Symmetry Nonrestoration at High Temperature and the Monopole Problem

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We show that there exists a range of parameters in SU(5) theory for which the grand unified theory symmetry remains broken at high temperature, thus avoiding the phase transition that gives rise to the overproduction of monopoles. The thermal production of monopoles can be naturally suppressed, keeping their number density below the cosmological limits.

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It has been known for a long time that the existence of magnetic monopoles (a single one would suffice) would lead to the quantization of electromagnetic charge. In grand unified theories (GUTs), based on a simple group (or their products), the electromagnetic charge is necessarily quantized and thus the magnetic monopoles are the necessary outcome of the theory. This, what should be a blessing, is, however, precisely what makes these theories incompatible with standard cosmology.

Namely, it is believed that at high temperature in the early Universe the spontaneously broken grand unified symmetry gets restored. If so, during the subsequent phase transition the monopoles get produced via the well-known Kibble mechanism [1] whenever the original symmetry based on a simple group G gets broken down to a subgroup H which contains (at least one) U(1) factor. The trouble is that the resulting monopole number density n_M would then be some 10 orders of magnitude bigger than the critical density of the Universe [2].

The crucial assumption in the above is the existence of a phase transition that separates the broken and the symmetric phases. The aim of this Letter is precisely to address this issue, namely, to see whether symmetry nonrestoration at high temperature [3,4] can avoid the monopole problem.

Previous approaches to the solution of these problems are well known. One is of course inflation [5]. Unfortunately, no satisfactory model of inflation resulting from a realistic particle physics theory exists at present, and in view of this it is of extreme importance to study alternative possibilities. Among "noninflationary" attempts we want to cite the one by Langacker and Pi [6] who have argued that a period of "temporarily" broken $U(1)_{em}$ in some high temperature interval may avoid the problem, due to a rapid annihilation of monopoles (produced in a phase transition at higher T) during this period.

In the present paper we want to take a more radical approach and argue that the phase transition which would produce the monopoles may not take place at all. The fact that symmetries may remain broken at high T was already noticed [3,4], and recently [7] it was shown that this effect may avoid the domain wall problem even in the minimal schemes of physically important discrete and continuous global symmetries, such as CP or Peccei-Quinn symmetry. However, symmetry nonrestoration is not *a priori* enough to solve the problem, since unwanted defects can be produced by thermal fluctuations. In the case of domain walls and global axionic strings, it was shown [7] that thermal production can be naturally suppressed for a wide range of parameters. However, there is a crucial difference in the case of monopoles: Domain walls (or axionic strings) are global defects and can be produced by gauge singlet fields; therefore there is a rather large choice of parameters for the suppression of their production rate. The scenario for monopoles turns out to be dramatically different and more restrictive, since it is controlled by the value of the gauge couplings.

The important question for us is whether or not (and under which conditions) the symmetry gets restored in the minimal realistic GUTs. Here we analyze the usual prototype grand unified theory based on the SU(5) gauge group in its canonical form. The heavy Higgs field responsible for the SU(5) breaking is taken to be in the **24**-dimensional adjoint representation H_{24}, and the light Higgs fields that break the standard model symmetry must belong to the **5**- and **45**-dimensional representations Φ_5 and Ψ_{45}. The minimal model is normally taken to consist of Φ_5 only, whereas the minimal realistic theory of fermion masses is believed to require the existence of Ψ_{45} too.

What is crucial for the monopole problem is whether or not the vacuum expectation value (VEV) of H_{24} vanishes at high temperatures. In the minimal model case we find that $\langle H_{24}\rangle \neq 0$ at high T seems to be in conflict with the validity of perturbation theory, whereas including Ψ_{45} we find that the symmetry nonrestoration is possible for a wide range of the parameters.

Of course, avoiding the phase transition with SU(5) nonrestoration does not automatically solve the problem. One has to suppose that the field is "initially" homogeneously distributed inside a region which is much larger than an instant horizon size, although smaller than a comoving scale of the size of the present horizon. This amounts to ask that the so-called horizon problem be solved by some mechanism (such as primordial inflation), and we leave it to the readers to choose their favorite. We emphasize, however, that such a mechanism must be invoked in any case for the standard cosmological model to be in agreement with observation, and that this requirement is not equivalent to the inflationary solution to the monopole problem: Whatever the solution is, it does not have to be related to the scale of symmetry breaking, as long as it is implemented at an earlier time.

Even without a phase transition and with uniform initial distribution, at high T monopoles can still be thermally produced in e^+e^- collisions, as was studied by Turner [8]. Fortunately, his analysis shows that for $m_M/T \gtrsim 35$ or so (where m_M is the monopole mass) the relic number density of monopoles is perfectly compatible with cosmology. We have studied the impact of this constraint on the broken SU(5) theory at high temperature and our analysis puts the minimal model in serious trouble, whereas once again the more realistic version with Ψ_{45} works out right.

