Bose and Vortex Glasses in High Temperature Superconductors

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A theory of the Bose glass transition is developed. Mapping vortex trajectories onto world lines of 2D bosons gives a glass transition line as the locus where bosons become localized in the random potential. We describe the glass-to-liquid transition induced by columnar disorder in both dilute and dense vortex systems. A vortex glass transition in the light of the developed description is discussed.

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The nature of the location of vortex and Bose glass transitions in the vortex state of high temperature superconductors (HTS) is an extensively debated question of much theoretical and practical interest; for a review see [1]. It has been demonstrated [2,3] that quenched point disorder drives the vortex crystal into a glassy state characterized by divergent energy barriers for vortex motion, but does not create infinite barriers in a vortex liquid. This suggests that in clean crystals the vortex glass melts into a liquid via a first-order thermodynamic transition close to the melting line $B_m(T)$ of the pure crystal. On the other hand, the smallness of the Lindemann number c_L controlling the position of the melting line makes it possible that even weak disorder can modify the melting significantly and transform it into a continuous second-order transition. The effect of strong pinning, in particular the pinning due to linear defects introduced by irradiation of the superconducting sample with heavy ions, may be more dramatic: Columnar defects expand significantly the irreversibility region [4] where a low temperature Bose glass phase with vortices localized near columnar defects was shown to exist [5].

In this Letter we address the question of the structure of the different vortex states and the nature of the transitions between the possible phases in the presence of quenched disorder. We adopt the description of the low temperature glassy phase as the state where vortices are localized in potential wells generated by disorder. We propose a phenomenological description of the glass transition and construct a phase diagram of the mixed state of HTS in the presence of columnar and point disorder.

We start with a discussion of the constitutive relation $B(H)$ as modified by disorder in the weak field regime. Next, we investigate the glass transition due to columnar defects and point disorder. We restrict our consideration of the Bose glass transition to moderate fields, where columnar defects outnumber the vortex lines, and explore the analogy between the statistical physics of vortices and the quantum mechanics of 2D bosons introduced by Nelson [6]. Mapping vortices onto world lines of 2D particles describes the Bose glass state as the phase where the 2D bosons are localized by the quenched 2D disorder (which is the image of the columnar defects) [5]. Within this approach the glass transition line is obtained as the transition from the localized into the superfluid state in the related Bose system at zero temperature. At small fields, $B \ll H_{c1}$, the transition occurs when the localized radius l of vortices (or the size of a vortex sheaf), localized by fluctuations in the distribution of pinning centers, becomes of order of the vortex spacing a. At large fields, $B \gg H_{c1}$, long range vortex-vortex interactions are essential and the transition line is determined by the balance between elastic/pinning energies and thermal fluctuations. As we will show below, both limiting cases can be described in a unique manner as the line where the vortex related part of the tilt modulus c_{44}^v of the vortex liquid diverges.

A model free energy for N flux lines in a sample of thickness L defined by their trajectories $\mathbf{r}_i$ is

$$F_N = \int_0^L dz \left[\sum_i \frac{\epsilon_1}{2} \left(\frac{\partial \mathbf{r}_i}{\partial z} \right)^2 + U(\mathbf{r}_i) + \sum_{i<j} V(\mathbf{r}_{ij}) \right]. \tag{1}$$

The magnetic field is aligned along the z axis, perpendicular to the CuO planes, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\epsilon_1 = \epsilon^2 \epsilon_0$ is the linear tension of the vortex lines, $\epsilon_0 = (\Phi_0/4\pi\lambda)^2$, ϵ is the anisotropy parameter, $U(\mathbf{r}_i)$ represents the disorder potential, and $V(\mathbf{r}_{ij})$ is the vortex-vortex interaction. We assume Gaussian disorder with the correlation function $\langle U(\mathbf{r}) U(\mathbf{r}') \rangle = \Delta_1 \delta(\mathbf{r} - \mathbf{r}')$, $\Delta_1 = U_0^2 r_0^4 / d^2$, where U_0 is the depth of the potential well of defects, and r_0 and d are the radius of and the distance between them. In the dilute limit $a \gg \lambda$, vortices can be treated as hard core interacting but otherwise independent lines. The related quantum mechanical system is a weakly nonideal gas of hard core 2D bosons, with interactions described by the renormalized dimensionless coupling constant $g_1 = g_0/[1 + g_0 \ln(1/\lambda^2 n_v)]$, $g_0 = (\epsilon_1/2\pi T^2) \int dr\, r V(r) = \epsilon_1 \epsilon_0 \lambda^2 / 2\pi T^2$ [7], where n_v is the vortex density. One finds [8] that a single vortex line is always localized by a columnar defect (this corresponds to the existence of bound states in the 2D potential well, however weak). The vortex localization radius grows with temperature and at $T \approx T^* = (2/\sqrt{\pi}) r_0 \sqrt{\epsilon_1 U_0}$

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compares to the mean distance between tracks. This implies that at $T > T^*$ the vortex is localized on the fluctuations in the spatial distribution of columnar defects rather than on individual pins [5], and the description of the impurity potential by a white noise function is justified.

