# Dirichlet Branes and Ramond-Ramond Charges

Joseph Polchinski*

*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030*
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We show that D-branes, extended objects defined by mixed Dirichlet-Neumann boundary conditions, break half the supersymmetries of the type II superstring and carry a complete set of electric and magnetic Ramond-Ramond charges. The product of the electric and magnetic charges is a single Dirac unit, and the quantum of charge is that required by string duality. This is strong evidence that D-branes are intrinsic to type II string theory and are the Ramond-Ramond sources needed for string duality. Also, we find in the IIa string a 9-form potential, which gives an effective cosmological constant.

The type II closed superstring has two kinds of gauge field, from the Neveu-Schwarz–Neveu-Schwarz (NSNS) and Ramond-Ramond (RR) sectors of the string Hilbert space [1]. The respective vertex operators are $j \bar\partial X^\mu A_\mu(X)$ and $\bar Q \Gamma^{[\mu_1} \dots \Gamma^{\mu_n]} Q F_{\mu_1 \dots \mu_n}(X)$. Here $j$ is a world-sheet weight $(1,0)$ current and $\bar Q_\alpha$ and $Q_\alpha$ are $(0,1)$ and $(1,0)$ spin fields, the world-sheet currents associated with space-time supersymmetry [2]. From the physical state conditions, $A_\mu(X)$ plays the role of a spacetime vector potential, while the physical state conditions for $F$ imply (in the notation of forms) $dF = d^*F = 0$. These are the Bianchi identity and field equation for an $n$-form field strength.

The NSNS and RR gauge fields are quite different in perturbation theory. String states carry the world-sheet charge associated with the current $j$, and this translates into a charge under the corresponding NSNS spacetime gauge symmetry. On the other hand, all string states are neutral under the RR symmetries because only the field strength $H$ appears in the vertex operator. Further, backgrounds with nontrivial NSNS gauge fields are well studied in conformal field theory, whereas backgrounds of RR gauge fields are not easily understood in this way: The spin fields depend on the ghosts, with the additional complication of picture changing, and they break the separate superconformal invariances of the matter and ghost theories.

One of the important lessons of string duality is that such world-sheet distinctions are artifacts of string perturbation theory, with no invariant significance. Various dualities interchange NSNS and RR states, and string duality requires that states carrying the various RR charges exist [3]. Previously it has been suggested that these are black $p$-branes, extended versions of black holes [4]. In this paper we will observe that there is another class of objects which carry the RR charges, the D(irichlet)-branes studied in Ref. [5].

Let us begin with a type II closed superstring theory. Add open strings with Neumann boundary conditions on $p + 1$ coordinates and Dirichlet conditions on the remaining $9 - p$,

$$n^a \partial_a X^\mu = 0, \quad \mu = 0, \dots, p,$$
$$X^\mu = 0, \quad \mu = p + 1, \dots, 9. \tag{1}$$

The open string end points thus live on a hyperplane, the D-brane, with $p$ spatial and one timelike dimensions. Only closed strings propagate in the bulk of spacetime, but sense the hyperplane through the usual open-closed interactions. This is a consistent string theory, provided $p$ is even in the IIa theory or odd in the IIb theory. The consistency conditions will be explained further below, but consistency can also be seen from the fact that these boundary conditions arise in the $T$ dual of the usual type I string theory [5,6].

One would not expect a perfectly rigid object in a theory with gravity, and indeed the D-brane is dynamical. In Ref. [5] it is shown that there are massless open-string excitations propagating on the D-brane, the $T$ duals of the photons, with precisely the properties of collective coordinates for transverse fluctuations of the D-brane. It is further shown that since the D-brane tension arises from the disk, it scales in string units as $g^{-1}$, $g$ being the closed string coupling. This is the same coupling-constant dependence as for the branes carrying RR charges [7]. Now let us take this further. Far from the D-brane we see only the closed-string spectrum, with two $d = 10$ gravitinos. However, world-sheet boundaries reflect the right-moving $Q_\alpha$ into the left-moving $\bar Q_\alpha$, so only one linear combination of the two supercharges is a good symmetry of the full state. In other words, in the type II theory coupled to the D-brane, half of the supersymmetries of the bulk theory are broken: this is a BPS state.