Thus our work seems to indicate that the monopole problem is not an inevitable consequence of grand unification, but rather a dynamical question which depends on the spectrum and the parameters of the theory.

SU(5) theory at low and high T.—We first study the high T behavior of the minimal SU(5) theory with H_{24} and Φ_5 Higgs fields (we drop their subscripts hereafter). At $T = 0$ the Higgs potential is

$$\begin{aligned} V = &- m_H^2 \mathrm{Tr} H^2 + \lambda_1 (\mathrm{Tr} H^2)^2 + \lambda_2 \mathrm{Tr} H^4 \\ &- m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\ &- \alpha \Phi^\dagger \Phi \mathrm{Tr} H^2 - \beta \Phi^\dagger H^2 \Phi\,, \end{aligned} \tag{1}$$

where $H = \sum_{a=1}^{24} H_a \lambda_a$ and $T_a = \lambda_a/2$ are the generators of SU(5) for a **5**-dimensional representation such as Φ. The desired symmetry breaking $\mathrm{SU}(5) \xrightarrow{\langle H\rangle} \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with $\langle H\rangle = v_H\,\mathrm{diag}(1,1,1,-3/2,-3/2)$ implies the conditions

$$\lambda_2 > 0\,, \quad 30\lambda_1 + 7\lambda_2 > 0\,, \quad \beta > 0\,. \tag{2}$$

When the final stage of symmetry breaking is turned on through $\langle \Phi^T\rangle = (0, 0, 0, 0, v_\Phi)$, the minimum conditions require further

$$\begin{aligned} \lambda_\Phi > 0\,, \quad &(30\lambda_1 + 7\lambda_2)(40\lambda_2\lambda_\Phi - \tfrac{9}{2}\beta^2) \\ &- 3(10\alpha + 3\beta)^2 > 0\,. \end{aligned} \tag{3}$$

Conditions (2) and (3) play a crucial role in the study of the SU(5) phase diagram at high T. The computation of the effective Higgs potential at high T is rather complicated, but our task is facilitated by focusing on the leading terms of order T^2. Namely, we are interested in the high T phase diagram of SU(5) for $T \gg m_H$, and then we need the form of the T^2-dependent mass terms for the H and Φ fields.

In the approximation of weak couplings, assuming the validity of perturbation theory, one can use the general expression given by Weinberg [3],

$$\Delta V(T) = \frac{T^2}{24}\left[\left(\frac{\partial^2 V}{\partial\varphi_i \partial\varphi^i}\right) + 3(T_a T_a)_{ij}\, \varphi^i \varphi^j\right], \tag{4}$$

where T_a are the group generators and φ_i are the real components of the fields. For our potential this gives

$$\begin{aligned} \Delta V(T) &= \frac{T^2}{24}\left\{\left(48\lambda_\Phi - 96\alpha - \frac{96}{5}\beta + \frac{36}{5}g^2\right)\Phi^\dagger\Phi + \left(208\lambda_1 + \frac{376}{5}\lambda_2 - 20\alpha - 4\beta + \frac{15}{2}g^2\right)\mathrm{Tr}H^2\right\} \\ &\equiv m_\Phi^2(T)\Phi^\dagger\Phi + m_H^2(T)\mathrm{Tr}H^2. \end{aligned} \tag{5}$$

The above form has already been given in Ref. [9]. Now, since $\beta > 0$ and α too is allowed to be positive, one cannot make any *a priori* statements about the signs of the mass terms above. Actually, it was already noticed [9] that (5) allows for a negative mass for Φ, thus enabling the nonrestoration of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ symmetry. Since this is achieved at the expense of α,β being positive, it is easily seen that the coefficients in (5) make the nonrestoration of H much harder to achieve.

Notice first that the conditions (2) and (3) cannot allow both mass terms in (5) negative; but what about the coefficient of H? It turns out that the nonrestoration of $\langle H\rangle$ seems to require $\lambda_\Phi > 1$ and thus invalidates the weak-coupling expression (5). To see what is going on let us look at the simplified problem with λ_2 and β small. The conditions (2) and (3) now read ($\lambda_H = \lambda_1$)

$$\lambda_H > 0\,, \quad \lambda_\Phi > 0\,, \quad 4\lambda_H\lambda_\Phi > \alpha^2, \tag{6}$$

and $m_H^2(T) < 0$ requires

$$\alpha > \tfrac{52}{5}\lambda_H + \tfrac{3}{8}g^2. \tag{7}$$

It is easy to see that (6) and (7) imply

$$\lambda_\Phi > \left(\tfrac{26}{5}\lambda_H + \tfrac{3}{16}g^2\right)^2/\lambda_H\,, \tag{8}$$

and λ_Φ as a function of λ_H has a minimum at $\lambda_H = \frac{15}{416}g^2$. Thus we have a lower limit for λ_Φ,

$$\lambda_\Phi \geq \tfrac{39}{10}\, g^2. \tag{9}$$

Taking a typical value $g^2/4\pi \simeq 1/50$, this means $\lambda_\Phi \geq 1$. Clearly, the weak-coupling limit of (5) ceases to be justified.

