Scaling of the Coulomb Energy Due to Quantum Fluctuations in the Charge on a Quantum Dot

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The charging energy of a quantum dot is measured through the effect of its potential on the conductance of a second dot. This technique allows a measurement of the scaling of the dot's charging energy with the conductance of the tunnel barriers leading to the dot. We find that the charging energy scales quadratically with the reflection probability of the barriers. The observed power law agrees with a recent theory.

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Charge fluctuations reduce the effects of the Coulomb blockade [1] on electron transport in nanostructures. For example, Coulomb oscillations in the conductance of a quantum dot—periodic oscillations as a function of the voltage on an external gate electrode [1,2]—can only be observed when the conductance G of the barriers connecting the dot to external leads is reduced below $2e^2/h$, i.e., in the tunneling regime [3,4].

The influence of quantum fluctuations in the Coulomb blockade regime has recently been studied theoretically by applications of scaling- and Luttinger-liquid theory [5–11]. One of these studies [7,8] explicitly considers the case of a split-gate defined semiconductor quantum dot, and predicts a scaling of the charging energy associated with the addition of an electron to the dot with the conductance of the point-contact barriers, according to

$$U^* \sim U(1 - T)^{N_c}. \tag{1}$$

Here, $U \equiv e^2/C$ is the bare charging energy (C is the self-capacitance of the dot), U^* is the effective (or "renormalized") charging energy observed for finite barrier conductance, N_c the total number of quantum point contacts leading to the dot, and T the transmission probability of the contacts [12]. The result (1) was derived by utilizing a mapping of the two-dimensional dot geometry to a one-dimensional model [13] with interactions equivalent to Ref. [6]. Observation of such a scaling behavior would thus be a strong support for the applicability of Luttinger-liquid theory in describing the influence of quantum fluctuations on charge transport in nanostructures.

Unfortunately, conductance measurements [3,4] are not well suited to test Eq. (1), because of the occurrence of complicating cotunneling processes [14,15] when the barrier conductance approaches $2e^2/h$. In this Letter, we report results obtained with a fully adjustable double quantum-dot structure, defined electrostatically in a (Al,Ga)As modulation doped heterostructure. This setup is designed to allow for a direct measurement of the charging energy U as a function of barrier transparency, and does not suffer from ambiguities in the determination of U due to cotunneling processes. We discuss an experiment that probes the role of quantum charge fluctuations in Coulomb-regulated transport. We use the device in an *electrometer* [16] configuration that allows a direct determination of U as a function of barrier conductance. We find good agreement between theory and experiment.

A schematic layout of the device is shown in Fig. 1(a). The hatched areas are the TiAu gates; crosses denote Ohmic contacts. The device consists of two adjacent quantum dots, 1 and 2. Gates A through F are used to define the barriers leading to the dots, and the electrochemical potential of the quantum dots can be adjusted through gates I and II. The lithographic diameter of each dot is about 1 μm. These relatively large dimensions are necessary to minimize cross talk between the gates; they also imply [15] that confinement effects on the transport properties are negligible, a prerequisite for the theory of Refs. [7,8]. During the experiments the samples are kept at 40 mK in a dilution refrigerator (we estimate the electron gas temperature to be 150 mK); the conductance $G \equiv G_{14}$ is measured between Ohmic contacts 1 and 4, using standard low-frequency lock-in techniques.

The mode of operation of the electrometer experiment is schematically depicted in Fig. 1(b). What is measured is the dependence of G on $V_{g\mathrm{II}}$, the voltage on gate II. In our double-dot device, scanning gate II induces Coulomb oscillations in both dot 1 and dot 2, but with a much shorter period (in $V_{g\mathrm{II}}$) in dot 2, because of the larger dot-to-gate capacitance. The top panel of Fig. 1(b) shows the stepwise increase of n_2, the number of electrons on dot 2, within a $V_{g\mathrm{II}}$-voltage range where n_1 only changes by 1. The stepwise increase of n_2 causes sawtooth-shaped oscillations in the energy ΔE needed to change the occupancy of dot 1 by one electron. As shown in the bottom panel of Fig. 1(b), these oscillations are reflected in the conductance of dot 1: An additional sawtooth behavior

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is superimposed on the (dashed) line shape one expects for the dot 1 Coulomb-blockade conductance peak without a charging energy for dot 2. The full curve in Fig. 1(b) is, in fact, a fit to an experimental trace (dotted curve), using a theory we will discuss below.

