Elastic Electron Scattering from the Deuteron Using the Gross Equation

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(Received 21 July 1995)

The elastic electromagnetic form factors for the deuteron are calculated in the context of a one-boson-exchange model using the Gross or Spectator equation. The formalism is manifestly covariant and gauge invariant, and provides a very good representation of the data.

PACS numbers: 25.30.Bf, 13.40.Gp, 13.75.Cs, 27.10.+h

With CEBAF now coming on-line, it will be routine to probe nuclear systems with electron scattering, where the energy and momentum transfers will be well in excess of the nucleon mass. Under such circumstances the usual nonrelativistic description of the nucleus is no longer reliable, and it is necessary to develop relativistically covariant models of the nuclear system. At such energies and momentum transfers it may also be necessary to take the underlying quark structure of the nucleon into account.

In this Letter we present a covariant calculation of the elastic electromagnetic form factors of the deuteron based on meson field theory. Carried out in the context of the Gross equation [1], our calculation is manifestly Lorentz covariant and has been constructed to be gauge invariant, and can be viewed as a covariant generalization of nonrelativistic potential models based on the physics of meson exchange. It includes some effects of the underlying quark-gluon structure of nucleons and mesons through the introduction of phenomenological form factors, but omits other effects, such as those which might arise from a possible six quark component of the deuteron wave function. A comparison of our results with the data may indicate the size of any quark effects not included in meson field theory. We will show that by carefully constraining the interaction model to fit the nucleon-nucleon phase shifts and by using only minimal exchange currents we are capable of obtaining an excellent description of the available data. This is due largely to the characteristics of small wave function components of relativistic origin.

The Gross equation [1] is a quasipotential equation [2,3] in which the relative energy is constrained by restricting one of the nucleons to its positive energy mass shell. The application of the Gross equation to the calculation of nucleon-nucleon scattering and the deuteron bound state is described in considerable detail in Ref. [4], which uses a one-boson-exchange (OBE) interaction kernel, explicitly antisymmetrized in order to ensure that the Pauli principle is exactly satisfied. The meson-nucleon couplings used in the OBE kernels include off-shell couplings and form factors which depend upon the invariant masses of the three virtual particles connected to the interaction vertex. For simplicity, we assume that these form factors can be written in a factorable form [4,7]

$$F(p'^2, p^2, \ell^2) = h(p'^2)h(p^2)f(\ell^2), \tag{1}$$

where p and p' are the initial and final nucleon four-momenta, $\ell = p - p'$ is the meson four-momentum, and $f(\ell^2)$ and $h(p^2)$ are meson and nucleon form factors, respectively. These form factors are given by Eqs. (3.9) and (3.13) of Ref. [4].

Four models for the *NN* interaction were presented in Ref. [4]. Each model was fitted to the *NN* phase shift data [5] and constrained so that the deuteron bound state mass is correct. The interaction model used in the calculations shown here is a variation on model IIB, in which the parameters of the model have been adjusted to fit the Nijmegen energy dependent *np* phase shifts [6]. This model uses a one-boson-exchange kernel containing six mesons: π, η, σ, σ_1, ω, and ρ, where the σ_1 meson is a scalar-isovector companion to the σ with a mass comparable to the σ mass. The pion mixing parameter was fixed at $\lambda_\pi = 0$ for pure pseudovector coupling. A total of 13 parameters were adjusted in the fitting procedure. The database in SAID [5] gives a χ^2 per datum of 1.89 for energies of 1 to 250 MeV and of 2.53 for 1 to 350 MeV.

The deuteron wave functions for this model are similar to those of model IIB shown in Fig. 2 of Ref. [4]. There are four wave functions, the usual S and D waves that appear in the nonrelativistic treatment of the deuteron and singlet and triplet P waves of relativistic origin. The contributions to the normalization of the wave function from these components are 92.979% for the S wave, 5.015% for the D wave, 0.049% for the triplet P wave, and 0.009% for the singlet P wave. The remaining 2% is associated with the derivative term in (2.71) of Ref. [4]. Note that the signs of the singlet and triplet P waves are opposite for this model.

