

Bubble Wall Velocity in a First Order Electroweak Phase Transition

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We calculate the velocity and thickness of a bubble wall at the electroweak phase transition in the minimal standard model. We model the wall with semiclassical equations of motion and show that friction arises from the deviation of massive particle populations from thermal equilibrium. We treat these with Boltzmann equations in a fluid approximation. Our technique allows us to account for particle transport, and to determine the wall thickness dynamically. We find that the wall is significantly thicker than at phase equilibrium, and that the velocity is fairly high, $v_w \approx 0.6c$ for $m_H = 50$ GeV.

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There has been a growing interest in the idea that the baryon asymmetry of the Universe may be created at a first order electroweak phase transition. However, ingredients needed to construct the whole picture are still missing. The exact nature of the Higgs mechanism is unknown, and the simplest model, the minimal standard model, apparently does not contain sufficient CP violation for baryogenesis; we must consider extensions, such as the two Higgs model. Even in the minimal standard model we have difficulties computing the finite temperature effective potential (which is needed to determine the strength of the phase transition) and the dynamics of the transition.

Recently there have been advances in calculating the effective potential; the two loop contribution has been evaluated [1,2], and there has been progress in understanding nonperturbative effects [3–5]. Both results support the view that the phase transition is first order and strong enough to proceed by bubble nucleation and growth (even when the Higgs is moderately large, $m_H \sim m_W$).

To model baryogenesis accurately one also needs to know the profile and velocity of an expanding bubble wall. The wall velocity is friction limited, but determining the strength of friction involves determining the nonequilibrium populations of massive particles in the vicinity of the wall, which is difficult [6–8]. The recent discovery that the top quark is very heavy [9] suggests that top quarks may be the dominant source of friction, in which case an approximation that models top quarks with good accuracy should improve our understanding of the wall motion. In this Letter we reanalyze the bubble wall's velocity and its shape, working in the minimal standard model and using the fluid approximation to model the particle populations, which should treat fermions fairly well. The technique also allows us to account for transport and to determine the wall thickness dynamically (taking a specific *Ansatz* for the wall profile).

We intend to study the dynamics of infrared condensates in the Higgs field Φ . Such condensates should be

have semiclassically to a good approximation. From the terms in the electroweak Lagrangian containing Φ ,

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - V(\Phi^\dagger \Phi) - \sum y(\Phi^\dagger \bar{\psi}_R \psi_L + \Phi \bar{\psi}_L \psi_R)$$

(where the sum runs over the quarks and leptons ψ , and y denotes the Yukawa coupling), we find the equations of motion for ϕ (where $\Phi^\dagger = [0, \phi/\sqrt{2}]$) to be

$$\square \phi + V'(\phi) - \frac{g_w^2}{4} \phi \text{Tr} A^2 + \sum \frac{y}{\sqrt{2}} \bar{\psi} \psi = 0, \quad (1)$$

plus terms linear in A . Here A and ψ are quantum operators. (For simplicity we have set the Weinberg angle $\tan \theta_W = 0$.) It is reasonable to take thermal averages of these operators using WKB wave functions. This is because the wall will be much thicker than the thermal length T^{-1} , which characterizes the reciprocal momenta of particles in the plasma. In this approximation we get

$$\square \phi + V'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3 k}{(2\pi)^3 2E} f(k, x) = 0, \quad (2)$$

where V is the renormalized vacuum potential, f is the phase space population density (in the background of a propagating wall), and the sum includes all massive physical degrees of freedom. Note the condensed notation $m = y\phi/\sqrt{2}$ for quarks and leptons and $m = g_w \phi/2$ for the gauge fields.

We write the population density f as the equilibrium population f_0 plus a deviation $f = f_0 + \delta f$. The vacuum contribution $V'(\phi)$ and the contribution from f_0 combine to give the finite temperature effective potential $V_T'(\phi)$. Thus we have [10]

$$\square \phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3 p}{(2\pi)^3 2E} \delta f(p, x) = 0. \quad (3)$$

We see that the friction force arises due to the departure from thermal equilibrium δf . We will use this equation,

an expression for V_T , and equations for δf to compute the wall velocity and its shape once it has reached a planar steady state $\phi = \phi(z + v_w t)$, $\delta f = \delta f(z + v_w t)$.