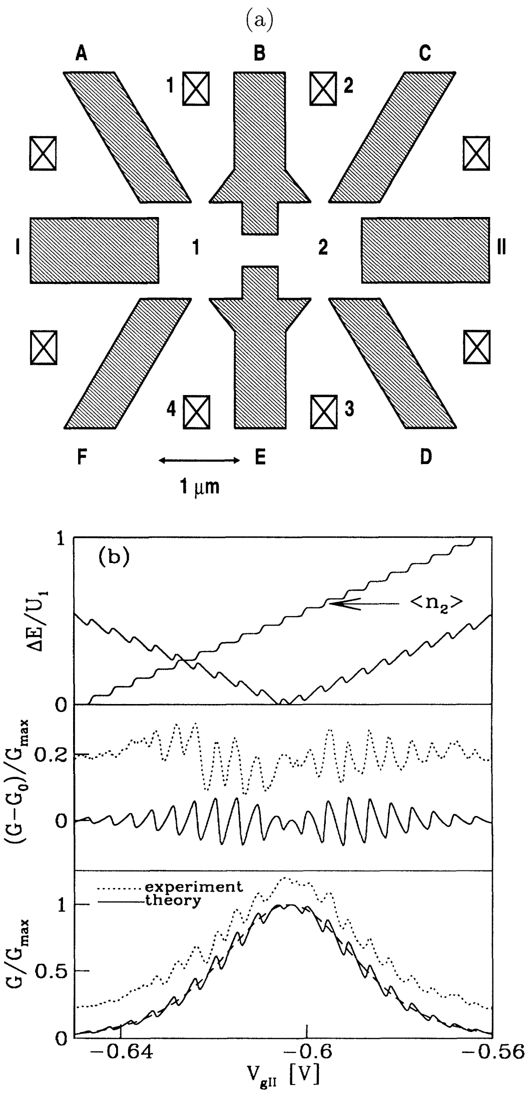

FIG. 1. (a) Schematic layout of the double-dot sample. The hatched areas denote gates, the crosses Ohmic contacts. (b) The operating principle of the electrometer experiment: scanning $V_{g\mathrm{II}}$ leads to an increase in n_2, the number of electrons on dot 2. The concomitant sawtooth oscillations in ΔE, the minimum energy required to change the occupancy of dot 1. Full lines are results of the model calculation and the bottom panel shows a fit to the experimental trace (dotted curve). The dashed line corresponds to the case where there is no charging energy for the second dot. In the middle panel we have subtracted the conductance of the first dot from the result obtained when there is no charging energy of the second dot. The size of the resulting oscillations thus measures the effective charging energy of dot 2, which is used for determining U_2^*, as plotted in Fig. 3.

The size of the sawteeth is a measure of the charging energy of dot 2. Dot 1 thus acts as an electrometer [16] that measures the changes in the potential of dot 2. In the absence of a charging energy of dot 2, the conductance is given by the usual Coloumb oscillation peaks, which we have indicated with the dashed line (obtained by setting $U_2^* = 0$ in the equations below). Subtraction of the dashed and the full line gives the trace shown in the middle panel of the figure. We have also subtracted the same background from the experimental trace. Using the subtracted trace, we are able to fit the experimental curves by adjusting the parameter U_2^* as explained below. This procedure allows us to determine the effective charging energy of dot 2 fairly accurately.

