Avalanches, Barkhausen Noise, and Plain Old Criticality

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We explain Barkhausen noise in magnetic systems in terms of avalanches of domains near a plain old critical point in the hysteretic zero-temperature random-field Ising model. The avalanche size distribution has a universal scaling function, making nontrivial predictions of the shape of the distribution up to 50% above the critical point, where two decades of scaling are still observed. We simulate systems with up to 1000^3 domains, extract critical exponents in 2, 3, 4, and 5 dimensions, compare with our 2D and $6 - \epsilon$ predictions, and compare to a variety of experiments.

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When materials are pushed, they can yield in different ways. Some crackle: they transform through a series of pulses or avalanches. In many systems, the behavior of these avalanches is unaffected by thermal fluctuations: one domain triggers, pushing some of its neighbors to trigger, in a deterministic process dependent on the static, quenched disorder in the material (and on the stress history). The statistics of Barkhausen noise (the avalanches seen in magnetic materials as the external magnetic field is ramped up and down) has been extensively studied experimentally [1–11]. We suggest that the zero-temperature random-field Ising model [12,13] provides a universal, quantitative explanation for many of these experiments.

A typical experiment will collect a histogram of pulse sizes, times, or energies. The distribution will follow a power law, which cuts off after two to several decades—much broader than any observed morphological feature in the materials. An explanation for the experiment must involve collective motion of many domains; it must provide an explanation for the power-law scaling regions, *and it must provide an explanation for the cutoff.*

Figure 1 shows the distribution $D_{\rm int}(S,R)$ of avalanche sizes integrated over the field H, for our model in 3D (discussed already in Ref. [12]), at several values of the microscopic disorder R. The model is a collection of domains $s_i = \pm 1$ coupled to an external field H, a local random field f_i (due to random anisotropies, grain morphology, or other defects) chosen from a distribution $\rho(f) = \exp(-f^2/2R^2)/\sqrt{2\pi}R$ of standard deviation R, and to its nearest neighbors s_j with an energy of strength $J = 1$. The domain s_i flips over when the net local field $F_i \equiv f_i + H + J\sum_{\rm nn} s_j$ seen at site i changes sign. Because of the nearest-neighbor interaction, a flipping spin often causes one or more neighbors to flip also, thereby spawning a whole avalanche of spin flips. Figure 1 shows the distribution of avalanche sizes found by integrating as the external field $H(t)$ is raised adiabatically from $-\infty$ to ∞ (the field is thus constant during the individual avalanches).

Notice three things about Fig. 1. (1) The distributions follow a power law, which cuts off after two to several decades. (2) The cutoff appears to diverge at a critical value of the disorder R_c, which we estimate in three dimensions to be $2.16J$. (3) The critical region is large. While the true power-law distribution is only obtained at $R_c = 2.16$, we get avalanches with more than a hundred domains all the way up at $R = 4$. This suggests that experiments can see decades of scaling without working hard to find the critical disorder. Several decades of scaling without tuning a parameter need not be self-organized criticality: it can be vague proximity to a plain old critical point.

Notice four more things about Fig. 1. (4) The straight line lying askew below the numerical data is our extrapolation to $R = R_c$ from scaling collapses, for the asymptotic power law $D_{\rm int}(S,R_c) \sim S^{-(\tau+\sigma\beta\delta)}$. The exponent

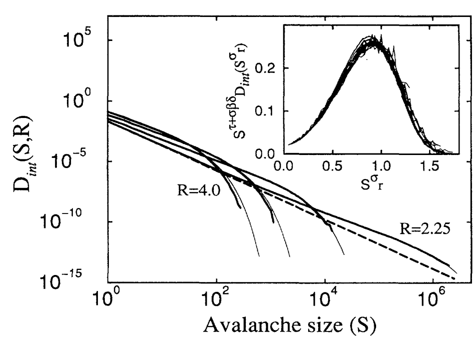

FIG. 1. Avalanche size distribution curves in 3 dimensions integrated over the external field. From left to right, the first three curves are for system size 320^3 and disorders 4.0, 3.2, and 2.6. They are averages over different intial random field configurations. The last curve is a 1000^3 run at $R = 2.25$, where $R_c = 2.16$ and $r = (R - R_c)/R$. The sizes plotted are large enough that there is no dependence of the cutoff position on the system size. The inset shows the scaling collapse of curves in 3D. The disorders range between 2.25 and 3.2; the top curves at $R = 3.2$ show noticeable 10% corrections to scaling. In the main figure, the avalanche size distribution curves obtained from the fit to this data (thin lines) are plotted alongside the raw data (thick lines). Notice that the scaling theory is predictive up to $R = 3.2$, 50% above R_c. The long-dashed straight line is the expected asymptotic power-law behavior $s^{-2.03}$. Notice that it does not agree with the measured slope of the raw data.

