Suppressed and Induced Chaos by Near Resonant Perturbation of Bifurcations

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We present experimental results which demonstrate for the first time that the onset of chaos in a nonlinear system can be either suppressed or induced by the application of a perturbation signal which is near resonant to a subharmonic of the fundamental system frequency. The technique represents a feedback independent method for stabilizing or destabilizing chaotic orbits. Shifts in the onset of chaos are demonstrated for perturbation signals which are near resonant to the period-2 orbit and the period-4 orbit of the experimental system.

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Manipulation of chaotic motion of deterministic dynamical systems has received considerable attention recently. Controlling the chaotic response of a nonlinear dynamical system by stabilizing unstable periodic orbits of a chaotic attractor has been proposed [1] and experimentally demonstrated in various systems [2–7]. Such control of chaos algorithms has been successfully exploited to improve the output of a laser system [3], to control cardiac arrhythmias [4], to control chaotic behavior of a nervous system [5], and to tame chaos in various other systems [6]. Another interesting example of the manipulation of complex motion of a nonlinear dynamical system to generate beneficial results concerns synchronized chaotic systems [7]. Such synchronized chaotic systems, which have been experimentally demonstrated [8], are expected to play a role in the area of private communications. In other words, manipulation of chaotic motion is not only of scientific interest, but has important practical ramifications as well.

In this work, we demonstrate a novel method for *suppressing* as well as *inducing* chaotic behavior in a nonlinear dynamical system. The approach is feedback independent and relies on the addition of a small periodic signal to the drive. More specifically, the approach exploits the effects of near-resonant perturbation [9] on subharmonic bifurcations which appear as precursors to chaotic behavior in many nonlinear dynamical systems. It should be pointed out that suppressing chaotic behavior by adding a second periodic force to the nonlinear system has been explored in previous work [10–12]. However, in this work we add a second periodic force to the drive which is specifically near resonant to one of the subharmonics of the system. Typically, near-resonant perturbation of bifurcations can either suppress or induce the bifurcation [13]. By suppressing the bifurcation, we show that it becomes possible to delay the threshold of chaos, thereby increasing the parameter space in which stable periodic behavior can be observed. Similarly, by choosing an appropriate near-resonant perturbation, we discovered that it becomes possible to shift the threshold point such that chaotic behavior can be *induced* in a nonlinear dynamical system. The feedback and model independent approach described in this work could have important practical ramifications in fields ranging from biology to optics. For instance, the performance of a particular nonlinear system could be considered optimal if it operates in the periodic mode (e.g., solid state laser [3]), while another system may operate best in the chaotic mode (e.g., certain neural functions [5]). However, drifts in the system could place it in undesirable parameter space where system performance is nonoptimal. In this work, we show that by applying perturbations which are near resonant to subharmonic bifurcations, a *stabilizing* as well as a *destabilizing* shift of the chaotic threshold of a nonlinear dynamical system can be produced. In other words, chaotic threshold of a dynamical system can be suppressed or induced by the addition of a small near-resonant perturbation, potentially resulting in improved device or system performance.

The experiment involved measuring the dynamic strain response of a magnetically driven $Fe_{78}B_{13}S_9$ amorphous magnetostrictive ribbon (Metglas 2605S-2) using a fiber-optic Mach-Zehnder interferometer. A small portion (< 5 mm) of the ribbon (50 mm × 12 mm × 25 μm) was bonded to the optical fiber comprising one arm of the interferometer. The phase shift of light propagating in the fiber attached to the ribbon is a direct measure of strain in the ribbon. The interferometer was contained in a solenoid which was driven by a two channel frequency synthesizer (HP 3326A), providing a longitudinal magnetic field $H = H_{dc} + h_0 \cos 2\pi f_0 t$, where H_{dc} is the applied dc field and h_0 is the amplitude of the sinusoidally varying field. The perturbing signal, $h_1 \cos 2\pi f_1 t$, where h_1 and f_1 are the amplitude and the frequency, respectively, of the perturbing signal, was added with the second channel of the synthesizer. The output time series of the strain response was digitized with a high speed digitizer (LeCroy 6810) for computing power spectral densities and reconstructing flow and Poincaré diagrams. The amplitudes of the applied magnetic fields were obtained by measuring the voltage drop across a 1 Ω resistor in series with the solenoid. The experimental arrangement has been described in previous work [14].