We will now discuss an experiment that uses this method to measure the scaling behavior of U_2 with the conductance of barriers BC and DE. In the left panel of Fig. 2 we plot G vs $V_{g\mathrm{II}}$ in a series of measurements where the barriers between dot 2 and the wide 2DEG are gradually adjusted from the metallic to the tunneling regime. From top to bottom we have $G_{\mathrm{BC}}, G_{\mathrm{DE}} \approx 1.3e^2/h$, $0.65e^2/h$, $0.43e^2/h$, $0.14e^2/h$, and $0.05e^2/h$, respectively, while in all traces $G_{\mathrm{AB}}, G_{\mathrm{BE}}, G_{\mathrm{EF}} \approx 0.05e^2/h$, so that dot 1 is always fully in the Coulomb blockade regime. One clearly observes the sawtooth structure on the dot 1 Coulomb oscillation due to the electrometer effect. In addition, one finds that for increasing conductances G_{BC} and G_{DE} the sawtooth feature is much less pronounced. In view of the arguments given above, it is obvious to attribute these observations to the scaling of the charging energy U_2^* as a function of the conductance of barriers BC and DE. Note that the period of the sawtooth feature is unaffected by the changes in G_{BC} and G_{DE}. We will explain below that the sawtooth *period* is determined solely by the classical electrostatics while its *depth* is a measure of thermal and quantum fluctuations.

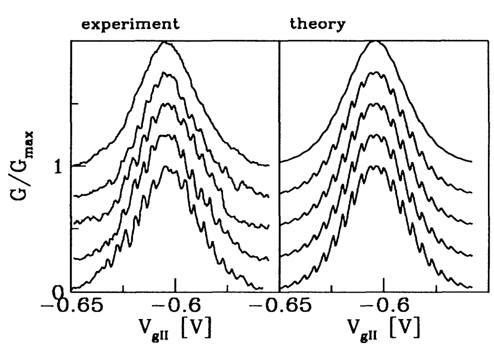

FIG. 2. Traces of G vs $V_{g\mathrm{II}}$ in an electrometer experiment, where the conductance of barriers BC and DE is varied. An offset of $0.25G/G_{\mathrm{max}}$ is used between consecutive curves. Left panel: Experimental data, where from top to bottom $G_{\mathrm{BC}}, G_{\mathrm{DE}} \approx 1.3e^2/h$, $0.65e^2/h$, $0.43e^2/h$, $0.14e^2/h$, and $0.05e^2/h$. In all traces $G_{\mathrm{AB}}, G_{\mathrm{BE}}, G_{\mathrm{EF}} \approx 0.05e^2/h$. Right panel: The results of model calculations using Eqs. (1), (4), and (5).

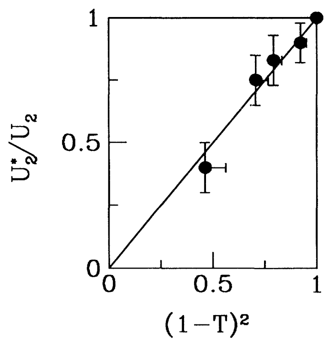

FIG. 3. Plot of the ratio U_2^*/U_2 between renormalized and bare charging energy of dot 2, vs $(1 - T)^2$, where T is the transmission of barriers BC and DE that control the coupling between dot 2 and the external leads. The linear dependence found in this plot indicates the validity of a scaling law of the type in Eq. (1).

In Fig. 3 we plot the values for U_2^*/U_2, obtained by the fitting procedure discussed above, in the different curves of Fig. 2, vs $(1 - T)^2$, where T is the conductance of barriers BC and DE, measured in units of $2e^2/h$. The fit obtained is strong evidence that the scaling law in Eq. (1) contains the correct physics—note that $N_c = 2$ indeed corresponds to the total number of quantum point contacts connecting to the dot.

