed will become constant when the torque produced by the doughnut laser beam is balanced by the drag torque exerted by the surrounding liquid.

The drag torque acting on a spherical particle rotating with angular velocity ω_a is given by [11]

$$\tau_v = -8\pi\eta r^3 \omega_a , \tag{3}$$

where r is the radius of the particle, η is the viscosity of the liquid, taken as 1.58×10^{-3} N s m^{-2} for kerosene.

For our particles, with radius about 1 μm, this leads to a rotation speed of around 4 Hz, consistent with our observations, bearing in mind the actual form of our particles, and the fact that we have neglected the effects of the nearby slide surface. Our assumption that the particles absorb 25% of the incident power is based on the case where the sizes of particles and the central dark spot are the same. We define the size of the dark spot to be the diameter of the maximum intensity ring of the doughnut beam. This estimate does not affect the relative sizes of the torques hypothetically present as they all would be proportional to the absorbed power.

Although absorbtion of angular momentum therefore accounts satisfactorily for the observed rotation, we briefly consider other explanations that might be advanced. It is possible that an unbalanced force, such as a thermal force, or a scattering force, may produce a torque acting upon a nonsymmetric particle. Among these forces, the scattering force is the biggest. However, since the direction of the scattering force is predominantly downward, the direction of the torque produced by the scattering force is therefore perpendicular to the beam axis. We believe that such torques are responsible for the irregular tumbling exhibited by asymmetric particles.

However, it is conceivable that reflections on the surface of an asymmetric particle may produce a torque in the same direction as that of the angular momentum from the beam.

Such a torque will be, at most,

$$\tau_a = k\gamma F r , \tag{4}$$

where F is the force acting on the particle, r is an effective radius, γ is the reflection coefficient, and k is an asymmetry index, expressing the ratio of the area of a typical irregularity to the total area.

The scattering force (radiation pressure) cannot exceed

$$F_s = \frac{P}{c} = 1.3 \times 10^{-11} \text{ N} .$$

The value of k can vary from 0 to 1. However, only if the particle has a perfect "propeller" shape will k have a value approaching 1. Since our particles are mostly spheroidal and the effect of the trap is to center them in the beam, as the main asymmetry is due to the bumps on the surface of the particle, the value of k is approximately equal to the ratio of the bump area to the particle cross sectional area. Using a scanning electron microscopy image of these particles, we estimate that the value of k will normally be less than 1×10^{-2} as the irregularities are smaller than 10%. It is safe to assume the reflection coefficient to be smaller than 0.1 as our particles are highly absorptive. Hence, the torque due to asymmetric scattering forces is less than 10^{-20} N m, which is much smaller than the angular momentum from the doughnut laser beam. Obviously, a particle with an irregular shape may have a large value of k for a short period of time during which the scattering torque may dominate. However, the scattering torque is not constant, and hence the rotation direction changes. Furthermore, such a scattering force would be independent of the sign of the doughnut and therefore not able to explain our experimental results.

It is very difficult to estimate the magnitude of possible thermal effects like the photophoretic forces known to act on illuminated particles in air [12], but they could be expected to be smaller in our experiments because of the much higher thermal conductivities of the liquid media and the particles used. We have been unable to find any reports of such forces in liquids. Even if such forces are acting, considering the high thermal conductivity of the material and the fact that irregularities are small relative to the particle size and the wavelength, it is unlikely that they would exert torques leading to regular rotation with direction depending on the helicity of the wave.

Finally, if the rotation we observe was due to some thermally mediated torque, in order that angular momentum be conserved, it would be necessary that the surrounding liquid circulate in the opposite direction to the particle. However, our observations of the motion of nearby particles clearly eliminates this possibility. We see nearby particles swept around in the same direction as the rotating particle, consistent with a picture of an externally driven trapped particle stirring the surrounding liquid.

In conclusion, it has been demonstrated that absorptive particles trapped in the dark central minimum of a doughnut laser beam are set into rotation. The rotational motion of the particles is caused by the transfer of angular momentum carried by the photons. Since the laser beam is linearly polarized, this must originate in the "orbital" angular momentum associated with the helical wave-front structure and central phase singularity. We have shown that the direction of the rotational motion is determined by the chirality of the helical wave front. With a laser power of a few milliwatts, the rotation speed of the particles lies between 1 and 10 Hz depending on their sizes and shapes. This is in agreement with a simple model of absorption of the angular momentum of the radiation field.