The BPS property and the scaling of the tension identify the D-brane as a carrier of RR charge, but we can also see this by direction calculation. The disk tadpole for a closed string state $|\psi\rangle$ can be written as $\langle \psi | B \rangle$, where $|B\rangle$ is the closed-string state created by the boundary [9–11]. The reader need not feel compelled to work through these rather detailed references: The essential points will be evident in the simple calculation (5). In Refs. [10,11] this is studied for the RR sector of the superstring with Neumann boundaries, and in Ref. [8] for fully Dirichlet conditions. The Ramond ground-state component of $|B\rangle$ is determined by a condition

$$(\psi_0^\mu - \tilde\psi_0^\mu)|B\rangle = 0, \quad \mu = 0, \dots, 9, \tag{2}$$

this being the superconformal partner of the Neumann condition on $X^\mu$. Call the ground state defined by these conditions $|0\rangle$. In Ref. [10] it is shown that this corresponds to a tadpole for an RR 10-form potential (there will be more on the 10-form below). Now go to the mixed boundary conditions (1). The boundary state satisfies

$$(\psi_0^\mu - \tilde{\psi}_0^\mu)|B\rangle = 0, \quad \mu = 0, \ldots, p,$$

$$(\psi_0^\mu + \tilde{\psi}_0^\mu)|B\rangle = 0, \quad \mu = p + 1, \ldots, 9, \quad (3)$$

and the ground state becomes

$$(\psi_0^{p+1} + \tilde{\psi}_0^{p+1})(\psi_0^{p+2} + \tilde{\psi}_0^{p+2}) \cdots (\psi_0^9 + \tilde{\psi}_0^9)|0\rangle. \quad (4)$$

In the formalism of Ref. [10] this removes $9 - p$ indices, leaving a $(p + 1)$-form potential, as appropriate for coupling to a $p$-dimensional object. Also, since only the even forms appear in the IIb theory, and only the odd forms in the IIa, consistency between the projections in the closed and open string sectors (the analog of modular invariance) gives the consistency condition stated earlier.

The actual value of the quantum of charge is of some interest. This can be determined from a calculation on the disk, but is more easily extracted from a one-loop vacuum amplitude by factorization. Consider parallel Dirichlet $p$-branes, at $X^\mu = 0$ and at $X^\mu = Y^\mu$ for $\mu = p + 1, \ldots, 9$, where $Y^\mu$ are some fixed coordinates. There are open strings with one end attached to each D-brane, and the one-loop vacuum graph from such states is a sum over cylinders with one end lying on each D-brane. This amplitude thus also includes the exchange of a single closed string between the two $p$-branes. The amplitude is given by (we will work in Euclidean spacetime)

$$A = V_{p+1} \frac{1}{2} 2 \int \frac{d^{p+1}p}{(2\pi)^{p+1}} \sum_i \int \frac{dt}{t} e^{-t(p^2 + m_i^2)/2}. \quad (5)$$

The factor $V_{p+1}$ is the volume of the D-brane, defined by putting the system in a large box, the $\frac{1}{2}$ is for real fields, and the 2 is from interchanging the ends of the oriented string. Alternately, the net symmetry factor $\frac{2}{2} = 1$ arises because the discrete part of the world-sheet differential invariance is completely fixed. The sum runs over the spectrum of open strings with ends fixed on the respective D-branes; this is given by the usual oscillator sum with an additional term $Y^\mu Y_\mu / 4\pi^2 \alpha'^2$ in the mass squared from the tension of the stretched string. Carrying out the oscillator sum and momentum integral gives

$$A = V_{p+1} \int \frac{dt}{t} (2\pi t)^{-(p+1)/2} e^{-tY^2/8\pi^2\alpha'^2} \prod_{n=1}^\infty (1 - q^{2n})^{-8}$$

$$\times \frac{1}{2} \left\{ -16 \prod_{n=1}^\infty (1 + q^{2n})^8 + q^{-1} \prod_{n=1}^\infty (1 + q^{2n-1})^8 \right.$$

$$\left. - q^{-1} \prod_{n=1}^\infty (1 - q^{2n-1})^8 \right\}, \quad (6)$$

where we define $q = e^{-t/4\alpha'}$. The three terms in large braces come, respectively, from the open string R sector

with $\frac{1}{2}$ in the trace, from the NS sector with $\frac{1}{2}$ in the trace, and the NS sector with $\frac{1}{2}(-1)^F$ in the trace; the R sector with $\frac{1}{2}(-1)^F$ gives no net contribution.