The dc magnetic field, H_{dc}, acted as the bifurcation parameter for the unperturbed system. That is, for a particular bifurcation sequence, f_0 and h_0 were held fixed while

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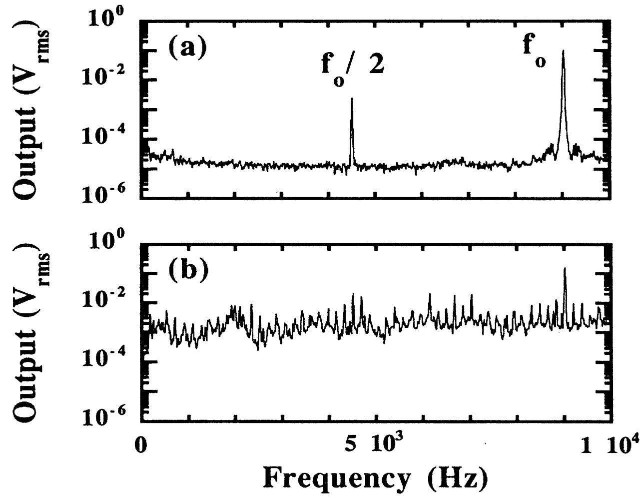

FIG. 1. Representative power spectra for system with $f_0 = 9.02$ kHz and $h_0 = 0.51\ \mathrm{Oe_{rms}}$. (a) Spectrum of period-2 orbit, for $H_{dc} = 0.75$ Oe. (b) Spectrum of chaotic orbit, for $H_{dc} = 1$ Oe.

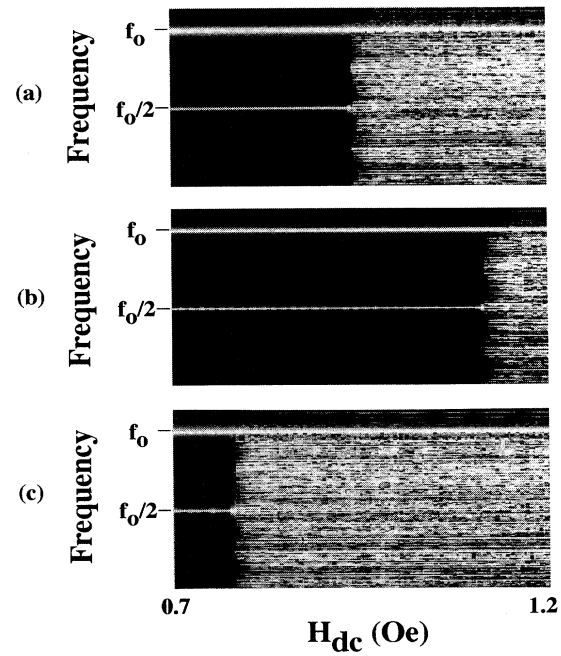

FIG. 2. Spectrograms showing shift in the bifurcation parameter (H_{dc}) for onset of chaotic behavior for three cases of near-resonant perturbation. (a) Unperturbed case, $h_1 = 0$. (b) Chaos suppressed for detuning $\Delta = 9 \times 10^{-4}$. (c) Chaos induced for detuning $\Delta = 6 \times 10^{-5}$. $h_1 = 0.1$ Oe for (b) and (c).

H_{dc} was varied. Different routes to chaos were observed depending on the values of f_0 and h_0. For instance, we were able to observe subcritical and supercritical period-2 bifurcations, Hopf bifurcations, and period-4 and period-8 bifurcations. Figures 1(a) and 1(b) show typical power spectra at two different values of H_{dc}, depicting period-2 and chaotic output, respectively. For the data of Fig. 1, the drive frequency $f_0 = 9.02$ kHz and the amplitude of the ac pump magnetic field $h_0 = 0.5\ \mathrm{Oe_{rms}}$.