The high temperature expansion of the one loop effective potential is [7]

$$V_T(\phi) = D(T^2 - T_0^2)\phi^2 - E\phi^3 T + (\lambda_T/4)\phi^4, \quad (4)$$

with $D = (2m_t^2 + 2m_W^2 + m_Z^2)/8v_0^2 \approx 0.167$, $E = (2m_W^3 + m_Z^3)/4\pi v_0^3 \approx 0.01$, λ_T the Higgs self-coupling at a scale roughly given by T , and $T_0 = [m_H^2 + 3(4m_t^4 - 2m_W^4 - m_Z^4)/8\pi^2 v_0^2]/4D$. Recently, the authors of [1] and [2] have computed the two loop expression. The most important changes are that E becomes $[4m_W^3 + 2m_Z^3 + 3(1 + \sqrt{3})\lambda^{3/2}]/12\pi v_0^3$ and a qualitatively new term $-B\phi^2 T^2 \ln(\phi/T)$ appears. The result of [2] is $B \approx (1.42g_w^4 + 4.8g_w^2\lambda - 6\lambda^2)/16\pi^2$. It may also be important to include nonperturbative effects. Shaposhnikov proposes a term $-(A_f g_w^6 T^4/12) \text{Pit}(\phi)$ [3]. The function Pit describes the contribution of a gauge condensate and it is roughly constant near $\phi = 0$ and falls exponentially as $\exp(-\phi/g^2 T)$ for large ϕ . We add such a term to parametrize our ignorance about the free energy of the symmetric phase; we use $-(A_f g_w^6 T^4/12) \text{sech}(\phi/\alpha_w T)$ because it is simple—the exact form of Pit will have little effect on our calculations—and we take λ_T and A_f as unknowns.

What should we use for T ? Most of space is converted to the broken phase by bubbles which nucleate when the critical bubble free energy reaches $S \approx 100T$ [11]. We have computed S using our form for V_T and the techniques used in [6–8] and find, for instance, that for $\lambda_T = 0.03$, $A_f = 0$, and $S = 100T$ that $T = 0.99980T_0$. (Note that because of the B term, T_0 is no longer the spinodal temperature.)

Next, we must determine δf , the deviation from equilibrium in the presence of the moving wall. Our starting point is the Boltzmann equation

$$\partial_t f + \frac{p_z}{E} \partial_z f + \dot{p}_z \partial_{p_z} f = -C[f], \quad (5)$$

where $C[f]$ represents the scattering integral, $E = (p^2 + m^2)^{1/2}$ is the particle energy, $v_z = p_z/E$ is the velocity (with the broken phase at positive z), and $\dot{p}_z = -\partial_z E$ is the force on the particle. In a complete description

each particle species in the plasma would be described with a Boltzmann equation. We allow ourselves the approximation that all species but top quarks and perhaps W bosons are in equilibrium. This is reasonable as the induced deviation from equilibrium goes as m^2 , and top quarks and W bosons are the heaviest particles. We also use a single f for tops and antitops of both helicity. In the minimal standard model this is reasonable as there is almost no CP violation, and the difference in transport properties arises only at the subleading level of weak scatterings. For simplicity we will also neglect the change in the background temperature due to the liberation of latent heat. This approximation may be weaker [12,13], but including this change would significantly complicate our analysis. We intend to return to this problem in [14].

The Boltzmann equations are nonlinear partial integro-differential relations and as such are analytically intractable. To solve them we make the fluid *Ansatz* assuming f to be of the form

$$f^{-1} = 1 + \exp \frac{E - E\delta T/T - p_z v - \mu}{T}, \quad (6)$$

where we have written explicitly three types of perturbations: chemical potential μ , temperature δT , and velocity v . This *Ansatz* is a truncation of an expansion in powers of momentum; it gives a reasonable description of the populations of thermal energy particles when the background varies slowly on the scale of the diffusion length. For top quarks this should be sufficient as the diffusion length is short and the influence of infrared particles is phase space suppressed. For W bosons, Bose statistics give large infrared particle populations, and the fluid approximation is unreliable unless W bosons thermalize on time scales short compared to their annihilation rate. We consider the fluid equations for W bosons as a guide for their importance and concentrate on top quarks. We will, furthermore, work to linear order in perturbations which are of order $(m/\pi T)^2$, and, therefore, naturally small.

