Dynamics of Runaway Electrons in the Magnetic Field of a Tokamak

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An energy cap of runaway electrons is found experimentally by observing their bremsstrahlung spectra in the ASDEX tokamak. This observation is explained by analyzing the dynamics of runaway electrons, including acceleration in the toroidal electric field, deceleration due to synchrotron radiation losses, collisions with plasma particles, and a resonance between gyromotion and magnetic field ripple of the tokamak. For the dynamics of runaway electrons a Fokker-Planck equation in momentum space is developed.

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The collisional drag force of the plasma particles, exerted on a moving electron, decreases for superthermal electrons with increasing electron velocity. Thus an electric force acting on the electron exceeds the collisional drag for velocities higher than the so-called "critical velocity" where both forces are equal. The electrons, which are faster than this critical velocity, are continuously accelerated; i.e., they run away [1]. The runaway electrons moving on toroidal paths in the tokamak reach a maximal energy when the synchrotron radiation loss due to their motion on the curved path balances the energy gain in the electric field. The radius of curvature depends on the pitch angle, which is the angle between the direction of electron velocity and magnetic field. For zero pitch angle the radius of curvature is maximal and equals the major radius of the tokamak, and the typical maximum energy of the runaway electrons for the ASDEX tokamak [2] is 65 MeV. However, much lower energies are observed.

In ASDEX runaway electrons were generated at the beginning of a plasma discharge at 0.1 ± 0.01 s because of a low thermal electron density and high electric field. Because of higher thermal electron density (3×10^{19} m^{-3}) and lower electric fields (0.1 V/m), the generation of runaways is lateron negligible. Since most of the runaways are generated in a time interval of only 20 ms, they do not populate a plateau between the critical energy and the maximal energy of the runaways but form a monoenergetic beam with an energy width of only about 300 keV. Investigation of the dynamics of these runaways was done by observing the hard x-ray bremsstrahlung spectra generated by runaways hitting a tungsten target outside the plasma [3]. With increasing time the energies of the emitted quanta increase also, until for times above 0.8 s the bremsstrahlung spectrum stays constant (Fig. 1). This stationary spectrum is clearly generated by monoenergetic runaway electrons with energy 9.4 ± 0.2 MeV. Therefore a plateau distribution for the runaways, as mentioned before, can experimentally be excluded. From a free-fall model an energy of 24 MeV would be expected at 0.8 s. So the observed lower energy can easily be reached. The confinement limit [5], arising from the balance between the centrifugal force of runaways and the Lorentz force generated by the poloidal magnetic field via plasma current, is an additional upper boundary of the energy of runaways. Since the confinement limit for runaways in Fig. 1 (plasma current 240 kA) is 85 MeV, the observed maximum energy of the runaways is not a confinement loss. Also the low final energy of 9.4 MeV cannot be explained by additional poloidal motion of runaway electrons on drift surfaces [5], neglected so far, because this changes only slightly the radius of curvature and thus the synchrotron losses. A dramatic lowering of the energy stems from the gyromotion of the runaways having a finite pitch angle. The collisions of runaways with plasma particles increase the pitch angle but are not effective enough, so that the runaways are decelerated due to enhanced synchrotron losses. A very efficient mechanism to increase the pitch angle, and thus the synchrotron losses, which finally leads to the observed energy blocking of runaways, is the resonance of gyromotion with the nth harmonic of the magnetic ripple (Fig. 2), as will be shown. The ripple perturbation of the

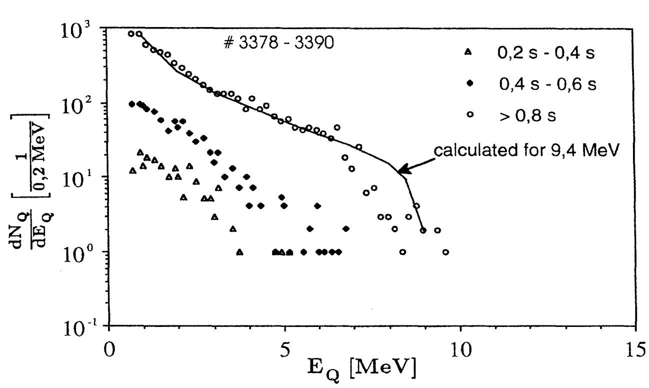

FIG. 1. Measured bremsstrahlung spectra of runaway electrons in the ASDEX tokamak (dN_Q/dE_Q the number of quanta per energy interval, E_Q the energy of the quanta). The stationary spectrum is clearly generated by monoenergetic runaway electrons [spectrum calculated with formula (2BS) of [4]].

