Energy Localization in Nonlinear Fiber Arrays: Collapse-Effect Compressor

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We analyze a collapse mechanism of energy localization in nonlinear fiber arrays. The nonlinear fiber array is suggested as a device to amplify and compress optical pulses. Pulse propagation in one-dimensional fiber arrays has features of collapse (self-focusing) dynamics. Collapse-type compression leads to the localization of all energy initially dispersed in array into a few fibers. Numerical simulations demonstrate the robustness of the suggested compression mechanism.

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The potential of achieving very high bit rates of fiber transmission lines has motivated development of new devices for processing, storage, and generation of ultrashort optical signals. Waveguide nonlinear couplers and nonlinear fiber arrays have been studied extensively in recent years because of their potential in all-optical signal processing (see, e.g., [1–4]). New effective techniques were described for producing multiport coupler devices [1,5]. Multiwaveguide couplers as well as fiber arrays demonstrate sharper switching characteristics and greater sensitivity to the input signal in comparison with two-mode systems. Recent investigations of the short optical pulse propagation in nonlinear fiber arrays (NFA) revealed novel interesting features of such devices. Stable multidimensional solitons have been discovered that may be of interest for all-optical information processing [4]. Particlelike dynamics of such stable structures makes them very attractive for efficient ultrafast light switching gates in time division multiplexed fiber devices. Solitons can switch nearly entirely or not at all from one channel to others. In this Letter we demonstrate that the NFA can be used also for *optical pulse compression*.

The basic model describing a pulse evolution in the NFA is an interesting (rather generic) example of continuous-discrete nonlinear equations combining features of the discrete models and continuum nonlinear systems. Continuous-discrete systems have been attracting attention recently in rather different physical problems, such as fluxon dynamics in the long Josephson junction arrays [6], semiconductor laser arrays [7], and others. Wave dynamics in NFA is a result of the interplay between nonlinearity, dispersion, and discreteness. It has been shown recently that the discreteness can modify significantly stability properties of the solitons and collapse phenomenon [8,9] in comparison with continuum models. Therefore, the investigation of a pulse dynamics in nonlinear fiber arrays is both of importance for applications and of interest as a fundamental physical problem.

In this Letter we demonstrate that the so-called collapse effect plays the role of the *energy localization mechanism* in NFA. Understanding this mechanism leads us to suggest the use of nonlinear fiber arrays for optical pulse compression and amplification.

It is well established that optical pulse compression may be achieved by nonlinear effects (see, e.g., [10] and references therein). There are two main types of devices commonly used for optical pulse compression. The first type is the fiber-grating compressor that consists of an optical fiber followed by a grating pair [11]. Compression via grating pair exploits the difference in path lengths for different frequencies contained in the pulse. In fact, such devices make use of a linear compression mechanism. A specifically chirped pulse passing through a dispersive delay line becomes narrower. The role of a nonlinear fiber is simply to create a chirp on the pulse as a result of the interplay between self-phase modulation and dispersion. The second type of compressor is based on the soliton properties of a nonlinear medium. In 1983 Mollenauer *et al.* [12] have demonstrated in a classic experiment how a two-soliton pulse dynamics in fibers can be used for the compression of optical pulses. This method is based on the principle of *stable soliton propagation* in optical fibers.

We would like to emphasize that stable soliton propagation is not the unique type of behavior of nonlinear systems. Solitons may be unstable and the features of a system with unstable solitons differ drastically from that in the stable case. There are, in fact, two main types of wave dynamics in a nonlinear medium, namely, *stable*

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soliton dynamics and the so-called *wave collapse*. Wave collapse is a process of finite-time energy localization into small scales where additional physical effects (for instance, the nonlinearity saturation or damping) stop collapse (see, e.g., for a review [13–15]). Wave collapse has been suggested recently as a mechanism of the optical pulse compression [16].

Wave collapse typically occurs in multidimensional nonlinear systems (see, e.g., [14]). The well-known example in optics is self-focusing of light beams in a nonlinear medium. Self-focusing is described by the two- (in the stationary case) and three-dimensional (in the nonstationary case) nonlinear Schrödinger equation (NLSE). On the other hand, it is known that the one-dimensional NLSE describing pulse propagation in optical fibers has stable solitons and collapse does not occur.

