Resistive Hysteresis and Nonlinear *I-V* Characteristics at the First-Order Melting of the Abrikosov Vortex Lattice

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We study a three-dimensional network of Josephson junctions in a magnetic field, which undergoes a first-order melting transition of the triangular vortex lattice. We perform a Langevin dynamics calculation of the resistance and current-voltage (*I-V*) characteristics. We find hysteresis in the resistance as a function of temperature as measured in untwinned $YBa_2Cu_3O_7$. Close to the melting temperature the *I-V* curves are *S* shaped with hysteresis and show a melting transition when *increasing the current,* driven by the blowing out of current nucleated vortex loops.

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Abrikosov demonstrated in 1957 [1] that there is a lattice of vortices or flux line lattice (FLL) in type II superconductors for magnetic fields such that $H_{c1} < H < H_{c2}$. In high-T_c superconductors, thermal fluctuations can melt the FLL at a temperature T_M well below the mean field critical temperature $T_c(H)$ [2]. Brézin *et al.* [3] have shown that this melting transition should be first order. This was recently confirmed by Monte Carlo simulations [4] in the three-dimensional (3D) *XY* model. The first experimental evidence of a first-order transition of the FLL was obtained by Safar *et al.* [5] in untwinned $YBa_2Cu_3O_7$ crystals: a sharp hysteretic jump in the resistance was measured at very low bias currents, as a function of temperature and magnetic field. This result was reproduced by others, including the effects of the direction of the magnetic field, twin boundaries, and disorder [6–8]. However, in principle, a resistive hysteresis can also be caused by dynamic processes or disorder [9,10]. Recently, Jiang *et al.* [11] have argued that current-induced nonequilibrium effects can be invoked to explain some of their experimental findings on the resistive hysteresis. In view of this, there is a need to make a connection between the thermodynamical melting phase transition and the behavior of nonthermodynamic quantities such as the resistance. Here we use Langevin dynamics simulation to compare the hysteresis in the internal energy and first-order transition seen in Ref. [4] with the resistive hysteresis and current-voltage (*I-V*) characteristics calculated in the same model. We find good correspondence when the bias current is low enough. A new result is the existence of a *current-induced melting* of the FLL and hysteresis in the *I-V* characteristics close to T_M, which explain the resistive hysteresis.

The high-T_c superconductors can be described, below a mean field transition temperature $T_c^{\rm MF}$, by the thermal fluctuations of the phase θ of the superconducting order parameter $\Psi = |\Psi|e^{i\theta}$. A lattice version of this approach leads to the 3D *XY* model [12–14], with the Hamiltonian

$$\mathcal{H} = -\sum_{\mathbf{r},\hat{\mu}} J_{\hat{\mu}} \cos[\Delta_{\hat{\mu}}\theta(\mathbf{r}) - A_{\hat{\mu}}(\mathbf{r})], \qquad (1)$$

which has been studied both for zero [12] and finite magnetic fields [14]. We consider a 3D stacked triangular lattice [4] given by $\mathbf{r} = r_1\hat{a}_1 + r_2\hat{a}_2 + r_3\hat{z}$, with r_1, r_2, r_3 integers, and $\hat{\mu} = \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{z}$, with $\hat{a}_1 = \hat{x}, \hat{a}_2 = -\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}, \hat{a}_3 = \frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}$. The phase difference is $\Delta_{\hat{\mu}}\theta(\mathbf{r}) = \theta(\mathbf{r} + \hat{\mu}) - \theta(\mathbf{r})$. The gauge factor $A_{\hat{\mu}}(\mathbf{r}) = \frac{2\pi}{\Phi_0}\int_{\mathbf{r}}^{\mathbf{r}+\hat{\mu}} \mathbf{A}\cdot dl$ ($\Phi_0 = h/2e$ is the quantum of flux) depends on the magnetic induction $\mathbf{B} = \nabla \times \mathbf{A} = B\hat{z}$ [15]. This gives $A_z(r) = 0$ and a frustration f in the xy planes: Σ_{xy} plaquette$A_{\hat{\mu}}(\mathbf{r}) = 2\pi f = 2\pi B a^2\sqrt{3}/4\Phi_0$, with the lattice constant $a = |\hat{\mu}|$. We consider an isotropic lattice with $J_{\hat{\mu}} = J$ for all $\hat{\mu}$, where $J = \Phi_0^2 a/16\pi^3\lambda^2$ with λ the London penetration depth.