First, we derive the constitutive relation $B(H)$ near H_{c1} in the presence of disorder. At low fields vortices occupy the states with the smallest free energies corresponding to the lowest energy states in the related Bose system. The density of states $n(E)$ for the particle in a Gaussian 2D random potential was found in [9–11]: $n(E) \propto \exp(-2.9E/E_0) \propto N(E) = \int_0^E dE' n(E')$, where $-E_0 = -\Delta_1\epsilon_1/T^2$ is the energy of the localized vortex ground state. The localization length $l = T^2/(\Delta_1)^{1/2}\epsilon_1$ [5], and can be larger than the penetration depth λ, in which case more than one vortex can be placed into the potential well formed by the ensemble of defects. Noninteracting vortices would have collapsed to the lowest state; interactions, however, give rise to a sequential filling of energy levels in the potential well. The number of particles N in each potential well is determined by minimizing their total energy $\mathcal{E} = -NE + gEN(N-1)$, where $g = 2.9\pi g_1$: N is the closest integer to $1/2 + 1/2g$ and depends on T and B. Different N can correspond to different kinds of Bose glasses.

The free energy density of the vortex system is $F = \int^E dE' n(E')\mathcal{E}(E')$, where E is defined by $n_v = N\int^E dE' n(E')$. Using the definition of the chemical potential $\mu = \Phi_0(H - H_{c1}) = \partial F/\partial n_v$, where $n_v = B/\Phi_0$, one arrives at

$$B \cong (\Phi_0/l^2 g)\exp[\mathcal{N}\Phi_0(H - H_{c1})/E_0], \quad H < H_{c1}, \qquad (2)$$

with the numerical factor $\mathcal{N} = 2.9/[1 - g(N-1)]$. Measuring the finite induction $B(H < H_{c1})$ one can scan the disorder-induced density of bound vortex states.

Next, we describe the glass-to-liquid transition in the dilute limit, $a \gg \lambda$, as the process of sequential filling of energy levels in the potential wells generated by disorder and the eventual overlapping of the sheaves with the delocalization of the vortices taking place. In the glassy region the mean distance between the sheaves is large and each sheaf contains an integer number of vortices. The transfer of a vortex from one sheaf to another increases the total energy $\mathcal{E}$ by $2gE$, and the vortex exchange between sheaves is unfavorable. However, such an exchange increases the positional entropy and the corresponding gain in the free energy can be estimated as $E\exp(-r/l)$, where $r \simeq a\sqrt{N}$ is the distance between the sheaves. As long as this entropy gain remains small as compared to gE, the number of vortices in each sheaf is preserved, the sheaves are isolated, and the system remains in a glassy state. Upon increasing the degree of filling the distance between the sheaves decreases, the vortex states become more extended, and the percolation-like transition into a vortex liquid occurs. The transition line is determined by the condition $E\exp(-r/l) = gE$ leading to $B_{\rm BG}^{(2)} = (\Phi_0/l^2)\mathcal{L}$, where $\mathcal{L} = \ln(1/\lambda^2 n_v)/[\ln\ln(1/\lambda^2 n)]^2$ for $g \ll 1$, and $\mathcal{L} = 1$ for $g \simeq 1$. Substituting $l = T^2/(\Delta_1)^{1/2}\epsilon_1$ one finds

$$B_{\rm BG}^{(2)} = B_\Phi(T^*/T)^4\mathcal{L} \propto [(T_c - T)/T]^4, \qquad (3)$$

which coincides up to a factor $\mathcal{L}$ with the low field Bose glass transition line found in [5] from the condition $l \simeq a$. The same qualitative picture and result hold for the localized- (glass-) to-superfluid state transition in a weakly nonideal 2D Bose gas placed on a random substrate.