The system of equations to be analyzed here consists of a set of linearly coupled 1D NLS equations and combines features of one- and two-dimensional NLSEs. As in the single fiber dynamics, there is temporal dispersion, but now there is also a discrete variable and a discrete dispersive term which arise due to the weak coupling between fibers. This additional feature is important because it makes possible a redistribution of energy among the fibers in the array in addition to the evolution of the shape of the pulse in each fiber. We show that the stable, quasicollapse energy localization phenomenon occurs only by the coexistence of *all* the dominant effects in the dynamics: nonlinearity, temporal dispersion, and linear coupling. In this sense, discreteness plays the role of effective saturation of nonlinearity, making singularity formation impossible in contrast to the continuum models. This is the principal difference between collapse dynamics in the continuum and discrete systems. The final state of quasicollapse is a highly compressed and amplified pulse propagating in a predetermined fiber, plus two satellite pulses in neighbor fibers [4].

The nonlinear dispersive propagation of light in an array of linearly coupled optical fibers can be described by a Hamiltonian system of equations, in the dimensionless variables taking the form

$$i\partial_z A_n = -A_{n+1} + 2A_n - A_{n-1} - A_{ntt} - 2|A_n|^2 A_n = \frac{\delta H}{\delta A_n^*}, \quad (1)$$

where the Hamiltonian $H = \sum_n(\int |A_n - A_{n-1}|^2\,dt + \int |A_{nt}|^2\,dt - \int |A_n|^4\,dt)$. In addition to H, Eq. (1) conserves total energy: $P = \sum_n \int |A_n|^2\,dt$ and momentum in t, $M_t = i\sum_n \int(A_n A_{nt}^* - A_n^* A_{nt}\,dt)$. These quantities play an important role in the dynamics and, in particular, P allows one to estimate compression rates for a wide range of input signals and the invariance of M_t accounts for the same Galilean invariance of the 1D and 2D NLSE.

Before we discuss why energy localization takes place in Eq. (1), let us notice that a particular solution describing condensation of all energy in one fiber can be found even in a linear problem. For instance, if we drop the nonlinear and dispersion terms in Eq. (1), the resulting equation $i\partial_z A_n + A_{n+1} - 2A_n + A_{n-1} = 0$ governs an optical pulse dynamics in the linear fiber array. In this case, an exact solution can be found (compare with [17]) having the form

$$A_n(z) = J_n(2(z_0 - z))\exp(-i2z - in\pi/2). \quad (2)$$

Here $J_n(x)$ is the Bessel function of nth order. Because of the properties of Bessel functions [$J_n(x) \rightarrow (1/n!)(x/2)^n$ as $x \rightarrow 0$] this particular solution describes energy concentration in the central ($n = 0$) fiber at the finite distance z_0. It can be shown, however, that in the linear problem the typical variant of dynamics is a redistribution of the energy among all fibers. This corresponds to the pulse broadening in a dispersive medium. This is not the case in the NFA, where energy localization is a typical variant of the evolution of most initial wave field distributions.

To understand why this is the case, if we consider broad initial field distributions and introduce a coordinate x along the array (perpendicular to the direction of propagation in the fibers), then by using the simplest continuum approximation of Eq. (1) we obtain

$$iA_z + A_{xx} + A_{tt} + 2|A|^2 A = 0. \quad (3)$$

This equation coincides with the NLS equation that describes, for example, light self-focusing. The integrals of motion mentioned above have continuous analogs. In particular, $H = \int(|A_x|^2 + |A_t|^2 - |A|^4)\,dx\,dt$.

For Eq. (3), a remarkable result is known [18]:

$$\partial_z^2 \int (x^2 + t^2)|A|^2\,dx\,dt = 8H. \quad (4)$$

It means that if H is negative at the input, then $A(z,x,t)$ develops a singularity in a finite propagation distance. In the array, however, near the collapse point discreteness comes into play and compression stops. A multidimensional localized pulse is formed as a result of quasicollapse. Practically all the input energy is concentrated in one fiber as a pulse whose analytical form is well approximated by $A_0 = (1/\tau_{\text{out}})/\cosh(t/\tau_{\text{out}})$. We would like to point out that this stable multidimensional soliton can be used as an elementary information carrier in optical computing. The nonlinear fiber array may then be considered as a generator of these elementary objects.