Current-driven *XY* models are usually studied by means of a current-conserving overdamped Langevin dynamics [13,16]. The current $I_{\hat{\mu}}(\mathbf{r})$ in each bond of the 3D lattice is given by the resistively shunted junction model

$$I_{\hat{\mu}}(\mathbf{r}) = \frac{\Phi_0}{2\pi\mathcal{R}}\frac{d\Delta_{\hat{\mu}}\theta(\mathbf{r})}{dt} + I_0\sin[\Delta_{\hat{\mu}}\theta(\mathbf{r}) - A_{\hat{\mu}}(\mathbf{r})] + \eta_{\hat{\mu}}(\mathbf{r},t), \qquad (2)$$

with $I_0 = 2\pi J/\Phi_0$ and $\mathcal{R}$ the shunt resistance, considered isotropic. The thermal noise term has correlations $\langle\eta_{\hat{\mu}}(\mathbf{r},t)\eta_{\hat{\mu}'}(\mathbf{r}',t')\rangle = (2k_BT/\mathcal{R})\delta_{\hat{\mu},\hat{\mu}'}\delta_{\mathbf{r},\mathbf{r}'}\delta(t-t')$. Together with the condition of current conservation,

$$\sum_{\hat{\mu}} I_{\hat{\mu}}(\mathbf{r}) - I_{\hat{\mu}}(\mathbf{r} - \hat{\mu}) = \Delta_{\hat{\mu}} \cdot I_{\hat{\mu}}(\mathbf{r}) = I_{\rm ext}(\mathbf{r}), \qquad (3)$$

this determines the full set of dynamical equations. The boundary conditions are periodic along the $\hat{x}$ ($\hat{a}_1$) and $\hat{z}$ directions, and open in the $\hat{a}_2$ direction, with a current bias I in the so-called $[01\bar{1}]$ current injection direction [17]. This corresponds to $I_{\rm ext}(\mathbf{r}) = I(\delta_{r_2,0} - \delta_{r_2,L_2})$. From (2)

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and (3) we obtain

$$\frac{d\theta(\mathbf{r})}{dt} = \frac{2\pi}{\Phi_0} \times \sum_{\mathbf{r}'} G(\mathbf{r},\mathbf{r}') \left[I_{\text{ext}}(\mathbf{r}') + \frac{2\pi}{\Phi_0} \frac{\delta \mathcal{H}}{\delta \theta(\mathbf{r}')} - \Delta_{\hat{\mu}} \cdot \eta_{\mu}(\mathbf{r}', t) \right]. \quad (4)$$

Here the 3D Green's function is the solution of $\Delta_{\hat{\mu}} \cdot \frac{1}{\mathcal{R}} \Delta_{\hat{\mu}} G(\mathbf{r},\mathbf{r}') = \delta_{\mathbf{r},\mathbf{r}'}$ under the given boundary conditions. We simulate Eq. (4) with the same algorithm as in Ref. [13] with a time step of $\Delta t = 0.05\tau_J$ with $\tau_J = \Phi_0/2\pi \mathcal{R} I_0$ and with an integration time $t_{\text{int}} = 5 \times 10^3 \tau_J$ after a transient of $10^3 \tau_J$.

The choice of a triangular network in the xy planes is the most convenient lattice discretization: (i) there is no frustration between the ideal triangular FLL and the periodic pinning potential of the Josephson lattice, as there is in cubic networks [4], (ii) it has the lowest single vortex pinning barrier with a critical depinning current of $I_c = 0.042 I_0$ ($I_c = 0.1 I_0$ in a square network) [17], and (iii) it has the largest Josephson current (above which the whole network dissipates, equivalent to the depairing current of continuous superconductors) $I_J = 2I_0$ for the $[01\bar{1}]$ current injection direction ($I_J = I_0$ in a square network) [17]. We note that the model considered here does not have any random pinning. The weak pinning present here is introduced by the periodic discreteness of the array geometry. We consider a field of $f = 1/6$ vortices per plaquette and a system of size $18 \times 18 \times 18$. These parameter values and model system are the same as in the Monte Carlo simulation of Ref. [4]: see their Fig. 1 for a plot of the unit cell of the vortex ground state. It was carefully determined in [4] that the FLL melting transition is first order, with hysteresis in the average energy around the melting temperature $T_M = 1.175$ (in units of J).