The above picture applies to large sheaves with $l > \lambda$. In the opposite limit of a dense system where $l < \lambda$, the vortex-vortex interactions change the character of the transition. As the density of vortex lines grows, the intervortex repulsion leads to correlations in their spatial arrangement. This cooperative behavior leads to the onset of short range order, and the long range correlations are destroyed by the random potential only on a large spatial scale [2]. To analyze the glass-to-liquid transition in a dense system, $a \ll \lambda$, we adopt the "harmonic oscillator Lindemann" approach developed in [5]. We consider a representative vortex in the solid phase as being localized by the rest of the lattice in the potential cage of transverse size a. In the related quantum description the representative particle is confined in a harmonic-oscillator potential well. The delocalization (i.e., melting for the pure system) occurs when the ground state energy becomes a fixed fraction (given by the Lindemann number c_L) of the saddle-point barrier energy.

In the spirit of collective pinning theory we first consider the clean lattice, and then find perturbatively the contribution of the disorder to the harmonic oscillator barrier energy. The characteristic transverse (u_T) and longitudinal (L_T) sizes of thermal fluctuations of the representative vortex in the cage are determined by the minimization of its elastic energy $\mathcal{E}_{\rm el} = c_{66}u^2L + \epsilon_1(u^2/L)$ under the condition $\mathcal{E}_{\rm el}(u_T) \sim T$. This gives rise to $u_T^2 \simeq T/\sqrt{c_{66}\epsilon_1}$, $L_T \simeq \sqrt{\epsilon_1/c_{66}}$, where $c_{66} \sim \exp(-a/\lambda)$ at $B < H_{c1}$, and $c_{66} \sim B\Phi_0/(8\pi\lambda)^2$ at $B > H_{c1}$ is the shear modulus describing the interaction of the representative vortex with its neighbors. The transition into the liquid occurs when the temperature matches the characteristic shear barriers, which is $c_L^2 c_{66}a^2L_T$ in the absence of disorder. Defects increase this elastic barrier by the pinning energy $\mathcal{E}_p = L_T\sqrt{\Delta_1/(u_T^2 + \xi^2)}$ in the volume $u_T^2L_T$. The term u_T^2 in the denominator accounts for the thermal reduction of the pinning potential [1]. The equation determining the transition line then reads

$$T = c_L^2 c_{66}a^2L_T + L_T\sqrt{\Delta_1/(u_T^2 + \xi^2)}. \qquad (4)$$

Generally speaking, the free energy barrier for the vortex has a purely energetic (elastic) part [the first term on the right-hand side (rhs) of the equation] and an entropic disorder-induced part (the second term). If the disorder term on the rhs of the equation is small compared to the elastic one, the localization line lies slightly above the melting line, and the resulting transition $B_{\rm BG}^{(1)} \gtrsim B_m^{(1)}$ is expected to remain a first-order transition. However, because of the smallness of the Lindemann number, $c_L \approx 0.1-0.2$, even weak disorder can distort significantly the elastic barrier and change the order of the transition. At the temperature T^* where $c_L^2 c_{66} a^2 L_T \simeq \mathcal{E}_p$, disorder starts to dominate the transition and the first-order line $B_{\rm BG}^{(1)}$ terminates. At lower temperatures $T < T^*$ Eq. (4) reduces to $T \simeq \mathcal{E}_p(T)$, which can be viewed as the generalization of the Lindemann criterion to the disorder-dominated case, and gives the second-order glass transition

$$B_{\rm BG}^{(2)} \simeq 4\Phi_0 \Delta_1 \epsilon^4 \epsilon_0^2 / T^6 \propto (T_c - T)^6, \qquad T > T_{dp}, \tag{5}$$

with $T_{dp} \simeq \epsilon \epsilon_0 \xi^2 / a$ defined by the condition $u_T = \xi$. At $T < T_{dp}$, $B_{\rm BG}^{(2)} \propto (T_c - T)^3$.

In the dual Bose representation the transition from the vortex liquid to the glass results from the suppression of the superfluid density n_s by quenched disorder, and the position of the Bose glass transition, where bosons (vortices) become localized, can be found from the condition $n_s = 0$. To explore this approach let us establish the correspondence between the vortex part of the tilt modulus and the superfluid density n_s in the related Bose system. Evaluating the longitudinal part of the correlator of the vector potential $\langle A_z(\mathbf{r}) A_z(\mathbf{0}) \rangle$ in the clean Bose system one can show that

$$c_{44} \simeq (B^2/4\pi)[1 + (4\pi\lambda^2 n_s)^{-1}]. \tag{6}$$

This defines the vortex related part of the tilt modulus $c_{44}^v \simeq B^2/(4\pi\lambda)^2 n_s$, which becomes infinite at the Bose glass transition line.