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TABLE I. Critical exponents obtained from numerical simulations [12,15] and experiments on Barkhausen noise in different magnetic materials (Fe, alumel, metglass [2], NiS [4], SiFe [5–7] 81% NiFe [8], AlSiFe [9], and FeNiCo [10]). The sample shapes were mostly wires. The quoted exponents were experimentally obtained from the pulse-area distribution in a small bin of the magnetic field H (exponent τ), the pulse-area distribution integrated over the entire hysteresis loop ($\tau + \sigma\beta\delta$), the distribution of pulse durations in a small bin of H [$(\tau - 1)/\sigma\nu z + 1$], the distribution of pulse durations integrated over the loop [$(\tau + \sigma\beta\delta - 1)/\sigma\nu z + 1$], the power spectrum of the pulses in a small bin of H [$(3 - \tau)/\sigma\nu z$], the power spectrum of the pulses integrated over the hysteresis loop {[$(3 - (\tau + \sigma\beta\delta)]/\sigma\nu z$}, and the distribution of pulse energies in a small bin of H [$(\tau - 1)/(2 - \sigma\nu z) + 1$]. Notice that these experiments are mostly done in geometries which minimize the effects of demagnetization fields.

Exponents	Simulation in 3 dimensions	Experiments in 3 dimensions
τ	1.6 ± 0.06	1.74, 1.78, 1.88 [2]; 1.5 ± 0.5 [4]; 1.33 [10]; 1.5–1.7 [5]
$\tau + \sigma\beta\delta$	2.03 ± 0.03	1.73–2.1 [8]
$(\tau - 1)/\sigma\nu z + 1$	2.05 ± 0.12	1.64, 2.1, 1.82 [2]; 1.7–2 [5]
$(\tau + \sigma\beta\delta - 1)/\sigma\nu z + 1$	2.81 ± 0.11	2.28 [8]
$(3 - \tau)/\sigma\nu z$	2.46 ± 0.17	Around 2 [2,9]
$[3 - (\tau + \sigma\beta\delta)]/\sigma\nu z$	1.70 ± 0.10	1.6 [6,7]; 1.8 [11]
$(\tau - 1)/(2 - \sigma\nu z) + 1$	1.42 ± 0.04	1.44, 1.58, 1.60 [2]

τ for the distribution at fixed H is shifted by $\sigma\beta\delta$ by the integration over H. (Only near a critical value H_c do large avalanches occur. The cutoff in the avalanche size at R_c goes as $|H - H_c|^{1/\sigma\beta\delta}$: σ gives the cutoff dependence with $R - R_c$, and β and δ, as usual [12], give the singularities of the magnetization with $R - R_c$ and $H - H_c$, respectively. The exponent for any quantity varying H at R_c is given by multiplying $1/\beta\delta$ by the exponent for the singularity varying R at H_c.) The obvious experimental method of taking the slope on the log-log plot gives the wrong answer until many, many decades of scaling are obtained. (5) The inset shows the collapse of the data onto a scaling function

$$\mathcal{D}_{\rm int}(s^{\sigma}(R - R_c)/R) = \lim_{R\to R_c} s^{\tau+\sigma\beta\delta} D_{\rm int}(S,R)\,, \tag{1}$$

which is a universal prediction of our model: real experiments rescaled in the same way should look the same (apart from overall vertical and horizontal scales). This scaling function is quite unusual: it grows by a factor of over ten before cutting off. This bump is the reason that the experiments take so long to converge to the asymptotic power law. To make a definite prediction, we have phenomenologically fit our curve [$x = s^{\sigma}(R - R_c)/R$]

$$\mathcal{D}_{\rm int}(x) = (0.021 + 0.002x + 0.531x^2 - 0.266x^3 + 0.261x^4)\exp(-0.789x^{1/\sigma})\,, \tag{2}$$