We calculate the normalized voltage drop along the direction of the current as $v = \frac{\tau_J}{N_1(N_2-1)N_z} \sum_{r_1,r_3} \langle \dot{\theta}(r_1, L_2, r_3) - \dot{\theta}(r_1, 0, r_3) \rangle$. In Fig. 1(a) we show the dc resistance $R = v/i$ for a low current $i = 0.01$ (currents are normalized by I_0) as a function of temperature. We increase the temperature in steps of $\Delta T = 0.01$ from the FLL ground state at $T = 0$, taking as initial condition of each step the final state of the previous step (adiabatic sweep). For very low T we have $R \approx 0$ since we are well below the critical current $i_c(T=0) = 0.042$. At higher temperatures $T \gtrsim 1$ there is a small finite R due to thermally activated vortex motion. At a temperature $T_+ = 1.20$ there is a sharp rise in R of about 1 order of magnitude. When we decrease the temperature from a disordered initial condition at $T = 1.5$, also in an adiabatic sweep with $\Delta T = -0.01$, we find a sharp drop in R at $T_- = 1.15$. Thus we obtain the hysteretic cycle in R shown in Fig. 1(b). A very similar hysteresis was seen in the experiments [5–8,11]. The hysteresis loop occurs around the *equilibrium* melting temperature, $T_- < T_M < T_+$. (In Ref. [11] the resistive hysteresis was interpreted to be below T_M; however, higher currents can shift T_+ very close to T_M; see below.) The resistive hysteresis has about the same size in temperature as the hysteresis in the internal energy in the equilibrium simulations of Ref. [4]. This shows that there is a strong correspondence between the first-order character of the equilibrium melting transition and the resistive hysteresis at low bias currents. We also show in Fig. 1(a) the helicity modulus Υ_z along the z direction (proportional to the superfluid density) [14,18]. It vanishes at about T_+ when increasing T, thus indicating the vanishing of superconducting coherence, and it has the same hysteresis as R when decreasing T [19]. We have also calculated the vortex structure function: $S(\mathbf{k}) = \frac{1}{L_1 L_2 L_z} \sum_{r_3} \langle n_z(\mathbf{k}, r_3) n_z(-\mathbf{k}, r_3) \rangle$.