In the liquid phase the disorder-induced renormalization of the inverse vortex related tilt modulus can be determined within the lowest order perturbation theory [12]:

$$\frac{1}{c_{44}^{vR}} = \frac{1}{c_{44}^{v}} - \frac{T^4 n \Delta_1}{(c_{44}^v)^2 \epsilon_1} \int \frac{d^2 q\, q^4}{\varepsilon^4(q)}, \tag{7}$$

where $\varepsilon(q)$ represents the spectrum of the 2D related bosons. In the dilute limit $\lambda < a$ the Bogoliubov spectrum is given by $\varepsilon(q) = [T^4 q^4/(2\epsilon_1)^2 + 4\pi T^4 n_v q^2/\epsilon_1^2 \ln(\lambda^2 n_v)]^{1/2}$ [6], the expression for a weakly nonideal Bose gas. In the dense limit, $a < \lambda$, the related Bose system is equivalent to a system of 2D bosons with Coulomb interactions, which drive the acousticlike Bogoliubov spectrum to the plasmon spectrum with $\omega(q = 0) = \omega_p$. The plasmon frequency ω_p can be easily derived from the table of correspondence between boson and vortex quantities: $\omega_p^2 = 4\pi n \epsilon_0/\epsilon_1$. The dispersion $\varepsilon(q)$ at small wave vectors q has been derived in [13]. We propose the interpolation formula for the spectrum as

$$\varepsilon^2(q) = T^2 \omega_p^2 - 2\epsilon_0 T^2 q^2/\pi\epsilon_1 + (Tq)^4/(2\epsilon_1)^2. \tag{8}$$

The spectrum has a roton dip at $q = q_0 \simeq \epsilon\epsilon_0/T$. The transition line is estimated from the condition $1/c_{44}^{vR} = 0$. Evaluating the integral in (7) with the appropriate spectrum and equating the inverse tilt modulus to zero one immediately recovers Eqs. (3) and (5). Thus both limiting cases corresponding to the dilute, $a \gg \lambda$, and dense, $a \ll \lambda$, vortex systems can be obtained in a unique way by using the appropriate formula for the spectrum $\varepsilon(q)$ in the general formula for c_{44}^{vR} (7).

Shown in Figs. 1(a) and 1(b) are the two possible realizations of the Bose glass transition, corresponding to high, $B_m(T^*) < B_\Phi$, and low, $B_m(T^*) > B_\Phi$, concentrations of columnar defects. The first-order line is shifted to higher temperatures $T_{\rm BG}$ as compared to the melting temperature T_m in the clean crystals [1,5]: $T_{\rm BG} = T_m[1 + (\pi^2 c_L a/2d)(T^*/T_m)^2]$, and is expected to terminate around $T \approx T^*$. In the strongly disordered system [see Fig. 1(a)] the first-order line $B_{\rm BG}^{(1)}$ continues as a second-order transition $B_{\rm BG}^{(2)}$ according to Eqs. (3) and (5) at $B < H_{c1}$ and $B > H_{c1}$, respectively. The low field

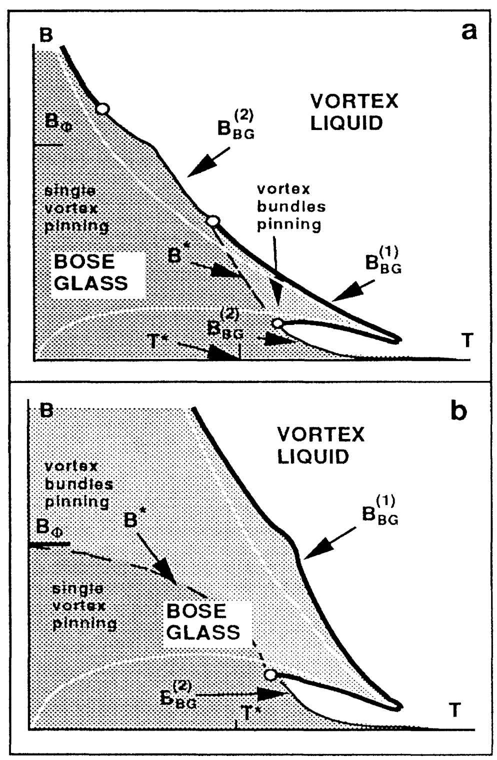

FIG. 1. Schematic phase diagram for superconductors with columnar defects $B_\Phi > B_m(T^*)$ (a) and $B_\Phi < B_m(T^*)$ (b). The thin white line is the pristine melting line $B_m(T)$. The dashed line denotes the accommodation field B^* [5] separating the single vortex and bundle-pinning regimes. Drawings are not to scale.