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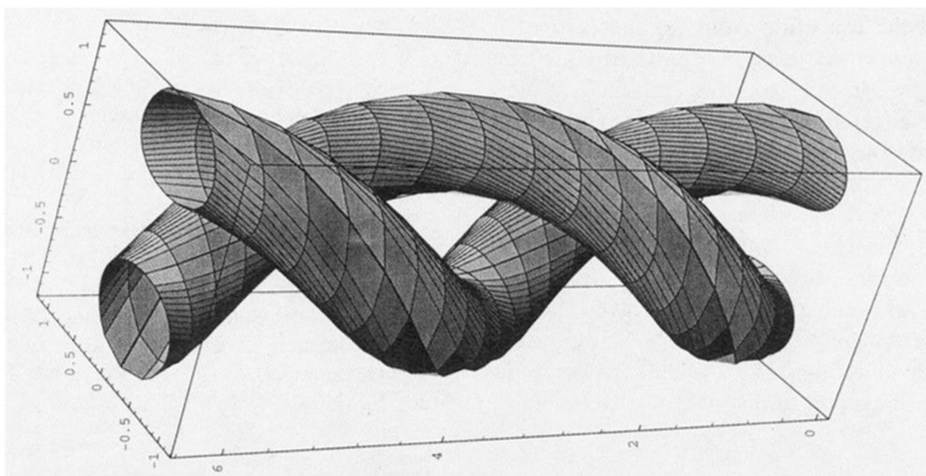
[1] R.A. Beth, Phys. Rev. **50**, 115 (1936).
[2] E. Santamato, B. Daino, M. Romagnoli, M. Settemlre, and Y.R. Shen, Phys. Rev. Lett. **57**, 2433 (1986).
[3] S. Chang and S. S. Lee, J. Opt. Soc. Am. B **2**, 1853 (1985).
[4] A. Ashkin, Science **210**, 4474 (1980); **210**, 1081 (1980).
[5] N.R. Heckenberg, R.G. McDuff, C.P. Smith, H. Rubinsztein-Dunlop, and M.J. Wegerner, Opt. Quant. Electron. **24**, S951 (1992).
[6] L. Allen, M.W. Beijersbergen, R.J.C. Spreeuw, and J.P. Woerdman, Phys. Rev. A **45**, 8185 (1992).
[7] S.M. Barnett and L. Allen, Opt. Commun. **110**, 670 (1994).
[8] H. He, N.R. Heckenberg, and H. Rubinsztein-Dunlop, J. Mod. Opt. **42**, 217 (1995).
[9] D. Marcuse, *Light Transmission Optics* (Van Nostrand, New York, 1972).
[10] C. D'Helon, E.W. Dearden, H. Rubinsztein-Dunlop, and N.R. Heckenberg, J. Mod. Opt. **41**, 595 (1994).
[11] S. Oka, in *Rheology,* edited by F.R. Eirich (Academic Press, New York, 1960), Vol. 3.
[12] L.R. Eaton and S.L. Neste, AIAA J. **17**, 261 (1979).



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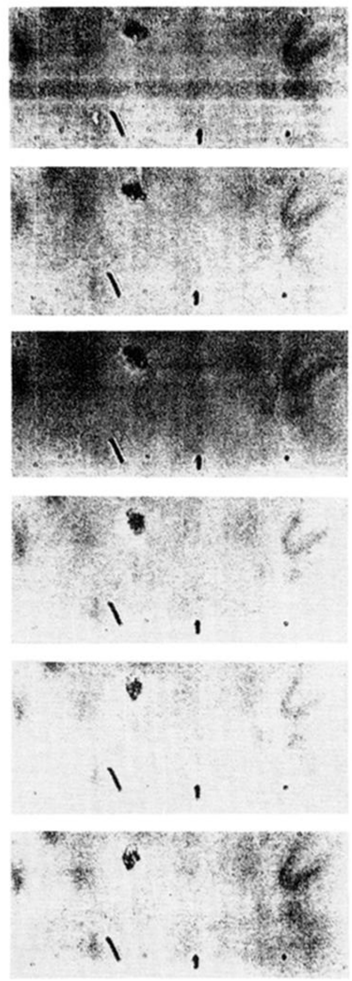

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