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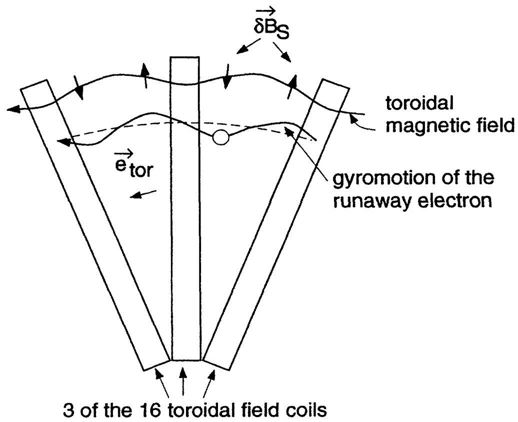

FIG. 2. Toroidal $n = 1$ ripple resonance: the runaways perform one gyrorotation between two toroidal field coils.

axisymmetric magnetic field of the tokamak is due to the finite number of coils generating the toroidal field. From the approximate resonance condition

$$\omega_{ce} = \frac{eB}{\gamma m_e} \approx \frac{nN_t}{R} c$$

(ω_{ce} is the gyrofrequency of the runaways, e the elementary charge, B the magnetic field, γ the relativistic gamma factor of the runaways, m_e the electron rest mass, n the toroidal harmonic number, $N_t = 16$ the number of toroidal coils, R the major radius, and c the speed of light) the toroidal resonance for the observed energy can be deduced. The maximum energy of runaways of 9.4 MeV ($B = 2.2$ T at $R = 1.65$ m) is resonant with the $n = 7$ toroidal harmonic of the magnetic ripple field. In another discharge with the same magnetic field and thermal electron density, but with a faster plasma current ramp-up (higher electric field), a final energy of the runaways of 13.4 MeV was reached, which is then the $n = 5$ toroidal ripple resonance energy.

Magnetic ripple field.—For the effect of energy blocking due to ripple resonance, only the component $\delta\vec{B}_s$ of the ripple perpendicular to the drift surfaces is important, since the components tangential to the drift surface only change the direction of the guiding center velocity [5] and not the partition of the energy between guiding center and gyromotion. $\delta\vec{B}_s$ can be described as a Fourier series in the toroidal and poloidal directions

$$\delta\vec{B}_s = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{\delta B_{nm}}{2} \sin(\vec{k}_{nm} \times \vec{r})\vec{e}_s$$

(δB_{nm} the Fourier components of the ripple magnetic field perpendicular to drift surfaces, $\vec{k}_{nm}$ the wave vector with toroidal component $k_t(n) = nN_t/R$, poloidal component $k_p(m) = (2\pi m)/C_d$, C_d the poloidal circumference of drift surface, $\vec{r}$ the positional vector, and $\vec{e}_s$ the direction perpendicular to the drift surface).

The strength of the magnetic ripple relative to the toroidal magnetic field is calculated to be typically of order $\delta B_{10}/B \approx 10^{-2}$ and $\delta B_{nm}/B \approx 10^{-7}$–$10^{-5}$ for $n \geq 2$, with δB_{nm} generally decreasing with increasing n, m.