The sum in large brackets vanishes by the usual "abstruse identity" of supersymmetric string theory. From the open string point of view this reflects the supersymmetry of the spectrum, while in terms of the closed string exchange it reflects the fact that there is no net force between BPS states. As in Ref. [10], it is straightforward to separate the two kinds of closed string exchange. Interchanging world-sheet space and time so as to see the closed string spectrum, the terms without $(-1)^F$ in the trace come from the closed string NSNS states (graviton and dilaton), while the term with $(-1)^F$ comes from the closed string RR states. The massless closed string poles arise from $t \to 0$; using standard $\vartheta$-function asymptotics in this limit, the amplitude becomes

$$A = \frac{1}{2}(1 - 1)V_{p+1} \int \frac{dt}{t}$$

$$\times (2\pi t)^{-(p+1)/2}(t/2\pi\alpha')^4 e^{-tY^2/8\pi^2\alpha'^2}$$

$$= (1 - 1)V_{p+1}2\pi(4\pi^2\alpha')^{3-p}G_{9-p}(Y^2). \quad (7)$$

Here $(1 - 1)$ is from the NSNS and RR sectors, respectively, and

$$G_{9-p}(Y^2) = \frac{1}{4} \pi^{(p-9)/2}\Gamma((7 - p)/2)(Y^2)^{(p-7)/2} \quad (8)$$

is the scalar Green function in $9 - p$ dimensions.

We compare the RR contribution with that from a $(p + 1)$-form potential $A_{p+1}$, $F_{p+2} = dA_{p+1}$, with action [12]

$$S = \frac{\alpha_p}{2} \int F_{p+2}^* F_{p+2} + i\mu_p \int_{\text{branes}} A_{p+1}. \quad (9)$$

For later convenience we have not chosen a normalization for $A_{p+1}$, so two constants $\alpha_p$ and $\mu_p$ appear. Calculating the amplitude from exchange of a $(p + 1)$-form between the Dirichlet $p$-branes, one finds a negative term as in the amplitude (7), with normalization

$$\mu_p^2/\alpha_p = 2\pi(4\pi^2\alpha')^{3-p}. \quad (10)$$

For branes with $p + p' = 6$, the corresponding field strengths satisfy $(p + 2) + (p' + 2) = 10$. These are not independent in the type II string but rather are related by Hodge duality, $F_{p+2} = {}^*F_{8-p}$. A Dirac quantization condition therefore restricts the corresponding charges [13]. Integrate the field strength ${}^*F_{p+2}$ on an $(8 - p)$-sphere surrounding a $p$-brane; from the action (9) one finds total flux $\Phi = \mu_p/\alpha_p$. One can take ${}^*F_{p+2} = F_{8-p} = dA_{7-p}$ except on a Dirac string at the pole. Then

$$\Phi = \int_{S_{8-p}} {}^*F_{p+2} = \int_{S_{7-p}} A_{7-p}, \quad (11)$$

where the latter integral is on a small sphere around the Dirac string. In order that the Dirac string be invisible to

a $(6 - p)$-brane, we need $\mu_{6-p}\Phi = 2\pi n$ for integer $n$. That is, the Dirac quantization condition is

$$\mu_p \mu_{6-p}/\alpha_p = 2\pi n. \tag{12}$$

The charges (10) of the D-branes satisfy this with minimum quantum $n = 1$ (it follows from $F_{p+2} = {}^*F_{8-p}$ that $\alpha_p = \alpha_{6-p}$).

From the point of view of the open string loop calculation this is a "string miracle," a coincidence in need of deeper explanation. Had the Dirac quantization condition not been satisfied, it would likely imply a subtle inconsistency in the *type I* superstring. That the minimum quantum is found strongly suggests that D-branes are actually the RR-charged objects required by string duality.

One can test this further. While the Dirac quantization condition constrains only the product (12), string duality makes specific predictions for the individual charges. Consider a $(p + 1)$-dimensional world-volume $\mathcal{M}$ with $p$-dimensional holes. Under a gauge transformation $\delta A_{p+1} = d\epsilon_p$, the action (9) changes by

$$\delta S = -i\mu_p \int_{\partial M} \epsilon_p. \tag{13}$$

This is the change in phase of a $p$-brane state under a gauge transformation. In Ref. [14], the fields are normalized so that the 2-brane wave functions are invariant for $\epsilon_2$ being $\alpha'$ times an element of the integral cohomology. In other words, $\mu_2 = 2\pi/\alpha'$. Adopting the same convention for the Dirichlet 2-branes, we would have $\alpha_2 = 1/2\pi\alpha'^3$. This is twice the value found in Ref. [14] (which would imply an incommensurate $\sqrt{2}$ in the charges of the Dirichlet and solitonic 2-branes), but agrees with the normalization in Ref. [15]. In comparing, note that the $B_{\mu\nu}$ field in Ref. [15] is twice that in Ref. [14], with other conventions the same. We have not succeeded in reconciling these calculations, but strongly expect that the RR charge is that required by string duality.