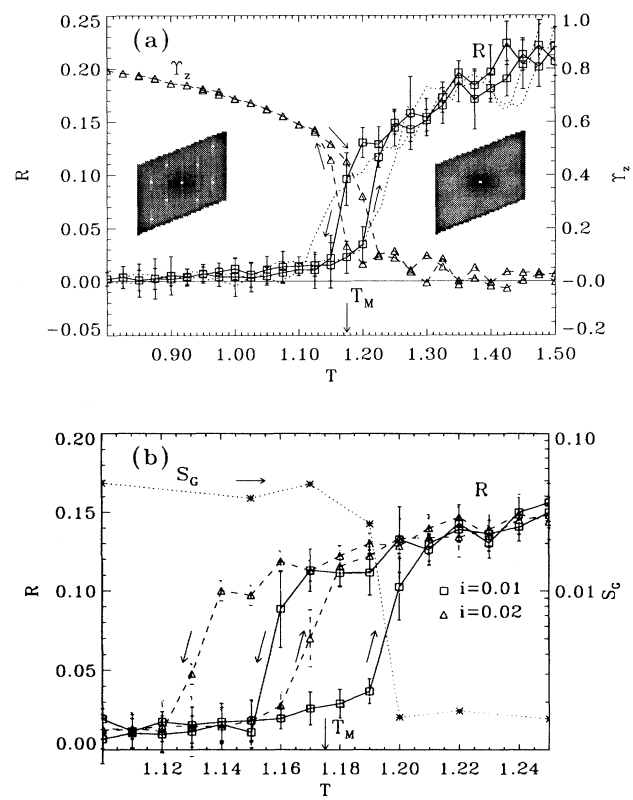

FIG. 1. (a) Left scale, squares, full line: resistance $R = v/i$ as a function of temperature T for a bias current $i = 0.01$ and integration time $t_{\text{int}} = 5000\tau_J$. Dotted line: R vs T for the same i but $t'_{\text{int}} = 500\tau_J$. Right scale, triangles, dashed line: helicity modulus along the z direction Υ_z vs T, for $i = 0.01$. Insets: intensity plots of the vortex structure factor $S(\mathbf{k})$, for $T = 1.0$ (left), and for $T = 1.35$ (right). (b) Left scale: R vs T, for $i = 0.01$ (squares, full line) and for $i = 0.02$ (triangles, dashed line). Right scale, stars, dotted line: intensity of the Bragg peak S_G vs T. Results for a $18 \times 18 \times 18$ lattice with field density $f = 1/6$ flux quanta per plaquette.

The vorticity along the direction $\hat{\nu}$ is $n_{\hat{\nu}}(\mathbf{R},t) = -\Sigma_{\text{plaquette}}\text{nint}\{[\Delta_{\hat{\mu}}\theta(\mathbf{r}) - A_{\hat{\mu}}(\mathbf{r})]/2\pi\}$ for $\hat{\nu}\perp\hat{\mu}$; nint$[x]$ is the nearest integer to x. Intensity plots of $S(\mathbf{k})$ are shown in the insets of Fig. 1(a) for temperatures below and above T_M. We see that below T_M there are Bragg peaks which disappear above T_M. In Fig. 1(b) we show the intensity of one of the Bragg peaks of the $f = 1/6$ vortex lattice, $S_G = S(\mathbf{k} = \mathbf{G})$, with $\mathbf{G} = \frac{4\pi}{\sqrt{3}}(\frac{\sqrt{3}}{6}\hat{x} + \frac{1}{2}\hat{y})$ when increasing T for $i = 0.01$. There is a sharp drop at T_+, indicating melting of the FLL. The simultaneous vanishing of S_G and Υ_z shows that the FLL melts into an entangled liquid [2,18]. The resistive hysteresis depends on the integration time (analogous to a time of "measurement") and the bias current. In Fig. 1(a) we show as a dotted line the resistance calculated with $t'_{\text{int}} = t_{\text{int}}/10$. For a fast integration the hysteresis loop around T_M is larger due to the metastability of the supercooled and overheated phases. In Fig. 1(b) we show the resistive hysteresis for a larger bias current $i = 0.02$. The loop shifts to lower temperatures (not longer centered around T_M, but yet $T_+(i) \gtrsim T_M$), and the size of the loop is slightly smaller than for $i = 0.01$. Larger currents further reduce the size of the loop, but then we are in a nonlinear current regime. Experiments by Jiang *et al.* lead to the conclusion that the resistive hysteresis is due to a current-induced effect, because of the absence of "subloops" in the hysteresis (see Ref. [11]) upon partial cooling and heating. However, subloops have been observed recently in very clean samples [20]. Our results on the hysteresis dependence on t_{int} suggest the existence of subloops in this model.

A better understanding of the origin of the resistive hysteresis can be obtained by studying the I-V curves. In Fig. 2 we show a set of I-V curves for different temperatures. This can be compared with the experiment of Kwok *et al.* [8] (see their Fig. 4). For $T \gg T_M$ our I-V curves show Ohmic linear behavior down to the lowest currents for the vortex liquid state. On the other hand, for $T \ll T_M$ the I-V curves show a downward curvature, evidencing a critical current. In this model, the vortex lattice is pinned at low currents by the periodic pinning potential of the discrete Josephson network. At temperatures very close to T_M (in the region of the resistive hysteresis loop) the I-V curves show an S shape also seen in the experiments [8]. Here there are two regions of linear "Ohmic" dissipation, for low and high currents. The linear region at low currents corresponds to the thermally activated flow of the vortex lattice. The linear region at high currents was interpreted as due to flux flow of the unpinned vortex lattice by Kwok *et al.* However, an analysis of the S-shaped I-V curves shows that it corresponds to the flow of the vortex liquid, as we now discuss.