Dynamics of runaway electrons in tokamaks.—When runaway electrons are being accelerated in the toroidal electric field, their energy increases, and their gyrofrequency ω_{ce} decreases. If one of the exact resonance conditions

$$\omega_{ce} = \frac{eB}{\gamma_{nm} m_e} = \vec{k}_{nm} \times \vec{v}_{gc}(\gamma, \vartheta)$$

[γ_{nm} is the relativistic gamma factor of runaways, resonant with the (n, m) component of the ripple and $\vec{v}_{gc}$ the guiding center velocity [5] of runaways] is fulfilled, the gyromotion of a runaway is in resonance with the nth toroidal harmonic of the ripple magnetic field. Since the gyrofrequency varies $\sim 1/R$ via the toroidal magnetic field and the ripple frequency $\vec{k}_{nm} \times \vec{v}_{gc}$ through k_t proportional to $1/R$, the runaways stay ripple-resonant during their motion along the drift surface, if they are so at one major radius. The Fourier components δB_{nm} do not vary along one drift surface. Thus, for a quantitative treatment of the ripple resonance effect, the toroidal drift surfaces can be transformed into planes in a slab geometry, where the magnetic field, independent of the major radius, is tangential and the ripple $\delta\vec{B}_s$ perpendicular to these planes. The dynamics of a runaway electron can then be described by the equation of motion

$$\frac{d\vec{p}}{dt} = -eE\vec{e}_{gc} - F_S\frac{\vec{p}}{p} - e\frac{\vec{p}}{\gamma m_e} \times \left[B\vec{e}_{gc} + \sum_{nm} \frac{\delta B_{nm}}{2} \sin\!\left(\vec{k}_{nm} \times \frac{\vec{p}}{\gamma m_e} t\right)\vec{e}_s\right]$$

($\vec{p}$ is the momentum vector of the runaways, E the toroidal electric field, $\vec{e}_{gc}$ the direction of the guiding center velocity, and t the time) with decelerating force due to synchrotron radiation [6]

$$F_S = -\frac{2}{3} r_e m_e c^2 \gamma^4 \left\langle \frac{1}{R_P^2} \right\rangle_t$$

(r_e is the classical electron radius) with the radius of curvature averaged over one gyrorotation,

$$\left\langle \frac{1}{R_P^2} \right\rangle_t = \frac{1}{R^2}\left[\cos^6(\vartheta) + \cos^2(\vartheta)\sin^4(\vartheta) + \cos^4(\vartheta)\sin^2(\vartheta)\left(\frac{5}{2} + \frac{r_g^2}{8R^2}\right)\right] + \frac{1}{r_g^2}\sin^4(\vartheta)$$

(ϑ is the pitch angle and r_g the gyroradius of the runaways).

Near a single ripple resonance the equation of motion can be simplified to

$$\dot{p}_{\rm gc} = eE - F_S \frac{p_{\rm gc}}{p} - e\frac{p_g}{\gamma m_e}\frac{\delta B_{nm}}{4}\cos(\delta),$$
$$\dot{p}_g = -F_S\frac{p_g}{p} + e\frac{p_{\rm gc}}{\gamma m_e}\frac{\delta B_{nm}}{4}\cos(\delta),$$
$$\dot{\delta} = k_{nm}\frac{p_{\rm gc}}{\gamma m_e} - \frac{eB}{\gamma m_e} =: \Delta\omega,$$

($p_{\rm gc}$ is the guiding center momentum, p_g the gyration momentum, and δ the phase difference between gyration phase and ripple phase).

For microscopic times $t \ll 2\pi/\Delta\omega$ the ripple resonance induces a reversible periodic exchange between the energy of gyromotion and of the guiding center motion. For macroscopic times $t \gg 2\pi/\Delta\omega$ the gyration momentum p_g changes in second order of $\cos(\delta)$ according to

$$\dot{p}_{g2} = -\frac{e\delta B_{nm}}{\gamma m_e}p_g F_S\frac{\cos^2(\delta)}{\Delta\omega}.$$