Although the previous analysis helps us understand why one has collapse-type dynamics for broad initial conditions with $H < 0$, it does not give an answer on the question: "What initial conditions lead to the quasicollapse dynamics?" We should point out that, contrary to the continuum limit, for Eq. (1) an analytic result like the virial theorem [18] is not known, and, therefore, there is no exact criterion of quasicollapse and eventual compression. It has been shown recently [8,9] that the discreteness changes the necessary conditions of the collapse. To understand which initial conditions will be compressed to a multidimensional soliton, we have analyzed different input signals. Our results demonstrate that a fast energy localization in a few fibers is a *generic feature* of NFA and, as a result, quasicollapse is a very *robust method of pulse compression.* In fact, we observe two types of compression in numerical simulations: either formation of the pure multidimensional soliton or a breathing final state (compare with Ref. [8]), when the input signal compresses in few fibers and then energy oscillates between the main fiber and two neighbors.

In order to elucidate this dynamics we consider results of numerical simulations. In all simulations we have used periodic boundary conditions. First, we study the evolution of an input signal localized in both directions. More specifically, we have analyzed an array of 15 fibers with input pulses of the form $A_n(t) = A\exp(-t^2/\tau^2)\exp[-(n - n_{cen})^2/M^2]$ and $A_n = \lambda/\cosh(\lambda t)$ for $n = 6$–10 and zero for all other n. The criterion used to specify the input parameters A, τ, M, and λ is that (sufficient) of collapse in the continuum limit, namely, $H < 0$. In fact, quasicollapse can be observed even for $H > 0$ (compare [8,14]). The results presented in Fig. 1 illustrate that the continuum criterion of compression works very well for the broad distributions.

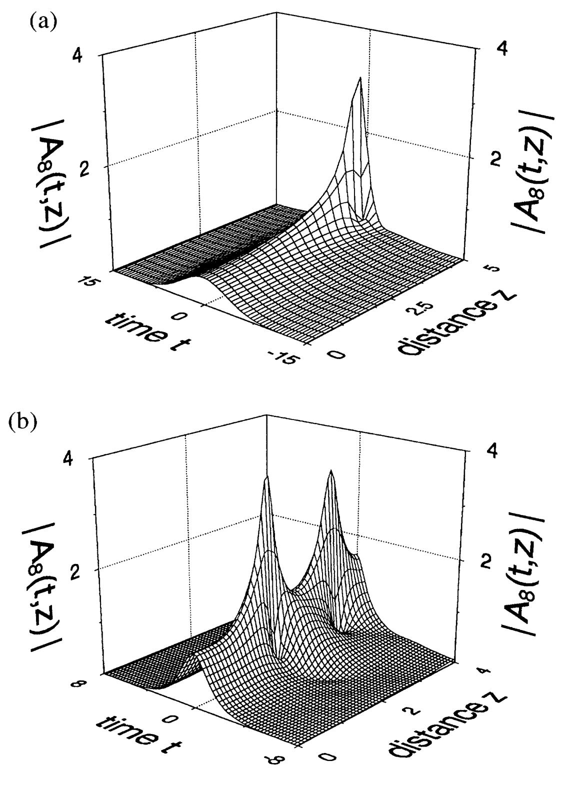

FIG. 1. Evolution of a two-dimensional initial distribution with $H < 0$. Numerical integration of Eq. (1), using the initial condition (a) $A_n(0,t) = A\exp(-t^2/\tau^2)\exp[-(n - n_{cen})^2/M^2]$; (b) $A_n = \lambda/\cosh(\lambda t)$ for $n = 6, 7, 8, 9, 10$, and zero for all other n in an array of 15 fibers; here the spatiotemporal evolution refers to the central fiber with $n = 8$.

