Dipole Moments on Dust Particles Immersed in Anisotropic Plasmas

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The process of dust particulate charging in plasmas is investigated in the presence of bulk drift velocities. An accurate self-consistent particle-in-cell simulation method is used. The results show the presence of dipole moments on dielectric dust particles. This effect is relevant to studies of dust coagulation, as it can enhance the coagulation rate. A theoretical analysis of the effect is presented. The theory represents correctly the relevant physical processes and provides a good estimation of the charge and the dipole moment on dust particles.

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The different mobility of ions and electrons causes objects immersed in plasmas to charge. At equilibrium, in the absence of secondary emission and photoemission, a net negative charge accumulates which repels electrons and attracts ions.

Different theories of dust charging have been presented [1,2], and reasonable agreement with experiments has been found for resting bodies in Maxwellian plasmas [2]. More recently, simulations of dust charging have been published [3]. The results are limited to Maxwellian plasmas with ion to electron mass ratio $m_i/m_e = 5$ and to resting bodies in an idealized 2D geometry where the dust particles are represented by infinite cylinders. Theoretical predictions and simulation results are in reasonable agreement, although differences as high as 15% have been found for the charge collected by the dust [3]. Similar agreement has been found in the simulations of dust charging in plasmas for industrial applications [4].

In the present Letter, we investigate the effects of relative motion between dielectric dust particles and the ambient plasma. We find that the relative motion is responsible for a dipole moment on the dust particles. Such an effect previously has not been investigated thoroughly. In studies of spacecraft charging, it has been proposed that nonuniform charging may alter the measurements of sensors mounted on space probes [1]. More recently, it has been suggested that dust in motion through a plasma develops dipole moments [5].

In the present Letter, we investigate the problem with accurate simulation methods for a realistic geometry and for the physical mass ratio of hydrogen. Conditions typical of glow discharges for industrial plasma applications are considered. The results show convincingly that a relative drift between objects and plasmas causes dipole moments. We will also discuss the mechanism responsible for the effect and provide a theoretical estimate based on a simple extension of existing theories.

The importance of the effect reported here can be understood, for example, in connection with the studies of coagulation of small dust particles to form large clusters. Evidently, the presence of charges of the same sign on the dust particles reduces the rate of coagulation. An intrinsic dipole moment could reduce the repulsion and account, at least in part, for the underestimation of the observed coagulation rates by current theoretical predictions [6]. Furthermore, dipole moments may be responsible for the formation of strings of spherical particles [7]. It has been also proposed that dipole moments can be responsible for the structures observed in crystallized dusty plasmas [8].

Numerical Experiments.—We investigate the charging of a spherical, perfectly dielectric, dust particle immersed in a collisionless plasma. In the initial configuration, the plasma has a uniform density n and the velocity distribution is Maxwellian with a drift velocity $\mathbf{w}$ relative to the dust particle. The ion to electron mass ratio is $m_i/m_e = 1836$. The temperature ratio is chosen to represent conditions relevant to plasma processing reactors: $T_i/T_e = 0.05$. We limit the study to isolated dust particles, i.e., to configurations where the interparticle distance is much larger than the electron Debye length $\lambda_{\rm De}$. No secondary emission or photoemission is considered.

The system evolves according to the Vlasov-Poisson model for ions and electrons. To represent correctly dielectric dust, the ions and electrons that hit the surface are stopped and kept still. During the evolution, the particle distribution function changes in the neighborhood of the dust particle and is calculated self-consistently from the Vlasov equation. The charge increases monotonically but with a rate decreasing exponentially. For the simulations described below, the charging time required to reach a steady state is of the order of $(50-100)\,\omega_{pe}^{-1}$.

The dust particle represents a continuous drain and, strictly speaking, only an infinite system can reach a steady state. We addressed the problem using a system of very large size or, alternatively, introducing an injection boundary. The two methods give similar results, when the system size is large enough. In the results below, a buffer plasma region, $20\lambda_{\rm De}$ in radius, surrounds the dust particle.

The simulation method is based on an implicit version of the particle-in-cell (PIC) algorithm [9]. Some enhancements have been introduced to address specific

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difficulties arising in the simulation of dust charging. Typically, the size of the dust particle is smaller than λ_{De}; however, the system size must be many Debye lengths to simulate the infinite medium.