In Fig. 3(a) we show the I-V characteristics for $T = 1.170$, very close to $T_M = 1.175$. We start with the $T = 0$ FLL as initial condition, then increase T to $T = 1.170$. From that state the current is increased in an adiabatic sweep with steps of $\Delta i = 0.005$. We see that when increasing the current the I-V has a kink at a characteristic current i_M (also seen in the experiments [8,12]), above which it becomes linear. In Fig. 3(b) we see that the intensity of the Bragg peak S_G has a sharp drop at this same current i_M. This means that *the vortex lattice melts*

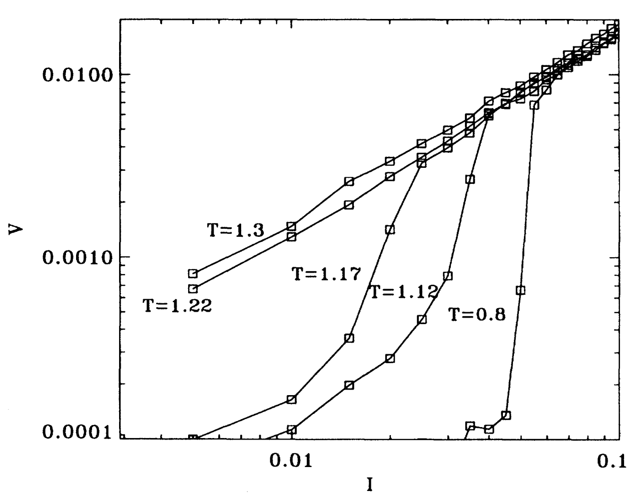

FIG. 2. Current-voltage characteristics for different temperatures and the same model parameters as in Fig. 1. From top to bottom $T = 1.3, 1.22, 1.17, 1.12, 0.8$ ($T_M = 1.175$).

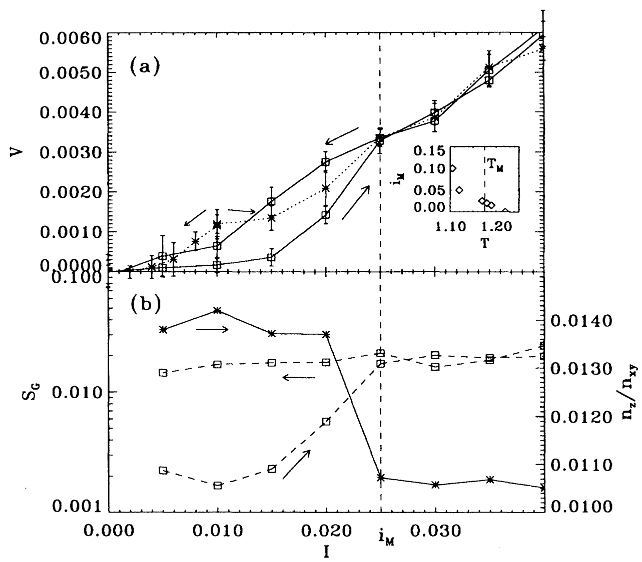

FIG. 3. (a) Voltage v as a function of current i for $T = 1.170$. Squares, full line: increasing the current from an ordered vortex lattice at $i = 0$, and then decreasing the current. Stars, dotted line: increasing and decreasing the current from an initial state at $i = 0.01$ corresponding to the upper branch of the R vs T calculation in Fig. 1(b). The inset shows the melting current i_M vs T. (b) Left scale, stars, full line: S_G vs i, when increasing the current. Right scale, squares, dotted line: vortex excitations n_z/n_{xy} vs i.

when increasing the current. Therefore, the linear part of the I-V curve at high currents corresponds to a flowing vortex liquid, for T close to T_M. When decreasing the current from $i \gg i_M$ to $i = 0$, we see hysteresis below i_M, since the voltage continues to be almost linear with i down to very low currents. Therefore the vortex liquid state is "supercooled" when decreasing the current below i_M. This result correlates with the hysteresis in the resistance in Fig. 1. For example, we take as initial condition the state in the supercooled branch of the resistance in Fig. 1(b) at $T = 1.170$ and $i = 0.01$, and from this point we both increase or decrease the current. This is shown as dotted lines in Fig. 3(a): the resulting points lie close to the ones obtained above when decreasing the current from $i \ll i_M$. Worthington *et al.* [21] have suggested a current-induced melting mechanism in their *S*-shaped I-V characteristics. They attribute the melting to a heating noise induced by random pinning forces. However, there is no randomness in the model considered here. Moreover, Koshelev and Vinokur [9] have shown that the effect of random pinning is to melt the FLL when *decreasing* the current. Note that this occurs at large currents in a nonlinear regime, and that it gives a clockwise hysteresis loop in the R vs T curve, instead of the anticlockwise loop seen here at very low currents.