With a three parameter *Ansatz* [6] we cannot ask that the full Boltzmann equations be satisfied, but only impose that three moments be satisfied, namely, the integrals over $\int d^3 p/(2\pi)^3$, $\int E d^3 p/(2\pi)^3$, and $\int p_z d^3 p/(2\pi)^3$. Working in the fluid frame and using $\partial_t f(z + v_w t) = v_w f'$, we obtain the following equations [15]:

$$\begin{aligned} v_w c_2 \mu' + v_w c_3 \delta T' + \frac{c_3 T}{3} v' + \Gamma_{\mu 1} \mu + \Gamma_{T 1} \delta T &= \frac{v_w c_1 (m^2)'}{2T}, \\ v_w c_3 \mu' + v_w c_4 \delta T' + \frac{c_4 T}{3} v' + \Gamma_{\mu 2} \mu + \Gamma_{T 2} \delta T &= \frac{v_w c_2 (m^2)'}{2T}, \\ \frac{c_3}{3} \mu' + \frac{c_4}{3} \delta T' + \frac{v_w c_4 T}{3} v' + \Gamma_v v T &= 0, \end{aligned} \quad (7)$$

where we have defined

$$c_i T^{i+1} \equiv \int E^{i-2} \frac{\exp(E/T)}{[\exp(E/T) + 1]^2} \frac{d^3 p}{(2\pi)^3}. \quad (8)$$

To lowest order in $(m/T)^2$ they are $c_1 = \ln 2/2\pi^2$, $c_i = i!(1 - 2^{1-i})\zeta_i/2\pi^2$, where ζ_i is the Riemann zeta function at i .

The Γ result from integrating over $C[f]$. We compute them in [14] to order $\alpha_s^2 \ln \alpha_s$. Using $\alpha_s = 0.12$, we find $\Gamma_{\mu 1} \approx 0.010$, $\Gamma_{T1} \approx 0.019$, $\Gamma_{\mu 2} \approx 0.019$, $\Gamma_{T2} \approx 0.082$, and $\Gamma_{v1} \approx 0.042$. The typical lifetime of a top quark before it annihilates against an antitop is $\sim 2c_2/\Gamma_{\mu 1} = 16/T$, and the typical time before it has suffered enough small angle collisions to randomize its direction is $\sim c_4/3\Gamma_v = 9/T$. For W bosons the Γ are about half as large.

The terms on the right-hand side of (7), which drive the plasma out of equilibrium, are proportional to m^2 , so in general $\delta f \propto m^2$. Note that the friction term in Eq. (3) is proportional to $m^2 \delta f \propto m^4$. For bosons the coefficients in Eqs. (7) differ, and, in particular, $c_1 \propto \ln(2/m)$. The integral in the equation of motion involving μ also contains a log enhancement, but because m_t^4/m_w^4 is very large, W bosons still produce less friction than top quarks.

With our *Ansatz* for f we can rewrite Eq. (3) for top quarks (with $\sum \rightarrow 12$) as

$$-(1 - v_w^2)\phi'' + V_T'(\phi) + 12T \frac{dm^2}{d\phi} \frac{c_1\mu + c_2\delta T}{2} = 0. \quad (9)$$

This equation and the fluid equations form a system of nonlinear differential equations for the wall profile and velocity. We will attempt to solve them in [14], but here we will content ourselves with an *Ansatz* for ϕ ,

$$\phi = \frac{\phi_0}{2} \left(1 + \tanh \frac{z + v_w t}{L} \right), \quad (10)$$

where ϕ_0 is the value of ϕ in the asymmetric phase and v_w the wall velocity and L the wall thickness in the plasma frame are treated as undetermined parameters. This *Ansatz* is chosen because the static equilibrium wall shape in the one loop approximation is of this form.