In order to observe the effects of near-resonant perturbation on the threshold of chaos, we have adapted the spectrogram [15,16] display technique, commonly used in processing speech and sonar signals, to our experimental data. In general, a spectrogram is a sequence of Fourier transforms, each taken over a finite window of the time series. The transform window is swept in time over the length of the time series to create the two-dimensional plot, where the horizontal axis corresponds to the location (generally in time, but in this case H_{dc}) of the window function. Shading is used to represent the amplitude of the power spectrum at a particular frequency and time. In this manner, spectral changes in nonstationary processes can be easily observed. The resulting spectrogram acts as a type of bifurcation diagram for our system. Figure 2(a) shows a typical example of a spectrogram where the system is depicted to be undergoing a period-2 to chaos transition as H_{dc} is adiabatically ramped. The route to chaos clearly showed that the system transitions from period-1 $\rightarrow$ period-2 $\rightarrow$ chaos. The transition from period-1 to period-2 was observed to be "soft" or supercritical, while the transition from period-2 orbit to chaos was abrupt.

We now describe the effects of adding a near-resonant perturbation, $h_1 \cos 2\pi f_1 t$, to the system on the threshold of chaos. A near-resonant perturbation can be defined as that signal whose frequency f_1 is close to the frequency of one of the subharmonics of the system. For instance, f_1 can be chosen to be near resonant to the period doubling frequency $f_0/2$, such that a detuning frequency can be defined as $\delta = |f_1 - f_0/2|$. In this work, the near-resonant signal has small detuning, $\Delta \leq 10^{-3}$, where $\Delta = \delta/f_0$. Typically, the effect of a near-resonant perturbation on a supercritical bifurcation is to suppress the bifurcation, shift the bifurcation point, and stabilize the behavior at the original bifurcation point [9,13]. The shift has been found to increase with smaller detunings and larger perturbation amplitude. We added a near-resonant perturbation (near resonant with respect to $f_0/2$) with amplitude $h_1 = 0.1$ Oe to the drive. As expected, the effect of the near-resonant perturbation was to shift the period doubling bifurcation point. However, two novel and surprising effects not predicted by existing theories, nor previously observed in experimental systems, were observed: (i) For detuning frequencies $10^{-4} \leq \Delta \leq 10^{-3}$, the effect of the near-resonant perturbation was to not only shift the period doubling bifurcation but to shift the threshold of chaotic behavior in such a way as to stabilize the global behavior of the system. This is clearly depicted in the spectrogram of Fig. 2(b) where the threshold of chaotic behavior is seen to be suppressed with respect to the behavior of the system with no applied near-resonant

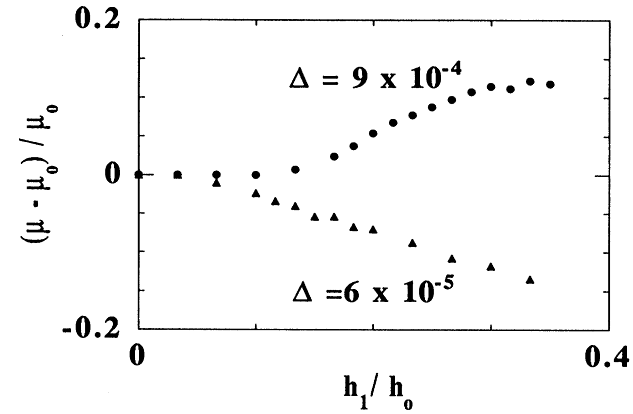

FIG. 3. Normalized shifted threshold of chaos $[(\mu - \mu_0)/\mu_0]$ versus relative amplitude of near-resonant perturbation (h_1/h_0), for frequency detunings $\Delta = 9 \times 10^{-4}$ and $\Delta = 6 \times 10^{-5}$.