The construction of appropriate current matrix elements for the Gross equation that maintain gauge invariance was discussed in Ref. [7]. In order to satisfy the Ward-Takahashi identities [8] in the presence of the form factors (1), an off-shell single-nucleon current operator must be introduced. A minimal form of the operator is given by [3,9]

$$J^{(i)\mu}(p',p) = F_1(Q^2) f_0(p'^2,p^2)\,\gamma^\mu + \frac{F_2(Q^2)}{2m} h_0(p'^2,p^2)\, i\sigma^{\mu\nu} q_\nu + F_3(Q^2) g_0(p'^2,p^2)\, \frac{\not{p}' - m}{2m}\,\gamma^\mu\, \frac{\not{p}' - m}{2m}\,, \tag{2}$$

where

$$f_0(p'^2,p^2) \equiv \frac{h(p^2)}{h(p'^2)}\,\frac{m^2 - p'^2}{p^2 - p'^2} + \frac{h(p'^2)}{h(p^2)}\,\frac{m^2 - p^2}{p'^2 - p^2}\,, \tag{3}$$

$$g_0(p'^2,p^2) \equiv \left(\frac{h(p^2)}{h(p'^2)} - \frac{h(p'^2)}{h(p^2)}\right)\frac{4m^2}{p'^2 - p^2}\,, \tag{4}$$

and $F_3(Q^2)$ and $h_0(p'^2,p^2)$ are arbitrary functions subject only to the constraints that $F_3(0) = 1$ and $h_0(m^2,m^2) = 1$. In the calculations presented here, $h_0(p'^2,p^2) = f_0(p'^2,p^2)$ and $F_3(Q^2) = G_{Ep}(Q^2)$, for simplicity. The sensitivity of the form factors to these choices will be explored elsewhere.

The use of factorizable form factors in (1) and (2)–(4) is a simple minimal assumption that allows for the construction of a current operator that is consistent with the interaction. This allows for an estimate of the possible size of the contribution of off-shell effects to the form factors. There is evidence from quark model calculations that this assumption may not be an accurate representation of the vertex function and that there may be some contributions to the current matrix elements which may not be readily represented in terms of meson exchanges with form factors [10].

In constructing the current matrix element it is necessary that the on-shell constraint used in the Gross equation be consistently applied to the calculation of the current matrix element. In the case of the Gross equation, the correct expression for the current matrix element can be obtained by keeping only the positive energy nucleon poles for particle 1 in the evaluation of the energy loop integrals of the Bethe-Salpeter current matrix element. For the elastic matrix elements this leads to the Feynman diagrams displayed in Fig. 1. Here the ovals represent the deuteron vertex functions, single lines represent nucleon propagators, lines with crosses denote on-shell nucleons, and the wavy lines represent virtual photons.

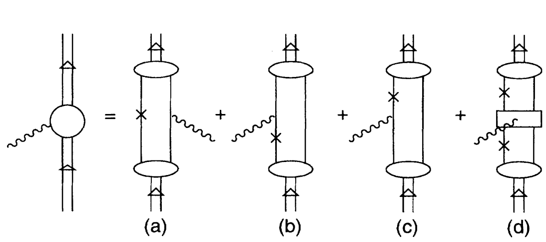

FIG. 1. Feynman diagrams representing the Gross current matrix element.

Diagram 1(a), where the virtual photon is absorbed on particle 2, has particle 1 constrained on-shell for both the initial and final state vertex functions and can be written in the form of a matrix element of the single-nucleon current operator between two Gross wave functions. However, if the virtual photon is absorbed on particle 1, the positive energy pole can be picked up for the propagator before the absorption of the virtual photon or the one after. This leads to diagrams 1(b) and 1(c). In these diagrams only the initial or final vertex function is on-shell and the other must be off-shell. These two diagrams do not have the simple form associated with the nonrelativistic impulse approximation as does diagram 1(a). The equation for the off-shell vertex function can be used to write the matrix elements for these diagrams in terms of the constrained Gross wave function [3]. The resulting diagrams may be viewed as interaction current contributions which are necessary to accommodate the on-shell constraint. It should be noted that only by calculating diagrams 1(a)–1(c) can the proper normalization of the charge be recovered from the charge form factor in the limit $Q^2 \to 0$. Diagrams involving two-body interaction currents (represented by the rectangle with attached photon) will have two internal energy loops which can be constrained independently to give diagram 1(d), provided that the meson exchange currents are explicitly symmetrized.

Prior to Ref. [7], it was assumed that the proper form of the Gross current matrix element was described by diagram 1(a) along with a symmetric diagram where the photon attaches to particle 1 and particle 2 is placed on mass shell [11]. Because of the symmetry of the matrix element, the contribution of the second diagram is equivalent to diagram 1(a). Thus this approximation is equivalent to simply calculating 2 × diagram 1(a). Since the form of this approximation looks like a matrix element of a single-nucleon current between spectator wave functions, it is referred to as the relativistic impulse approximation (RIA). Since the combination of diagrams 1(a)–1(c) are related to the relativistic impulse approximation but represent a complete gauge invariant description of the Gross one-body current matrix elements, we will refer to it here as the complete impulse approximation (CIA).