reentering Bose glass line $B_{\mathrm{BG}}^{(2)}$ reflects the localization of almost isolated individual vortices. The disorder effects increase when one follows the transition line towards lower temperatures as long as $B < B_\Phi$. For $B > B_\Phi$, the vortex system is increasingly dominated by vortex-vortex interactions, and we expect the first-order Bose glass line to recover and approach smoothly the original melting line. The maximum effect of columnar disorder is expected then around $B \approx B_\Phi$ [1,5].

We discuss briefly the effect of point defects. At small fields, $a > \lambda$, considerations analogous to those used in deriving Eq. (3) give rise to the transition line from a vortex liquid into a vortex glass as $B_{\mathrm{VG}} \simeq (H_{c2}\epsilon_1 \xi/T) \exp[-(T/T_{dp}^s)^3]$. The corresponding transverse localization length is $l \simeq TL_c/\epsilon_1$, where $L_c \simeq \epsilon\xi\sqrt{j_0/j_c}\exp[(T/T_{dp}^s)^3]$ is the pinning correlation length, and $T_{dp}^s = (\gamma\epsilon_1\xi^2)^{1/3}$ is the single vortex depinning temperature [1]. Note that with exponential accuracy, the same B_{VG} was obtained in [14] as the line where the dilute vortex liquid becomes unstable with respect to point disorder. The discrepancy in preexponential factors originates from the simplifying assumption of [14] that the vortex core radius coincides with the range of the interaction potential. The vortex glass transition line cuts the low temperature branch of the melting transition and then converts into a crossover line between single vortex and vortex bundle pinning regimes, traversing the solid domain almost vertically along T_{dp}^s. In the derivation of the intermediate field ($\xi < a < \lambda$) glass line, the disorder term on the rhs of Eq. (4) has to be substituted by $\sqrt{\gamma L_T/(\xi^2 + u^2)}$. As long as this term remains small as compared to the elastic term, point defects produce a small correction to melting. At higher fields, where analogously to the case of linear defects, the impurity contribution becomes of the order of the Lindemann term, $c_L^2 c_{66} a^2 L_T \sim \sqrt{\gamma L_T/(u_T^2 + \xi^2)}$, i.e., around the $T \simeq T_{dp}^s$, the first-order line terminates. Then the equation $T = \sqrt{\gamma L_T/(\xi^2 + u_T^2)}$ describes the crossover between the "conventional" and the very viscous pinned vortex liquid [3] (or, equivalently, vortex slush [15]). The shear modulus becomes finite at the second-order line according to the scenario proposed in [15].

Note that disorder fluctuations may localize vortices far from their equilibrium positions in the lattice. As a result the melting line (the onset of the quasi-long-range order), as detected, for example, by the onset of the neutron diffraction pattern or by the jump in the magnetization, may appear *below* the melting transition in the clean system. We expect this downward shift of the melting line to be especially significant for point disorder.

In conclusion, we have constructed a phase diagram for the Bose glass systems. We find an upward shift of the irreversibility line due to columnar defects at $B \lesssim B_\Phi$. We predict that the first-order transition terminates at critical points at temperatures where entropy barriers match the elastic ones and continue as a second-order glass transition in the regions where entropy barriers dominate the vortex elastic free energy. The critical points have to move to higher temperatures with increasing disorder. For sufficiently strong disorder (or in strongly anisotropic compounds where the melting line lies in a low temperature region) a lower second-order line lies *above* the turning melting point and merges with the dense limit transition line. Such a system experiences the second-order transition into a single-vortex glass and the low field glass reentrance disappears. For weaker disorder a double-reentrant transition can exist: Upon decreasing the field near the turning point the vortex liquid solidifies into a glass and then melts again according to the reentrant melting behavior of the pristine system. At lower fields the interactions between vortices become exponentially small and they get localized again (the isolated vortices are always localized). The low field glass corresponds to the localization of the individual vortices by disorder, whereas the glass near the turning point arises due to *collective* vortex bundles pinning.

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