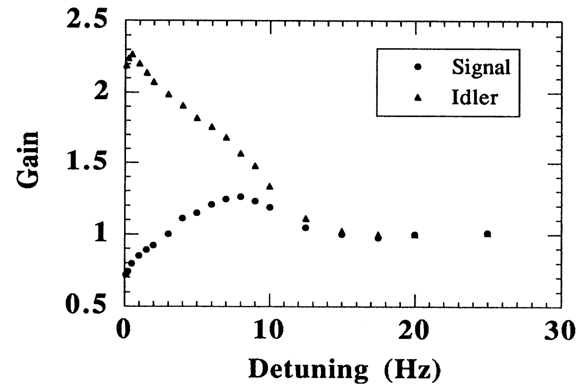

FIG. 4. Gain of signal and idler versus detuning frequency of perturbation. Relative perturbation amplitude $h_1/h_0 = 0.35$.

perturbation [i.e., Fig. 2(a), $h_1 = 0$]. (ii) For $\Delta \leq 10^{-4}$ the same amplitude near-resonant perturbation created a completely different behavior, such that the near-resonant perturbation was found to *induce* chaotic behavior. This phenomenon is depicted in the spectrogram of Fig. 2(c) (where $\Delta = 6 \times 10^{-5}$), which can be compared to the unperturbed case [Fig. 2(a)].

The novel and interesting aspects of the above results are as follows. We expect that a perturbation which is near resonant to a period doubling bifurcation will shift the period doubling bifurcation point. However, to our knowledge, this is the first result which shows that the detuning frequency of such a near-resonant perturbation can be used as a control parameter to *suppress* as well as *induce* chaos in a nonlinear dynamical system. That is, the data of Fig. 2 show that near-resonant perturbations tend to have a global effect on the system stability.

The amount of shift in the threshold of chaos is a function of both the detuning and amplitude of the near-resonant perturbation. Figure 3 shows the normalized shifted threshold of chaos as a function of the strength of the perturbation amplitude for two detunings. The normalized shifted threshold of chaos is simply defined as $(\mu - \mu_0)/\mu_0$, where μ_0 is the threshold parameter with zero applied perturbation (i.e., $h_1 = 0$) and μ is the shifted threshold parameter for $h_1 \neq 0$. It is clear from Fig. 3 that the shift in the threshold of chaos is positive, corresponding to suppression of chaos, for detuning $\Delta \approx 10^{-3}$. For smaller detuning, $\Delta \approx 10^{-4}$, the shift in threshold is negative, corresponding to induced chaos. For $10^{-4} \leq \Delta \leq 10^{-3}$, a smooth transition from suppressed to induced could be observed. The behavior of the shifted chaos threshold could be further characterized by monitoring the response of the near-resonant signal at f_1 as the system approaches the threshold of chaotic behavior. Previous understanding of the behavior of small signals in systems on the verge of an instability lead us to believe that the signal at f_1 will undergo net amplification as the system approaches the threshold of instability [17]. The theory of small signal amplification near a bifurcation also predicts that the net amplification of the signal, which in this case is the near-resonant perturbation signal, increases as its detuning frequency decreases, with the gain curve taking a Lorenztian shape [17]. The amplification effects due to the variation in the detuning frequency of the near-resonant signal in our system, which is biased close to but not at the threshold of chaos, are depicted in Fig. 4. For detuning frequencies $\delta > 15$ Hz the near-resonant signal experiences little amplification (gain ≈ 1) even though the system is close to an instability. For $5 \leq \delta \leq 15$ Hz the near-resonant signal undergoes a small net amplification (gain > 1), as expected. However, for $\delta \leq 5$ Hz the near-resonant signal undergoes a net *deamplification.* Such effects have been observed in the Lorenztian noise structure of parametric amplifiers operating near instabilities [18], but have not previously been observed in periodic signals. It is very interesting to note that the detuning frequency range in which the near-resonant signal undergoes amplification approximately

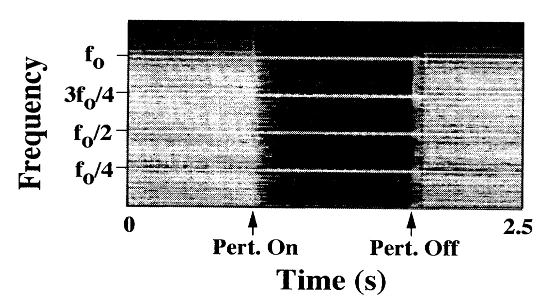

FIG. 5. Spectrogram showing suppression of chaotic behavior by application of perturbation near resonant to $f_0/4$. Perturbation is turned on at $t = 0.8$ s and is turned off at $t = 1.8$ s.

coincides with the range in which suppression of chaotic behavior was also observed [e.g., Fig. 2(b)], while the range in which deamplification takes place approximately coincides with the range in which induced chaos was observed [e.g., Fig. 2(c)].