where we guess the error in the curve to be less than 10% within the range $0.2 < x < 1.2$. (6) In the main figure, the scaling form passes through our data quite well, even far from R_c. The scaling theory is predictive for curves with only two or three decades of scaling. The critical region starts when the correlation length (and hence the avalanche size cutoff) becomes large—not only when the pure power law takes over. Using Eqs. (1) and (2) and the values $\sigma = 0.24 \pm 0.02$ and $\tau + \sigma\beta\delta = 2.03 \pm 0.03$, an experimentalist should be able to fit any *single* histogram of avalanches, shifting only the overall vertical and horizontal scale factors. (7) Only by collapsing curves at several values of the disorder R were we able to extract accurate critical exponents. We suggest that experimentalists try varying some parameter of their system (annealing time or temperature, grain size, impurity concentration, etc.) and observe the resulting cutoff dependence. Any *family* of histograms thus generated should, near the critical point, be fit with three parameters (horizontal scale, vertical scale, and R_c).

A comparison of our predicted exponents with power laws extracted from a number of experiments on magnetic Barkhausen noise in bulk three-dimensional systems is shown in Table I. Only one of the experiments [8] varied a parameter; they varied the annealing time and saw a shift in the cutoff, but did not extract a critical annealing time or do scaling collapses. The range of values for $\tau + \sigma\beta\delta$ observed in the experiments is compatible with the range of log-log slopes we observe due to the unusual scaling form for the integrated avalanche size distribution $D_{\rm int}$ discussed above.

The largest set of experiments measure the avalanche size distribution in a narrow range of fields (i.e., without averaging over the entire hysteresis loop): their power laws fits are a measure of the critical exponent τ. Thescaling function for the nonintegrated avalanche size distribution [12], we find, does not have a bump. The experimental measurements for τ are close to our numerical estimate.

The other experiments (pulse durations, power spectra, and pulse energies) introduce a new exponent combination $\sigma\nu z$. The correlation length exponent ν gives the divergence of the characteristic spatial extent of avalanches with $R - R_c$, and νz gives the divergence of the avalanche durations with $R - R_c$. The critical exponent z occasionally can depend on the details of the spin dynamics [15]; it is not even clear whether our simulation must have the same value of z as our ϵ expansion. Nonetheless, the agreement between our predictions and the measured values are about as good as the agreement between the different measurements.

We have also investigated the application of our model to other systems. Many experiments are done in effectively two-dimensional systems (magnetic hysteresis [16], avalanches as the field is swept in superconductors [17], and avalanches as helium is injected into Nuclepore [18]); our 2D explorations are still rather preliminary. Our model does not fit the avalanche size distributions measured in 3D martensitic transitions as the temperature is ramped [19]: Their measurement of the pulse duration exponent $(\tau + \sigma\beta\delta - 1)/\sigma\nu z + 1 \sim 1.6$ is significantly different from our prediction of 2.81 ± 0.11, possibly due to long-range elastic interactions. Full explanations about the various exponent combinations measured in different systems and detailed discussions of experiments and systems [20] are forthcoming.

Figure 2 shows the results for five of our exponents ($\tau + \sigma\beta\delta$, τ, $1/\nu$, $\sigma\nu$, and $\sigma\nu z$) measured in 2, 3, 4, and 5 dimensions. (From these one can get β and δ separately using the exponent equality [21] $\tau + \sigma\beta\delta = 2 + \sigma\beta$.) Our previous simulations in 3 dimensions [12] for sizes up to 200^3 gave estimates of τ and $1/\nu$ which are significantly different from the values in Fig. 2 because of large finite size effects. We measure the exponents in the (unphysical) dimensions four and five in order to test our renormalization-group predictions [13,21] for the behavior near six dimensions. First, notice that the numerical values converge nicely to the mean-field predictions as the dimension approaches six, and that the predictions of our $6 - \epsilon$ expansion do remarkably well. (The primary role of the renormalization-group treatment, of course, is to explain why scaling and universality might be expected in these systems.) The short-dashed lines for $1/\nu$ show three different Borel resummations [22] of the series to fifth order in ϵ: By mapping our model to all orders [21] onto the regular Ising model in two lower dimensions [23], we have been able to use [22] the series known to order ϵ^5 for ν. The long-dashed lines are predictions for the other exponents to first order in ϵ. These exponents have no equivalent in the equilibrium model: We have developed [21] a new method for calculating them using two replicas of the system.