The effects of the various elements of the calculation for the impulse approximation can be seen for $B(Q^2)$ in Fig. 2. The relativistic impulse approximation of Hummel and Tjon [12] with Höhler single-nucleon electromagnetic form factors [13] is shown for reference and is labeled "Tjon, RIA." Three versions of the impulse approximation are calculated using our model with the

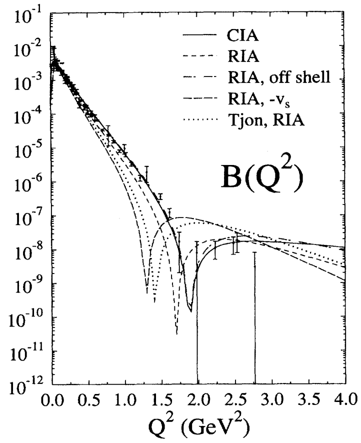

FIG. 2. $B(Q^2)$ in the impulse approximation.

dipole parametrization of the single-nucleon form factors of Galster *et al.* [14]. The calculation labeled "RIA," as described above, uses an on-shell form of the single-nucleon current operator obtained by setting $f_0 = h_0 = 1$ and $g_0 = 0$ in (2). The curve labeled "RIA, off shell" is the same as the first but with the full form of the off-shell single-nucleon current operator, and the curve labeled "CIA" is the complete impulse approximation corresponding to diagrams 1(a)–1(c) with the completely off-shell single-nucleon current operator. The use of the off-shell current operator in the RIA moves the minimum to even larger values of Q^2. The CIA is very close to the off-shell RIA (showing that the use of the RIA with off-shell currents very closely approximates the CIA) and both are in remarkably close agreement to the data. These curves show that only small contributions from the exchange currents are required to bring the CIA into good agreement with the data.

Note that for our calculation the minimum of $B(Q^2)$ is at larger Q^2 than in the calculation of Hummel and Tjon. This appears to be the result of dynamical differences in the interaction models used. In particular, the position of the minimum of $B(Q^2)$ is particularly sensitive to the sign of the singlet P wave v_s as can be seen by simply changing the sign of this wave function component in the calculation of the RIA as is shown in the curve labeled "RIA, $-v_s$." The effect of this change is to produce a large downward shift in the position of the minimum, without changing $A(Q^2)$ or T_{20} significantly.

The origin of the sensitivity in the minimum in $B(Q^2)$ can be seen from the expansion of the magnetic form factor to order v^2/c^2 given in Ref. [11]. The magnetic form factor is given by

$$G_M = G_{ES} D_M^E + G_{MS} D_M^M, \tag{5}$$

where G_{ES} and G_{MS} are the isoscalar electric and magnetic single-nucleon Sachs form factors and

$$D_M^E = \int_0^\infty dr \left\{ \frac{3}{2} w^2 + \frac{2mr}{\sqrt{3}} \left[v_t \left(\frac{1}{\sqrt{2}} u - w \right) - v_s \left(u + \frac{1}{\sqrt{2}} w \right) \right] \right\} [j_0(\tau) + j_2(\tau)], \tag{6}$$

$$D_M^M = \int_0^\infty dr [(2u^2 - w^2) j_0(\tau) + (\sqrt{2} uw + w^2) j_2(\tau)], \tag{7}$$

with $\tau = Qr/2$. Here u, w, v_t, and v_s are the S, D, triplet P, and singlet P radial wave functions. All terms quadratic in the P waves can be shown to be very small in this region. The position of the zero in $G_M(Q^2)$ and thus the zero in $B(Q^2)$ is sensitive to the interference terms between P waves and the larger S and D waves in the second term of (6).