To handle the disparity of length scales, we used an adaptive grid [10]. The computational grid used to solve Poisson's equation is nonuniform, with grid spacing much finer ($\lambda_{De}/20$) around the dust particle where a sheath is formed. Furthermore, the number of particles used to simulate the Vlasov equation is controlled to focus the attention around the dust particle. In its neighborhood, the computational particles are split to reduce the noise [11]. This method reduces the noise without altering the physical properties of the system (e.g., temperature, density, or Debye length).

Finally, the dust particle is treated with the immersed boundary method [12]. The immersed boundary method can be used in plasma simulations to represent complex surface processes and to describe solid objects of arbitrary shape [13].

The present code has been compared with the existing results in Ref. [3], for a 2D Cartesian geometry. Complete agreement has been found.

Next we studied 3D spherical dust particles using cylindrical coordinates. The vertical coordinate is chosen along the direction of the relative motion, $\hat{\mathbf{z}} = \mathbf{w}/w$. The problem is axisymmetric and the azimuthal coordinate becomes immaterial. The simulation method allows us to explore different plasma conditions and system configurations.

Figure 1 reports the charge accumulated by a spherical object of radius $a/\lambda_{De} = 0.4$, as a function of the Mach number $M = wm_i^{1/2}/(kT_e)^{1/2}$. The numerical experiments (circles in Fig. 1) show very clearly that the size of the negative charge accumulated by the dust increases first, reaching a maximum for $M \approx 2$, and then decreases.

Figure 2 shows, for the same simulations as in Fig. 1, the vertical component of the dipole moment D with respect to the center of the dust particle. The other two components are zero, thanks to the axisymmetry of the system. The results (circles in Fig. 2) show that a dipole moment is present in the vertical direction. The sign is negative, corresponding to a less negative side on the hemisphere facing the drifting plasma. The size of the dipole moment is zero at $w = 0$ and increases with increasing w.

To gain more insight into the processes described above, a comparison with probe theory is provided. The total charge accumulated by a dust particle can be compared with theoretical estimates.

Reference [14] reports an approximate expression for the ion and electron currents to a sphere, as a function of the drift velocity w and of the electrostatic potential ϕ on the surface of the object. The expression is derived assuming small dust particles ($a \ll \lambda_{De}$) and spherical symmetry of the electric potential. Clearly, dipole moments break spherical symmetry, and the theoretical results must be regarded as a rough estimate. The surface potential ϕ is obtained imposing the balance of ion and electron currents:

$$J_e(\phi, w) + J_i(\phi, w) = 0. \quad (1)$$

Using the analytic expressions in Ref. [14] for J_e and J_i, the equilibrium potential ϕ can be derived for any drift velocity w from the algebraic Eq. (1) [14,15]. The charge on the dust particle is linearly related to the surface potential: $Q = C\phi$. The capacitance C depends on the

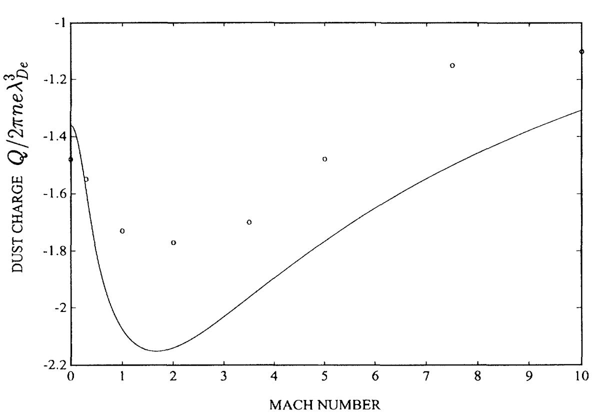

FIG. 1. Charge Q accumulated on a spherical dielectric dust particle immersed in a hydrogen plasma with $T_i/T_e = 0.05$, as a function of the Mach number M. Simulation (circles) and theoretical (solid line) results are compared.