Thus, for $F_S \neq 0$, which is always fulfilled in a tokamak, the gyration momentum changes according to the position of $p_{\rm gc}$ compared to $p_{{\rm gc}\,nm}$, the guiding center momentum resonant with the (n,m) component of the ripple. The gyromomentum increases for $p_{\rm gc} < p_{{\rm gc}\,nm}$ ($\Delta\omega < 0$) and decreases for $p_{\rm gc} > p_{{\rm gc}\,nm}$ ($\Delta\omega > 0$). The effectiveness of this energy exchange can be quantified by the toroidal electric field necessary to cross the ripple resonance: Simulating the particle dynamics with the above equations, electrical fields of about 1 V/m are found to be necessary for crossing relative ripple strengths of $\Delta B/B \approx 10^{-7}-10^{-6}$. These are much higher than the typical toroidal electric field of 0.1 V/m. Since the effect of the coherent gyromotion in the ripple field is so strong, one would not expect runaways with energies as high as 10 MeV. A lowering of the ripple resonance efficiency is reached via an incoherent gyromotion of the runaways with respect to the ripple phase. For a diffusion of the gyromomentum, for which the phase difference between gyromotion and ripple must be sufficiently random, it was suggested [7] that neighboring ripple resonances should overlap, so that the runaway motion becomes chaotic and can be described by quasilinear diffusion. Since, e.g., at ASDEX and ASDEX Upgrade [8] the resonances do not overlap due to small ripple, this criterion cannot be applied there. However, the runaways collide with the plasma particles: This changes the gyration phase randomly. The Lorentz force accelerating the gyromotion then imposes a random walk on the gyromomentum. The effect of the collisions [9] is simulated by [10]

$$p(t+\Delta t) = p(t) + k_C\left\{-\left[1+\left(\frac{m_e c}{p}\right)^2\right] + \frac{2\epsilon m_e c}{p}\right\}\Delta t \pm \sqrt{2k_C\epsilon m_e c\Delta t},$$

$$\mu(t+\Delta t) = \mu(t)(1-2D_{\vartheta\vartheta C}\Delta t) \pm \sqrt{[1-\mu^2(t)]2D_{\vartheta\vartheta C}\Delta t},$$

with p the momentum of the runaways, $\mu = \cos(\vartheta)$, t the time, and Δt a small time interval;

$$k_C = \frac{e^4 n_e \ln\Lambda}{m_e c^2 4\pi\epsilon_0^2},$$

n_e the density of thermal electrons, $\ln\Lambda$ the Coulomb logarithm, c the speed of light, and ϵ_0 the dielectric constant;

$$D_{\vartheta\vartheta C} = k_C\frac{Z_{\rm eff}+1}{2q^3 m_e c}\sqrt{q^2+1},$$

$Z_{\rm eff}$ the effective ion charge and $q = p/m_e c$.

With these collisions the minimum electric field for crossing a ripple resonance of $\delta B_{nm}/B = 10^{-6}$ is lowered to about 0.1 V/m (Fig. 3). This is equal to the typical toroidal electric field in a tokamak. Thus runaways cannot cross a ripple resonance of $\delta B_{nm}/B > 10^{-6}$ for an electric field of 0.1 V/m, or, if the electric field is smaller than 0.1 V/m, they cannot overcome a ripple harmonic of $\delta B_{nm}/B = 10^{-6}$. Since for the observed seventh and fifth toroidal ripple resonant energies $\delta B_{nm}/B$ and E are of the values discussed above, the ripple resonance mechanism can quantitatively account for the observed energy gap of the runaways.

Fokker-Planck equation.—When collisions are taken into account, the critical electric field for crossing the ripple resonance is equal to the one obtained by simulating the dynamics of the ripple resonant runaways via a diffusion coefficient (Fig. 3), which is deduced below. Thus, with the above direct simulation of the particle dynamic, the use of a diffusion coefficient for the ripple resonance mechanism is justified. The diffusion coefficient for the gyromomentum in the ripple resonance is calculated by

$$D_{p_g p_g} = \lim_{dt\to\infty}\left[\left\langle\frac{dp_g(dt)dp_g(dt)}{dt}\right\rangle_{\delta(0)}\right]_{\Delta p_{\rm gc}}$$

with dt the time interval, $\langle\rangle_{\delta(0)}$ the averaging over the random phases $\delta(0)$ at $dt = 0$, and $[\,]_{\Delta p_{\rm gc}}$ the averaging over the resonance width,

$$\Delta p_{\rm gc} = \sqrt{\frac{\delta B_{nm}}{B}p_{\rm gc}p_g},$$

which yield

$$D_{p_g p_g} = \begin{cases} \dfrac{\pi}{32} \dfrac{eB}{\gamma m_e} \left(\dfrac{\delta B_{nm}}{B} \right)^2 \dfrac{p_{\mathrm{gc}\,nm}^3}{\Delta p_{\mathrm{gc}}} & \text{if } |p_{\mathrm{gc}} - p_{\mathrm{gc}\,nm}| < \Delta p_{\mathrm{gc}} \\ 0, & \text{else} \end{cases} .$$