The spectrogram of Fig. 5 shows that suppression of chaotic behavior by near-resonant perturbation of higher periodic orbits is also possible. For drive frequency, $f_0 = 9.54$ kHz, the system showed a period-4 bifurcation prior to transitioning to chaos. By applying a signal which is near resonant to the period-4 orbit, it was possible to suppress chaotic behavior. The effects of turning the near-resonant perturbation on/off on the response of the system operating in the chaotic regime ($f_0 = 9.54$ kHDz) are shown in Fig. 5. The system transitions to a period-4 state when a signal which is near resonant to the period-4 orbit is turned on and switches back to chaotic behavior when the near-resonant signal is turned off. It should be pointed out that once the near-resonant perturbation is turned on to suppress chaotic response, the system remains in the suppressed (periodic) state as long as the near-resonant perturbation is kept on. In other words, with the presence of a near-resonant signal, the system does not wander off in phase space toward any other attractor, showing the robustness of the technique.

As described earlier, the method has the potential to benefit systems in fields ranging from biology to optics. Nonlinear systems which prefer to operate in the chaotic or complex region of phase space, but drift off onto periodic attractors, could be made to remain in the chaotic region by the application of an appropriate perturbation to the system, while systems which have a tendency to drift away into chaotic regions of parameter space could be made to remain in periodic region by the application of a suitable near-resonant signal.

In conclusion, we have demonstrated for the first time that near-resonant perturbations affect the global stability of a nonlinear dynamical system. We have observed that near-resonant perturbations of subharmonic bifurcations can suppress as well as induce chaotic response in certain nonlinear dynamical systems. The experimental work could have important ramifications in numerous fields of study.

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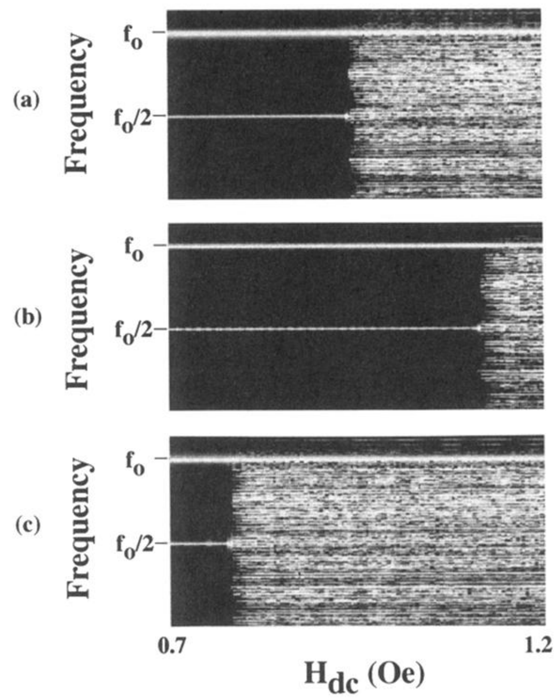

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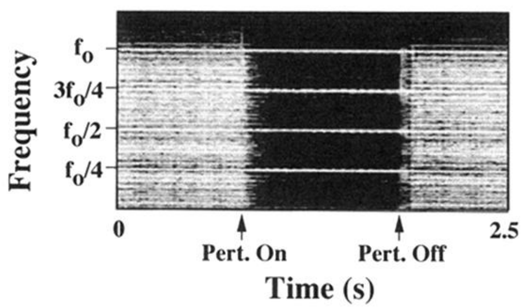

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