A second type of relevant initial field distribution presents a set of similar pulses in each fiber. This is a rather natural input for a fiber array organized in a circle. We have considered input pulses having soliton form $A_n(t) = A\lambda/\cosh(\lambda t)$. Quasicollapse is a typical variant of the evolution in this case. There is a transversal instability (in the "n" direction) of the initial field distribution [19] resulting in a creation of clusters that evolve typically to the multidimensional soliton found in [4]. It should be pointed out that in contrast to the instability of the one-dimensional soliton against transversal modulation, in the discrete system small amplitude solitons are stable. There is a threshold in amplitude for instability. The instability results in pulse compression. The criterion of compression of initial field distribution is $\lambda > \lambda_{\rm cr}$ where $\lambda_{\rm cr}^2 = \frac{4}{3}\sin^2(\pi/M)$, M being the number of fibers in the array. If λ is above the threshold, but not too much, then only one lump forms. The second critical value of intensity is $\lambda_{\rm cr2}^2 = \frac{4}{3}\sin^2(2\pi/M)$, in the sense that if $\lambda > \lambda_{\rm cr2}$, two symmetrical lumps with opposite phases may appear. The compression factor that we define as the ratio of the width of the compressed pulse to that of the input pulse $F_c = \tau_{\rm in}/\tau_f$ may be found analytically for input signals described above. For soliton input pulses, at the input $P_{\rm in} = M2\lambda_{\rm in}$ here $\lambda_{\rm in}$ is the input soliton amplitude. After passing the fiber array, most of the energy will be concentrated in the main fiber, and we may use for estimation $P_f = 2\lambda_f$, here λ_f is the amplitude of the final soliton, hence $F_c = \lambda_f/\lambda_{\rm in} = M$. For Gaussian input pulses $[A_n(t) = A\exp(-t^2/\tau^2)]$ the compression factor depends on parameters of the initial distribution: $F_c = (M/2)\sqrt{\pi/2}\,A^2\tau_{\rm in}^2$. We find that our estimations are in a good agreement with the results of numerical simulations. Details will be presented in a future publication.

A third type of initial condition that leads to compression is a uniform background, $A_n = A$. In this case compression starts if A exceeds the critical amplitude $A_{\rm cr}^2 = 4\sin^2(\pi/M)$; here M is the number of fibers in the array. When A grows, perturbations with finite ω become unstable and 2D structure forms. In the next stage it collapses to a compressed state. It is important to mention also that if the amplitude does not exceed the next critical value, namely, $A_{\rm cr2}^2 = 4\sin^2(2\pi/M)$, then only one mode is unstable, leading to the creation of one soliton. By varying the amplitude of the initial signal, in principle, one may produce several solitons. It is possible to select the fiber in which the final state will appear; this is done by varying the phase of the input perturbation so that the brightest field intensity coincides with the location of the desired fiber.

The compression rate may be increased substantially, if a two-dimensional nonlinear fiber array is used instead. The model equation describing pulse evolution in 2D

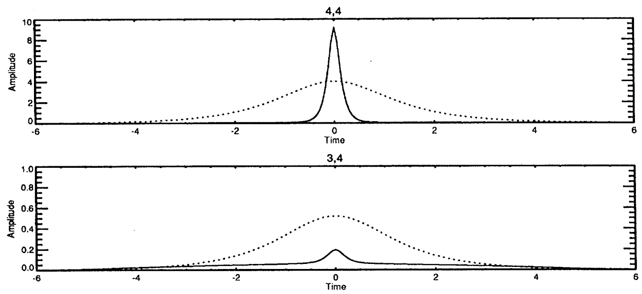

FIG. 2. Pulse dynamics in 2D NFA. Initial (dashed lines) and final (solid lines) pulse profiles in a two-dimensional fiber array, for the fibers with indices (n, m) equal to (4,4) (top figure); (3,4) (middle figure). The overall solution is symmetric with respect to that at (4,4) where the compressed and amplified pulse comes out.

NFA may be written in the form

$$i\partial_z A_{n,m} + A_{n+1,m} + A_{n-1,m} + A_{n,m+1} + A_{n,m-1} - 4A_{n,m} + \partial_t^2 A_{n,m} + 2|A_{n,m}|^2 A_{n,m} = 0\,. \quad (5)$$

It may be easily shown that the Hamiltonian of the two-dimensional discrete NLSE is bounded from below. This minimum is attained by a three-dimensional soliton, whose analytical form may be found by perturbation theory in the same way as in [4]. In the main fiber, the pulse profile is the same as in one-dimensional NFA, $A_0 = \lambda/\cosh(\lambda t)$, but the neighbors differ. The most important point that we would like to emphasize is that the compression factor in two-dimensional NFA is proportional to the square of M, the number of fibers in the array: $F_c \approx M^2$. Figure 2 shows the compression in 2D NFA. Again, this is a robust phenomenon, but a detailed analysis of overall pulse dynamics in two-dimensional arrays will be published elsewhere.

In conclusion, we have demonstrated by numerical modeling that nonlinear fiber arrays may be used for compression and amplification of optical pulses. We have found that it is a rather robust method that does not require precise adjustment of input pulses. Compression and amplification have been found for a wide range of input pulses. By means of modulation of initial pulses one may provide concentration of all energy in a specific fiber.

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