Again, because we have restricted the form of ϕ we cannot ask that the full equations of motion be satisfied; we can only enforce two moments. The natural choices are the space integral of the equation of motion times $\partial\phi/\partial v_w$ and $\partial\phi/\partial L$. Note that $\partial\phi/\partial v_w = t\phi'$ and $\partial\phi/\partial L = -(x/L)\phi'$, so an equivalent set of conditions is

$$\int [\text{Eq. (9)}] \phi' dz = 0, \quad \int [\text{Eq. (9)}] \frac{x}{L} \phi' dz = 0. \quad (11)$$

These equations have a simple physical interpretation. The first equation is the total pressure on the wall in its rest frame [6]; if it were nonzero, the wall would accelerate, changing v_w . The second equation is the asymmetry in the pressure between the front and back edges of the wall; if it were nonzero, the wall would be compressed or stretched, changing L .

The integrals for the first two terms in (9) are

$$\int [\square\phi + V_T'(\phi)] \phi' = V_T(\phi_0) = V_T(0) \equiv -\Delta V_T, \quad (12)$$

$$\int [\square\phi + V_T'(\phi)] \frac{z}{L} \phi' = \frac{(1 - v_w^2)\phi_0^2}{6L^2} - \frac{1}{2} [\Delta V_T + \Xi], \quad (13)$$

$$\Xi \equiv B\phi_0^2(\zeta_2 - 1)T^2 + \frac{E\phi_0^3 T}{2} - \frac{5\lambda_T \phi_0^4}{24} + \frac{A_f g_w^6 T^4}{12} \left(2.79 + \frac{1}{2} \ln \frac{\phi_0}{T} \right). \quad (14)$$

Note that the $\square\phi$ term acts to stretch the wall (increase L) while V_T acts to accelerate and compress the wall. The coefficient 2.79 in the last term is the only place where our choice for the function Pit enters our computation.

We will first get a rough estimate of the wall velocity and thickness by solving Eqs. (7) and (11), ignoring transport, by which we mean we will ignore the derivative terms on the left-hand side of the fluid equations. Transport reduces friction because particles tend to flow off the wall, where they contribute less to Eq. (11). We will also ignore δT , which turns out to be a good approximation. The expression for μ now becomes rather simple:

$$\mu = \frac{v_w c_1 y_t^2 \phi \phi'}{2\Gamma_{\mu 1} T}. \quad (15)$$

Note that μ does not depend on other decay rates apart from $\Gamma_{\mu 1}$.

Using $\int (\phi \phi')^2 = \phi_0^4/10L$, $\int x(\phi \phi')^2 = \phi_0^4/24$, and (12)–(14) one can solve for $L_w = L/(1 - v_w^2)^{1/2}$ and v_w :

$$\frac{1}{L_w^2} = \frac{1}{\phi_0^2} \left(\frac{\Delta V_T}{2} + 3\Xi \right), \quad \gamma_w v_w = \frac{5L_w \Gamma_{\mu 1} \Delta V_T}{6c_1^2 m_t^4}. \quad (16)$$

For $m_t = 174$ GeV, $\lambda_T = 0.03$, and $A_f = 0$ these give $L_w \approx 29T^{-1}$ and $\gamma_w v_w \approx 1.1$, a mildly relativistic and fairly thick wall. Note that $L \approx 20T^{-1}$ is much thicker than the top quark diffusion constant $D \approx 3T^{-1}$, so the fluid approximation is in good shape.

Now we solve the problem including transport. First, we must find the contributions to (11) involving μ and δT . This is most easily accomplished by Fourier analysis. Let us write Eqs. (7) in a matrix notation

$$A\delta' + \Gamma\delta = F\phi\phi', \quad (17)$$

where δ is a column vector $[\mu \delta T v T]^T$, A is the matrix of coefficients for the derivative terms, Γ is a matrix of the decay constants, and F is a column vector of the coefficients for the force terms. Note that A is velocity dependent, and F is linear in velocity. In Fourier space (17) becomes

$$ik\delta + A^{-1}\Gamma\delta = A^{-1}F\widetilde{\phi\phi'}, \quad (18)$$

which may be solved by eigenvalue methods. Denoting the eigenvalues and eigenvectors of $A^{-1}\Gamma$ as λ_i and ξ_i and expanding $A^{-1}F = \alpha_i \xi_i$, we find

$$\delta = \sum_i \frac{\alpha_i}{\lambda_i + ik} \xi_i \widetilde{\phi\phi'}, \quad (19)$$

$$\widetilde{\phi\phi'} = (\phi_0^2/2) \left(1 - \frac{ikL}{2}\right) \left(\frac{kL\pi}{2}\right) \operatorname{csch}\left(\frac{kL\pi}{2}\right). \quad (20)$$

This gives an explicit expression for $\widetilde{\mu}$ and $\widetilde{\delta T}$.