Second, notice the exponents in two dimensions. We conjecture [21] that the 2D exponents will be $\tau + \sigma\beta\delta = 2$, $\tau = 3/2$, $1/\nu = 0$, and $\sigma\nu = 1/2$. It is likely that two is the "lower critical dimension" for our system, below which all avalanches will be finite except at zero disorder. We must mention that our firm conjectures about the exponents in two dimensions must be contrasted with our lack of knowledge about the proper scaling forms. We have used three different scaling forms [14], and have obtained values for the critical disorder that are either 0.54 or 0, and values for $1/\nu$ of 0.15 and 0, respectively. Amazingly enough, the exponents plotted in Fig. 2 were largely independent of which scaling form we used. The error bars shown span all three *ansätze*, and are compatible with our conjectures above. Vives *et al.* [24] have measured the critical exponents and disorder in 2 dimensions for this model. Their values differ significantly from ours for R_c and substantially for $1/\nu$ (in particular, we found that all the avalanches are finite well below their value of R_c). This is easily explained: At the lower critical dimension, things can asymptote to their thermodynamic values very slowly so finite size effects can be large. Their simulation was done for system sizes of 100^2, while our system size is typically 5000^2.

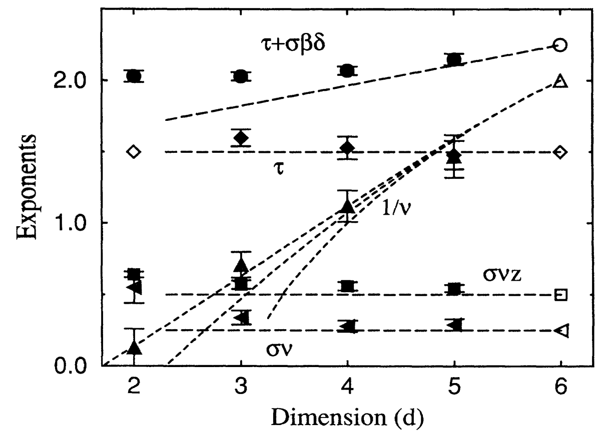

FIG. 2. Numerical values (filled symbols) of the exponents $\tau + \sigma\beta\delta$, τ, $1/\nu$, $\sigma\nu z$, and $\sigma\nu$ (circles, diamond, triangles up, squares, and triangle left) in 2, 3, 4, and 5 dimensions. The empty symbols are values for these exponents in mean field (dimension 6). Note that the value of τ in 2D was not measured. The empty diamond represents the conjectured value (see text). We have simulated sizes up to 7000^2, 1000^3, 80^4, and 50^5, where for 320^3, for example, more than 700 different random field configurations were measured. The long-dashed lines are the ϵ expansions to first order for the exponents $\tau + \sigma\beta\delta$, τ, $\sigma\nu z$, and $\sigma\nu$. The short-dashed lines are Borel sums [22] for $1/\nu$. The lowest is the variable-pole Borel sum from LeGuillou and Zinn-Justin [22], the middle uses the method of Vladimirov *et al.* to fifth order, and the upper uses the method of LeGuillou *et al.* (but without the pole and with the correct fifth order term). The error bars denote systematic errors in finding the exponents from collapses of curves at different values of disorder R. Statistical errors are smaller.

We are not the only ones to model avalanche behavior in disordered magnets. There has been much work on depinning transitions and the motion of individual interfaces [25,26]; our system, with many interacting interfaces, perversely seems much simpler to analyze. Many have studied related models with random bonds [24,27] and random anisotropies; random fields are actually rather rare in experimental systems. We now believe on symmetry grounds that all these systems are in the same universality class (as argued numerically [24] and previously shown for depinning [26]). The external field H_c at the critical point breaks the rotational and up-down symmetries of these models (and of the experiments), and the spins which flip far from the critical point [roughly $M(H_c)$] act as random fields. Long-range forces [10] and correlated disorder (e.g., dislocation lines) likely lead to closely related but distinct universality classes.

This paper gives definitive quantitative measurements from simulations of up to 10^9 spins for the model described in [12]. We show that these results are consistent with the predictions of the ϵ expansion [13,21]. The comparison to experiments showing Barkhausen noise is promising, and we argue that although the microscopics varies for different experiments, they all belong to the same universality class.

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