In order to understand the physical origin of this current induced melting, we examined the thermally induced vortex excitations. We calculate the average number of extra vortices along the z direction, $n_z = \langle |n_z(R,t)| \rangle - f$, where the average is over space and time, and the average vortex excitations in the xy directions $n_{xy} = \Sigma_{i=1}^{3} \langle |n_{\hat{a}_i}(R,t)| \rangle$ [$\langle n_z(R,t) \rangle = f$ and $\langle n_{\hat{a}_i}(R,t) \rangle = 0$ because of vortex number conservation]. In Fig. 3(b) we plot n_z/n_{xy} as a function of current. We see that n_z/n_{xy} increases when increasing i up to i_M, and then is almost constant for $i > i_M$. Then as the current is decreased below i_M, n_z/n_{xy} remains constant at its vortex liquid value, reflecting the hysteresis seen in the I-V curve; n_{xy} and n_z show similar behavior independently. This indicates that the effect of the current is to blow out thermally induced vortex loops, and to orient the loops in the plane perpendicular to the current, thus increasing both n_z and n_{xy}. These two effects increase with increasing current and tend to disorder the FLL, inducing melting at $i_M(T)$. We have observed this phenomenon for T close to T_M, in the range $1.12 < T < 1.19$. Even when $i_M(T)$ decreases with temperature [see inset in Fig. 3(a)], it does not vanish at T_M but at a higher temperature $T \approx 1.20$ in the overheated region. On the other hand, at $T \ll T_M$ and for large currents $i \gg i_c$, there is a flow of an ordered vortex lattice. We find that this flowing lattice melts when increasing T at $T_+^{\mathrm{flow}} = 1.15$, and then it freezes when decreasing T at $T_-^{\mathrm{flow}} = 1.10$. This suggests that $i_M(T)$ should diverge at T_-^{flow}.

In conclusion, our results show how a first-order vortex lattice melting transition correlates with a hysteresis in the resistance. Our results compare very well with the experimental measurements [5–8,11] in untwinned $YBa_2Cu_3O_7$ crystals [22]. Moreover, we find a new current induced vortex lattice melting caused by the blowing out of vortex loops. A measurement of hysteresis in the I-V characteristics close to the melting temperature, complemented with neutron diffraction measurements of the vortex structure, would confirm this scenario.

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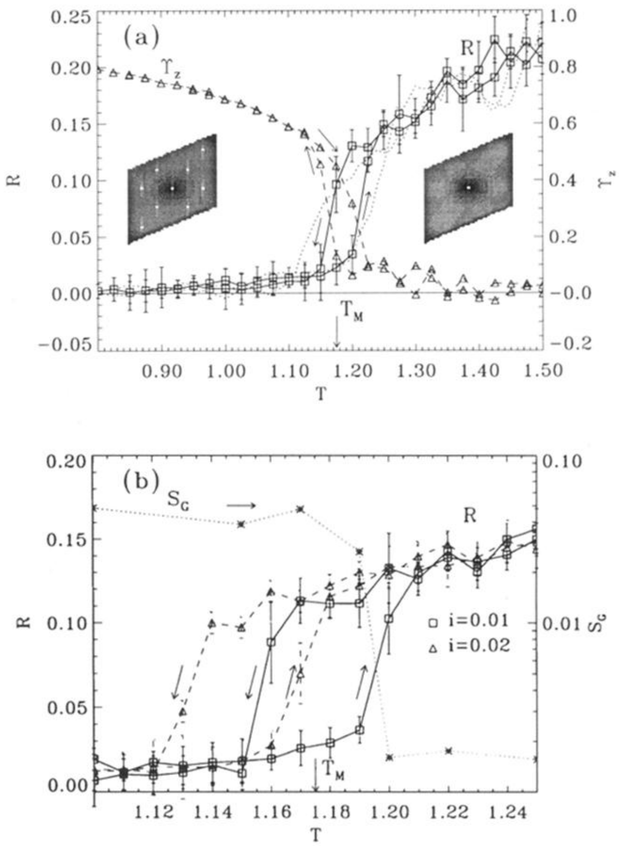

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