Finally, we can convert the relevant integrals of the friction term

$$6y_i^2 T \int [c_1 \mu(x) + c_2 \delta T(x)] \phi \phi'(x) dx, \quad (21)$$

and (21) with an x/L inserted into k -space integrals using $\int f_1(x)f_2(x) dx = \int \tilde{f}_1(-k)\tilde{f}_2(k) dk/2\pi$ and $xf(x) \Rightarrow id\tilde{f}(k)/dk$. This yields integrals of the form

$$\int \frac{1}{\lambda + ik} \left(1 + \frac{k^2 L^2}{4}\right) \left(\frac{kL\pi}{2} \operatorname{csch} \frac{kL\pi}{2}\right)^2 \frac{dk}{2\pi}, \quad (22)$$

which may be converted to a rapidly converging infinite sum by residue integration, or performed numerically.

This completes the evaluation of all terms in Eqs. (11). These equations each define a curve in the space of v_w and L . The intersection of these curves is a self-consistent solution for the wall shape and velocity within the *Ansätze* and approximations we have made.

We have solved these simultaneous conditions for some representative values of λ_T and A_f . We find that the friction from W bosons, calculated in the fluid approximation, is about half that from top quarks. Though our techniques are different than those of [8], we get a similar numerical value for the friction from W 's. We have included them in our analysis. We present the velocity and thickness of the wall in Table I. Here m_H is the physical Higgs mass computed from the one loop relations in [7].

The last two rows are the velocity and thickness of the wall when we treat v_w as a free parameter but fix L to the value derived from (13) without the contribution from friction. This value of L is slightly less than the thickness at phase equilibrium. We see that the wall is significantly deformed by the effects of friction, and that this increases its velocity. This is because the source for δf , which goes as $\phi\phi'$, peaks high on the wall, and so does the sensitivity of the wall to δf . Furthermore, at large wall velocities, the particles are swept further backward on the wall. Hence, most of the force from δf occurs near the back of the wall, stretching it out. Of course, if the deformation is large then we have little reason to believe that our wall shape *Ansatz* is accurate—

TABLE I. Wall velocity, etc., at several effective potential parameters.

| | | | | | |
|-----------------------|-------|------|------|------|-------------|
| λ_T | 0.023 | 0.03 | 0.05 | 0.05 | 0.03 |
| A_f | 0 | 0 | 0 | 0.1 | 0.1 |
| m_H (GeV) | 34 | 50 | 80 | 80 | 50 |
| ϕ_0/T | 0.98 | 0.80 | 0.58 | 0.79 | 1.04 |
| v_w | 0.78 | 0.62 | 0.58 | 0.97 | No solution |
| LT | 44 | 33 | 27 | 12 | No solution |
| $n_{\text{no str}}$ | 0.30 | 0.33 | 0.39 | 0.61 | 0.49 |
| $L_{\text{no str}} T$ | 15.9 | 16.5 | 16.6 | 8.0 | 9.7 |

one should model the shape more carefully than we have done here. The conclusion that the wall is fast and thick should be reliable, however.

When we include a sizable value of A_f , the parameter describing nonperturbative symmetric phase effects, we find no solution. The two Eqs. (11) turn out to be incompatible; the wall runs away, but maintains finite plasma frame thickness. This result probably comes from neglecting friction from the gauge condensate responsible for A_f , which would compress the wall and prevent runaway. To remedy this shortcoming we need a model for the nonequilibrium dynamics of nonperturbative infrared condensates.

The situation in two doublet models may be quite different from what we have found here. In these theories there are several new massive (Higgs) bosons. The ones which do not couple to the top quark have quite long half-lives and sizable diffusion constants, and may be a major source of friction.

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