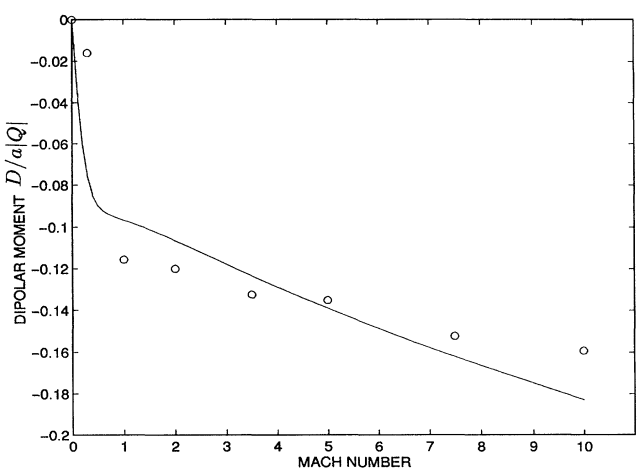

FIG. 2. Vertical component of the dipole moment D accumulated on a spherical dielectric dust particle immersed in a hydrogen plasma with $T_i/T_e = 0.05$, as a function of the Mach number M. Simulation (circles) and theoretical (solid line) results are compared.

potential distribution around the dust particle. For drifting plasmas, C cannot be evaluated exactly; in the limit of small a/λ_{De}, the vacuum expression $C = 4\pi\epsilon_0 a$ can be assumed [1,14].

Figure 1 compares the simulation results (circles) with the solution of Eq. (1) (solid line). The qualitative agreement is satisfactory; the theory represents correctly the dependence of the charge Q upon the drift velocity w. Equation (1) tends to overestimate the maximum of the negative charge (at $M \approx 2$), but the difference never exceeds 20% over the range considered.

We further note that the literature offers only intuitive explanations of the behavior of Q as a function of w [15]. Figure 3 shows the ion flux to the surface, from Ref. [14], as a function of the drift velocity w and of the surface potential ϕ. Two counteracting effects are observed.

First, in the absence of surface potentials, the ion flux increases monotonically with the drift velocity w, which tends to decrease the imbalance between ion and electron fluxes and ultimately to reduce the negative charge on the dust particle.

Second, the slope of J_i with ϕ, $\partial J_i/\partial\phi$, decreases with w. The drift reduces the ability of the potential to bend the trajectories and attract more ions [2]. This effect is dominant at first, leading to an increase of the size of the dust charge with w. At higher velocities, the first effect becomes more relevant and the magnitude of the negative charge decreases after reaching a maximum at $M \approx 2$.

The theory presented above assumes spherical symmetry and cannot predict dipole moments. To include this effect, Eq. (1) must be modified to include the variation of ϕ on the surface of the sphere. The axisymmetry of the problem allows us to consider only variations with the latitude variable μ (cosine of the angle between the normal $\hat{\mathbf{n}}$ to the surface and the unit vector $\hat{\mathbf{z}}$ along the vertical coordinate).

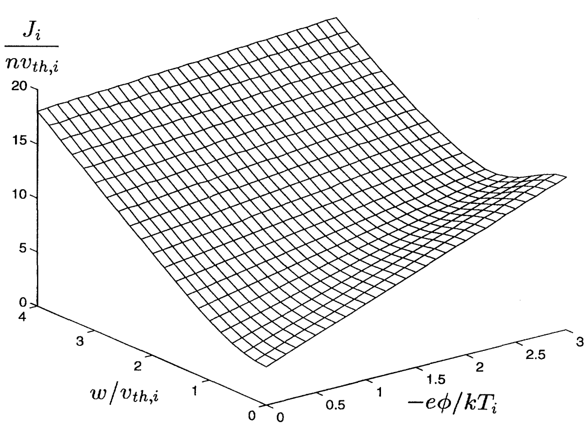

FIG. 3. Ion flux J_i to the surface of a spherical dust particle as a function of the drift velocity $w/v_{th,i}$ and of the surface potential $e\phi/kT_i$.

The flux j_s of particles of species s (ions or electrons) entering the dust surface at a latitude μ is

$$j_s = \int_{\mathbf{v}\cdot\hat{\mathbf{n}}<0} \mathbf{v}\cdot\hat{\mathbf{n}} f_s(\mu,\mathbf{v})d\mathbf{v}\,. \tag{2}$$

To evaluate Eq. (2), the distribution function f_s in the proximity of the surface is required and the solution of the Vlasov-Poisson system must be performed. Simplifying hypotheses are required to derive an analytical approximation of Eq. (2). The simplest approach capable of describing correctly the presence of dipole moments is to assume that the dust charging does not affect the angular dependence of the current. This assumption is only approximately correct but is a considerable improvement over standard theories which assume isotropy.