For elastic scattering from the deuteron, only isoscalar two-body exchange currents can contribute. The only possible isoscalar contributions for the one-boson-exchange model used here are of the type $\rho\pi\gamma$, $\omega\eta\gamma$, $\omega\sigma\gamma$, etc. These currents have couplings that are individually gauge invariant and therefore require no complicated modification of the off-shell behavior of the vertex functions and form factors in order to maintain gauge invariance. The $\rho\pi\gamma$ exchange current is related to the AAV anomaly [15], and the coupling and size of the contribution to the form factors at $Q^2 = 0$ are reasonably well constrained. The form factor for the $\rho\pi\gamma$ vertex is not known experimentally, however, and is a source of uncertainty in the calculations. The coupling constant for $\omega\eta\gamma$ can be extracted from existing data but with less accuracy than in the previous case. The couplings and form factors for the other possible exchange currents can be predicted by quark models, but are not otherwise constrained. The contributions of the exchange currents to the elastic form factors of the deuteron have been calculated by Hummel and Tjon [12] in an approximate fashion using the Blankenbecler-Sugar equation. The form factors for all contributions were taken to be given by the vector dominance model (VMD). It was found that $\rho\pi\gamma$ and $\omega\sigma\gamma$ exchange currents were needed to obtain any agreement with the data and that contributions of the $\omega\eta\gamma$ exchange currents were small. These calculations for $A(Q^2)$, $B(Q^2)$, and $T_{20}(Q^2) = \tilde{t}_{20}(Q^2)$ are shown in Fig. 3 and labeled as "Tjon, RIA+$\rho\pi\gamma$ + $\omega\sigma\gamma$." The CIA calculation is also shown for reference. Calculations of the contributions of the $\rho\pi\gamma$ exchange current are shown for our model as calculated with the VMD form factors [labeled "CIA+$\rho\pi\gamma$ (VMD)"], and quark model form factors as calculated by Gross and Ito [16] [labeled "CIA+$\rho\pi\gamma$ (Gross-Ito)"] and by Mitchell and Tandy [17] [labeled "CIA+$\rho\pi\gamma$ (Mitchell-Tandy)"]. Recent calculations of the $\rho\pi\gamma$ form factor by Cardarelli *et al.* [18] also give results very similar to those of Mitchell and Tandy. Both of the quark model form factors are softer than the VMD. The $\rho\pi\gamma$ exchange currents tend to increase the size of

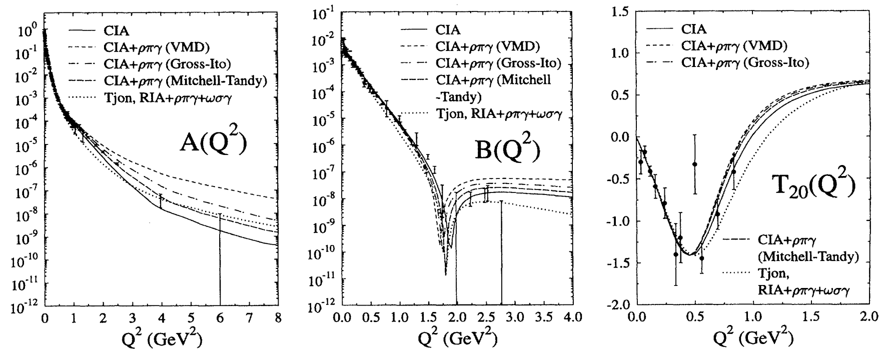

FIG. 3. $A(Q^2)$, $B(Q^2)$, and $T_{20}(Q^2)$ with exchange currents.

$A(Q^2)$ and to move the minimum of $B(Q^2)$ to lower Q^2. In both cases the VMD form factors produce much too large an effect, while the softer quark model form factors give smaller effects. Indeed the calculation with the Mitchell-Tandy form factor is remarkably close to the data. The tensor polarization $T_{20}(Q^2) = \tilde{t}_{20}(Q^2)$ shows some sensitivity to the exchange current contributions. The quality of the data is not yet sufficient to distinguish among the various models, however.

In summary, we have constructed a complete, relativistically covariant and gauge invariant model of elastic electron scattering from the deuteron using the Gross equation. The calculation includes the complete impulse approximation and $\rho\pi\gamma$ exchange currents. We find that the structure function $B(Q^2)$ is extremely sensitive to the presence of small P wave components of the deuteron wave function of relativistic origin. By using a soft $\rho\pi\gamma$ electromagnetic form factor we have been able to obtain an excellent description of the data. We are presently looking for more accurate interaction models (the D/S ratio and quadrupole moment of model IIB are a little too small). The Gross equation is also being applied to the calculation of the triton binding energy [19], and we expect that this will result in some additional constraints on acceptable models. It is possible that our best interaction models may produce different results for the deuteron form factors.

Supported by DOE Contracts No. DE-AC05-84ER40150 and No. DE-FG05-94ER40832

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