The motion of runaway electrons can now be described by the Fokker-Planck equation in momentum space

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \vec{p}} \times [(\vec{F}_E + \vec{F}_C + \vec{F}_S + \vec{F}_R) f] = 0$$

(f is the distribution function), with the electrical force in polar coordinates [9]

$$\vec{F}_E = eE(\mu \vec{e}_q - \sqrt{1 - \mu^2} \vec{e}_\vartheta)$$

($\vec{e}_\vartheta$ pointing in the direction of increasing pitch angle ϑ and $\vec{e}_q = \vec{p}/p$), the friction force due to collisions of runaway electrons with plasma particles [9],

$$\vec{F}_C = \left[-k_C \left(1 + \frac{1}{q^2} \right) \vec{e}_q + q D_{\vartheta\vartheta C} \sqrt{1 - \mu^2} \frac{\partial}{\partial \mu} \vec{e}_\vartheta \right],$$

the retarding force due to incoherent synchrotron radiation,

$$\vec{F}_S = -\frac{2}{3} r_e m_e c^2 (q^2 + 1)^2 \left\langle \frac{1}{R_P^2} \right\rangle_t \vec{e}_q ,$$

and the diffusive flow of the pitch angle due to ripple resonance,

$$\vec{F}_R f = m_e c \frac{\sqrt{1 - \mu^2}}{q} \frac{\partial}{\partial \mu} (q^2 D_{\vartheta\vartheta R} f) \vec{e}_\vartheta ,$$

with $D_{\vartheta\vartheta R} = D_{p_g p_g} / p^2 \mu^2$.

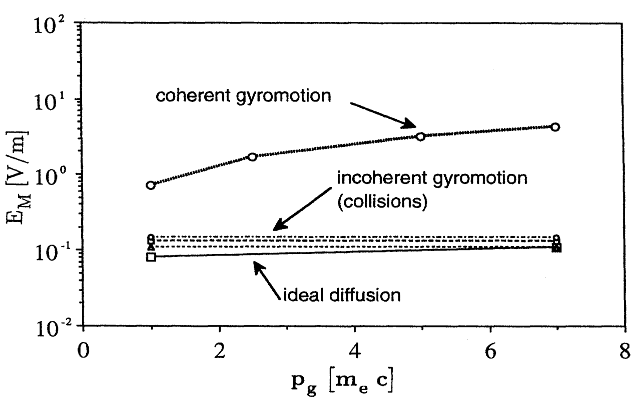

FIG. 3. Minimal electric field E_M for crossing a ripple resonance ($\Delta B/B = 10^{-6}$) depending on the gyration momentum p_g for incoherent motion due to collisions (thermal electron densities $n_e = (1-3) \times 10^{19}$ m^{-3}, effective ion charge equal to 1), for ideal diffusion and for coherent motion.

With the developed Fokker-Planck equation the dynamics of runaway electrons in a tokamak can be simulated if the time evolution of thermal electron density and electric field (corrected for the skin effect [5]) are known for a particular discharge.

Forming of an extremely monoenergetic electron beam.—When runaways accelerated in the toroidal electric field cannot cross a particular ripple resonance, they pile up at this resonance energy. Because of the isotropization of the pitch angle in a ripple resonance, runaway electrons are both accelerated and decelerated, depending on their momentary pitch angle. The energy spread of the resulting stationary distribution is smaller than 75 keV for the 9.4 MeV runaways, as calculated with the stationary Fokker-Planck equation. This is so small that the energy blocked runaways fulfill the conditions needed for a free-electron maser, which is indeed observed in ASDEX Upgrade [11].

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