Of course, the full computation must include the couplings α and β, and this requires a numerical analysis. We have performed it, and the end result is that (9) is not modified much. The point is that the couplings λ_1, λ_Φ, and α enter with the largest coefficients in (5), and thus it is more or less their role to determine whether or not the SU(5) symmetry may remain broken at high T ($T \gg m_H$).

We have seen above that the requirement of the validity of the perturbation theory points towards the usual assumption of the restoration of the SU(5) symmetry. Now, the analysis was performed for the minimal SU(5) model with the light Higgs Φ being **5** dimensional. But the minimal theory suffers from the problem of the fermionic spectrum being nonrealistic, namely, whereas $m_b \simeq m_\tau$ can be considered a success, this relation fails badly for the first two generations. It is generally believed that the realistic SU(5) theory must contain at least a **45**-dimensional multiplet (Ψ) needed to cure this problem. This prompted us to perform the above analysis for this, what should be considered a minimal realistic theory. Now, from the expression for the high T mass term in (5), it is clear (as we already remarked) that it is easier to keep the VEV of the smaller representation nonrestored, since α enters in its mass term with a much larger coefficient.

The analysis with **45** parallels the one performed above, and of course it gets even more messy. For the sake of space and since it worked well above, we present the computation in the limit of λ_2 and β small (and the analogous couplings for Ψ_{45} also small), i.e., we keep only α, λ_H, and λ_Ψ with λ_Ψ defined as in (1). More precisely, if we decompose Ψ into 90 real (45 complex) fields Ψ_i, we can write $V(H,\Psi)$ as in (1) with $\Phi^\dagger\Phi$ substituted by $\sum_{i=1}^{90}\Psi_i^2$.

Again, from the general form in [3], one can easily deduce the mass terms for Ψ and H at high T:

$$\begin{aligned} m_\Psi^2(T) &= \left(368\lambda_\Psi - 96\alpha + \frac{96}{5}g^2\right)\frac{T^2}{24}, \\ m_H^2(T) &= \left(208\lambda_H - 180\alpha + \frac{15}{2}g^2\right)\frac{T^2}{24}. \end{aligned} \tag{10}$$

Our point about the dimension of the representation and the nonrestoration of its VEV is manifested in (10): It is clearly much easier to keep $\langle H\rangle$ nonzero at high T (than $\langle\Psi\rangle$). With the condition for the boundedness of the potential

$$\lambda_H > 0, \quad \lambda_\Psi > 0, \quad 4\lambda_H\lambda_\Psi > \alpha^2, \tag{11}$$

we now obtain [instead of (8)]

$$\lambda_\Psi > \left(\frac{26}{5}\lambda_H + \frac{3}{16}g^2\right)^2 \frac{1}{81\lambda_H}. \tag{12}$$

Thus we get [instead of (9)]

$$\lambda_\Psi \geq \frac{13}{270}\, g^2. \tag{13}$$

Clearly λ_Ψ is allowed to remain small, while having $\langle H\rangle \neq 0$ at $T > m_H$.

Switching on other couplings in the potential does not change the results drastically. The bottom line is that SU(5) may remain broken at high T, thus avoiding the phase transition which leads to the disastrous overproduction of monopoles.

The monopole density.—As we mentioned in the introduction, the nonrestoration of symmetry, although necessary, is not sufficient to guarantee the nonoverabundance of monopoles. Monopoles can be thermally produced in e^+e^- (and other charged particles) collisions, and from the analysis by Turner [8] we know that their density depends crucially on m_M/T at these high temperatures. He finds out that in order to be consistent with cosmology we need

$$m_M/T \geq 35. \tag{14}$$

More precisely, for $m_M/T \geq 20$, he finds out

$$n_M/n_\gamma \simeq 3 \times 10^3 (m_H/T)^3 e^{-2m_M/T}, \tag{15}$$

where n_γ is the photon density; and from the upper limit $n_M/n_\gamma \leq 10^{-24}$ one obtains (14).