This result for the D-brane charge is new evidence both for string duality and for the conjecture that D-branes are the RR-charged objects required by string duality. That is, although it appears that we have modified the type II theory by adding something new to it, we are now arguing that these objects are actually intrinsic to any nonperturbative formulation of the type II theory; presumably one should think of them as an alternate representation of the black $p$-branes. This conjecture was made earlier and with less evidence in Ref. [5] (the argument there being that any object that can couple consistently to closed string must actually be made of closed strings) and in Ref. [7] [based on the $(2n)!$ behavior of string perturbation theory [16]].

As an aside, this would also imply that the type I theory is contained within the type II theory as a sector of the Hilbert space. The argument (the same as given in Ref. [5] but now presented in reverse order) is as follows. Periodically identify some of the dimensions in the type II

string,

$$X^\mu \sim X^\mu + 2\pi R, \quad \mu = p + 1, \ldots, 9. \tag{14}$$

Now make the spacetime into an orbifold by further imposing

$$X^\mu \sim -X^\mu, \quad \mu = p + 1, \ldots, 9. \tag{15}$$

To be precise, combine this with a world-sheet parity transformation to make an orientifold [5,11,17]. This is not a consistent string theory. The orientifold points are sources for the RR fields (by the analog of the above arguments for D-branes, but with the boundary replaced by a cross cap), but in the compact space these fields have nowhere to go. One can screen this charge and obtain a consistent compactification with exactly 16 D-branes oriented as in Eq. (1) [18]. Now take $R \to 0$. The result is the type I string [5,6].

A puzzling feature of the Dirichlet $p$-branes has always been their diversity, with $p$ ranging from $-1$ to 9, the case $p = -1$ being the D-instanton [7,8]. This now finds a satisfying explanation in terms of the diversity of RR forms: The D-branes comprise a complete set of electric and magnetic RR sources. The IIa theory has field strengths of rank $2, 4, 6, 8$ (with $n$ and $10 - n$ dual), which are the curls of potentials of rank $1, 3, 5, 7$ and so couple to $p$-branes for $p = 0, 2, 4, 6$. The IIb theory has field strengths of rank $1, 3, 5, 7, 9$, which are the curls of potentials of rank $0, 2, 4, 6, 8$ and couple to $p$-branes for $p = -1, 1, 3, 5, 7$.

The reader will notice that we have two extra branes, $p = 8$ and 9, coupling to 9- and 10-form potentials. While these forms do not correspond to propagating states, they are present in the IIa and IIb theories, respectively, and have important dynamical effects. The 10-form has been discussed previously [10]. It couples to a 9-brane, but what is that? A 9-brane fills space, so the open string end points are allowed to go anywhere: This is simply a Neumann boundary condition. If there are $n$ 9-branes (which must of course lie on top of one another), the end points have a discrete quantum number: This is the Chan-Paton degree of freedom. The total coupling of the branes to the 10-form is

$$in\mu_9 \int_{\text{spacetime}} A_9. \tag{16}$$

The equation of motion from varying $A$ implies that $n$ must equal zero. We cannot readily cancel this with branes of the opposite orientation and charge because we would no longer have a BPS state, but we can cancel it by again orientifolding (with a trivial spacetime transformation) to make the type I string. The cross cap gives a 10-form source of the opposite sign, giving in all

$$i(n - 32)\mu_9 \int_{\text{spacetime}} A_9. \tag{17}$$

Thus the equation of motion requires the group SO(32). It is worth recalling the logic of Ref. [10]: The spacetime

anomaly for other groups must arise from some world-sheet superconformal anomaly, but this must in turn correspond to some spacetime equation that is not being satisfied.

The 9-form potential in the IIa string has not been previously noted. The action $\int F_{10}{}^*F_{10}$ gives the equation of motion $d^*F_{10} = 0$, which for a 10-form field strength implies that $^*F_{10}$ is constant. There are thus no solutions at nonzero momentum, explaining why this is easily overlooked, but the constant solution is quite interesting: It is like a background electric field and so gives a contribution to the cosmological constant proportional to the square of the field. Note that if one simply substitutes a constant $^*F_{10}$ into the action one obtains a cosmological constant of the wrong (negative) sign because of the neglect of a surface term. It is obvious on physical grounds that the cosmological constant is positive, and this is what one finds from the equations of motion.

Thus the IIa superstring has a cosmological constant of undetermined magnitude (nucleation of 8-branes shifts the 10-form field strength by a large discrete unit, by analogy with two-dimensional massive electrodynamics). This is surprising, but has been partially anticipated by Romans [19], who found the corresponding supergravity theory for fixed cosmological constant. This 10-form background would appear to be a continuous parameter which breaks supersymmetry, a phenomenon not previously found in string theory. However, in a separate paper [20] it will be shown that the value of $F_{10}$ is quantized; the implications for compactification to four dimensions will also be discussed.

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*Electronic address: joep@itp.ucsb.edu

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