Under this hypothesis, the angular dependence can be factorized from the effects of surface charges:

$$j_s(\mu,\phi,w) = J_s(\phi,w)F_s(\mu,w)\,. \tag{3}$$

The total current J_s has the expression used in Eq. (1) [14]. The angular shape F_s is considered independent of the surface charge and can be calculated from Eq. (2) assuming the initial unperturbed distribution. Using in Eq. (2) the appropriate expression for a drifting Maxwellian, the unperturbed particle flow j_{s0} is

$$j_{s0}(\mu,w) = \frac{nv_{th,s}}{2\sqrt{\pi}}\exp\left(-\frac{\mu^2w^2}{v_{th,s}^2}\right) - \frac{n\mu w}{2}\,\mathrm{erfc}\left(\frac{\mu w}{v_{th,s}}\right), \tag{4}$$

where $v_{th,s}$ is the thermal velocity. From its definition, Eq. (3), the angular shape F_s is then obtained from Eq. (4) dividing by the total current: $F_s = j_{s0}/\int j_{s0}d\mu$.

Figure 4 shows F_s as a function of the drift velocity w. Clearly, the anisotropy increases with the ratio between the drift velocity and the thermal speed: $w/v_{th,s}$. For the values of the Mach number considered here ($M = 0-10$), the ions are anisotropic but the electrons remain essentially isotropic. Since there is a higher ion flux to the hemisphere facing the flow, a negative dipole results.

The balance of ion and electron currents requires that

$$J_e(\phi,w)F_e(\mu,w) + J_i(\phi,w)F_i(\mu,w) = 0\,. \tag{5}$$

Equation (5) replaces Eq. (1) and gives the value of the surface potential as a function of the latitude μ. The dipole moment can be evaluated explicitly, assuming that the effect of the anisotropy is small. From Eq. (5), with a Taylor series expansion around the average surface potential ϕ_0 given by Eq. (1), the dipole moment can be written as

$$D = -aC\frac{J_i(w,\phi_0)\int F_i\mu d\mu + J_e(w,\phi_0)\int F_e\mu d\mu}{2(\partial J_i/\partial\phi|_{\phi_0} + \partial J_e/\partial\phi|_{\phi_0})}\,. \tag{6}$$

Figure 2 compares the dipole moments from the simulations (circles) with the results from Eq. (6) (solid line). The correct dependence upon M is obtained, and a remarkably good quantitative agreement is shown.

Dipole moments affect the coagulation rate. An approximated expression for the coagulation rate A is derived in [6]: $A = A_0(1 - W/2kT_D)$, where A_0 is the coagulation rate in absence of dust charging, W is the interaction energy between two dust particles in contact, and T_D is the temperature of the dust. The interaction energy W must not include the effect of the Debye shielding because coagulation implies close encounters at distances where the shielding is not effective. The effect of the dipole moment on the dust particles is to alter the interaction energy. Assuming the chain configuration where the dust particles are aligned tail to head [7], the ratio of the dipole to Coulombic energy is $(D/aQ)^2/2$. Using the results in Fig. 2, the ratio is usually small, but it has a great relative effect for large dust particles where the coagulation rate A is small: in that case the effect of the dipole moment is strong. Furthermore, the dipole moment can align the dust particles preferentially to form chains, as observed in experiments [7].

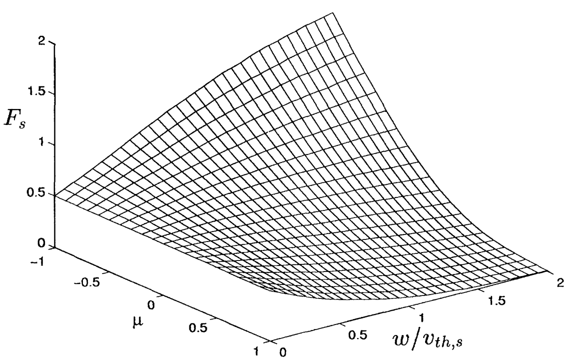

FIG. 4. Angular shape function F_s for species s (ions or electrons) as a function of the drift velocity $w/v_{th,s}$ and of the latitude μ.

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