Now, in SU(5) the lightest monopoles weigh [10]

$$m_M = (10\pi/\sqrt{2}g)\, v_H. \tag{16}$$

For $g^2/4\pi \simeq 1/50$ or $g \simeq 1/2$, $m_M \simeq 40 v_H$, and thus the consistency with the cosmological bound (14) implies

$$v_H/T \geq 1. \tag{17}$$

From (1) and (5), we get for $T \gg m_H$

$$\frac{v_H^2}{T^2} = -\frac{208\lambda_1 + \frac{376}{5}\lambda_2 - 20\alpha - 4\beta + \frac{15}{2}g^2}{12(30\lambda_1 + 7\lambda_2)}. \tag{18}$$

Obviously (17) and (18) will put even more restrictive conditions on the parameters of the theory [than just (9) or (13)]. In any case, the analysis is straightforward and we quote the results for the simplified models with only λ_Φ (λ_Ψ), λ_H, and α couplings in the Higgs potential (1).

Let us see first what happens for the minimal model with Φ_5. For $\lambda_1 = \lambda_H$, from (6), (17), and (18) we get

$$\alpha > \tfrac{142}{5}\lambda_H + \tfrac{3}{8}g^2, \tag{19a}$$

$$\lambda_\Phi > \tfrac{213}{20}\, g^2. \tag{19b}$$

For $g^2 \simeq 1/4$, $\lambda_\Phi \geq 2.7$ and the perturbation theory clearly fails.

We repeat the same for the more realistic version with the Ψ_{45} representation. As before [compare with (9) and (13)], the condition (19b) relaxes by a factor of $1/81$, and we get

$$\lambda_\Psi > \frac{213}{1620} g^2, \tag{20}$$

which for $g^2 \simeq 1/4$ would give $\lambda_\Psi > 1/30$. Thus the largest coupling of the theory λ_Ψ is still quite small and the perturbation theory is operative.

In summary, whereas in the minimal model, at least in perturbation theory, the monopole problem persists; in the more realistic version we see that it may not be there. Since the realistic theory requires the existence of *both* Φ_5 and Ψ_{45}, the nonrestoration of $\langle H \rangle$ and the nonoverabundance of monopoles produced becomes only easier to achieve.

Unfortunately, from the exponential nature of the monopole density in (15), it is clear that due to the uncertainty in the Higgs couplings we cannot predict precisely the monopole density.

Summary and outlook.—Our results seem to indicate that the problem of monopoles may not be generic to GUTs. Whether or not there is an overabundance of monopoles is directly tied to whether the GUT symmetry is restored or not, and our analysis shows that the symmetry nonrestoration is in general allowed, but it depends on the spectrum and the couplings of Higgs scalars.

We have studied this issue in the prototype theory of all GUTs, the SU(5) model, and found out that the problem persists in its minimal version with the **5**-dimensional light Higgs, but that the more realistic variant with a **45**-dimensional Higgs included eliminates (potentially) the problem.

We wish to say a few words about the generality and the meaning of our results.

(i) Unlike inflation, the symmetry nonrestoration scenario does not result in a negligible present-day value of the monopole number density. Thus, the possibility remains open for monopoles to be the required dark matter. Whether or not the monopole density is large enough to allow for experimental detection is again related to the spectrum of the theory.

(ii) The important question is what happens in the supersymmetric version of the theory, which is favored from the point of view of the hierarchy problem and the unification of couplings. Unfortunately, at the level of the leading T^2 analysis for small gauge couplings, it has been shown [11] (in the context of global supersymmetry) that internal symmetries get restored at high T. This would imply the existence of the monopole problem in supersymmetric GUTs. It is worth investigating, though, the generality of these results, with, for example, the nonleading "daisy" diagram contributions to the high T behavior of the theory, but this is beyond the scope of this paper.

(iii) What about other GUTs, such as SO(10), E(6), etc.? It should be clear from our discussion that the situation will depend on the Higgs spectrum of the theory. In many popular models one assumes the existence of a large representation, such as, say, **126** in SO(10), used to provide the mass for the right handed neutrino. Obviously, the presence of such a large number of fields will help the nonrestoration of the GUT symmetry. We leave the analysis of the extended theories (with more detail on the high T analysis) for a longer paper in preparation.

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Note added.—After this paper was accepted for publication, we learned that a similar idea has been put forward (and completely ignored in the literature) 10 years ago by Salomonson, Skagerstan, and Stern [12]. These authors, however, ignored the effect of the gauge coupling, which as is clear from our analysis plays an important role. The recent work of Bimonte and Lozano [13], in which they compute the next-to-leading order corrections to the effective potential for the models considered here, indicates that symmetry nonrestoration may require either a smaller gauge coupling, or a complete analysis of the general Higgs potential including all the couplings (this work is now in progress).

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