We have developed a theoretical model of transport through a quantum dot coupled to a second dot, which includes the effects of charge fluctuations. Using this model we have produced the calculated traces in the right-hand panel of Fig. 2. We start by considering the electrostatic energy of the coupled dot system [17],

$$E(n_1, n_2) = U_1 n_1^2 + U_2 n_2^2 + U_{12} n_1 n_2 + e \sum_{i=1,2} \sum_{j=\mathrm{I,II}} n_i a_{ij} V_{g,j} \,, \tag{2}$$

where n_i is the number of electrons on dot i. The constants U_i, U_{12}, and a_{ij} can be expressed in terms of the elements of the capacitance matrix of the system. Let us define as n_{i0} the (generally noninteger) number of electrons on dot i that minimizes $E(n_1, n_2)$. We now may write the dependence of $E(n_1, n_2)$ on small deviations $\delta n_i \equiv n_i - n_{i0}$ as

$$E(n_1, n_2) = E(n_{10}, n_{20}) + \delta E(n_1, n_2) \,, \tag{3a}$$

$$\delta E(n_1, n_2) = U_1 \delta n_1^2 + U_2 \delta n_2^2 + U_{12} \delta n_1 \delta n_2 \,. \tag{3b}$$

Here $\delta E(n_1, n_2)$ is the quadratic term that controls the fluctuation away from the optimum charge configuration (n_{10}, n_{20}). All fluctuation-dependent properties (such as the transport properties) are thus periodic functions of n_{10} and n_{20}. Consequently, the periodicity of the Coulomb oscillations is *unaffected* by number fluctuations. An increase in number fluctuations due to the lowering of the tunnel barriers may be thought of as a decrease of the charging energies that enter $\delta E(n_1, n_2)$. (Note that this formalism treats occupation-number fluctuations of quantum-mechanical and thermal origin on an equal footing.) With this in mind, we model the effect of quantum fluctuations of the charge on dot 2 by invoking a renormalized charging energy of Eq. (1), as follows. First we replace the deviation $\delta E(n_1, n_2)$ in Eq. (3) by its renormalized counterpart $\delta E^*(n_1, n_2)$. The charging energy that controls the deviation away from the optimum number of electrons on dot 2 (for a fixed number of electrons on dot 1) is thus assumed to be renormalized by the quantum fluctuations. We then have

$$\delta E^*(n_1, n_2) = [U_1 - U_{12}^2/4U_2] \delta n_1^2 + U_2^* (\delta n_2 - \delta n_1 U_{12}/2U_2)^2 \,. \tag{4}$$

Within this model we can generalize the rate equation approach of Ref. [18] to our double-dot system and then solve for the linear conductance. We obtain

$$G = \tfrac{1}{4} G_0 \beta \sum_{n_1, n_2} W_0^*(n_1, n_2) \times f[\delta E^*(n_1, n_2) - \delta E^*(n_1 - 1, n_2)] \,, \tag{5a}$$

$$W_0^*(n_1, n_2) = \frac{\exp[-\beta \delta E^*(n_1, n_2)]}{\sum_{n_1, n_2} \exp[-\beta \delta E^*(n_1, n_2)]} \,, \tag{5b}$$

$$f(E) = E/(1 - e^{-\beta E}) \,, \tag{5c}$$

where $\beta = 1/k_B T$ and $G_0 = G_{\mathrm{AB}} = G_{\mathrm{EF}}$.

The right panel of Fig. 2 shows line shapes calculated from Eqs. (1), (4), and (5). In order to obtain a consistent set of fits, we first determine the parameters $U_i = 0.13$ meV, $U_{12} = 0.009$ meV, $a_{ii} = -0.20$, and $a_{12} = -3.12$ from a fit of Eqs. (2) and (5) to the bottom trace of the left panel of Fig. 2—which is the same as the experimental (dotted) trace in the bottom panel of Fig. 1(b)—where both dots are fully in the tunneling regime. The upper curves are then obtained from Eqs. (5) and (4), keeping the same values for U_1, U_{12}, and a_{ij} while varying U_2^* with $G_{\mathrm{BC}}, G_{\mathrm{DE}}$ according to Eq. (1). As is evident from the theoretical curves, this procedure yields a very good agreement with the experiments.

Reduction of the charging energy due to quantum fluctuations has also been studied theoretically for the case of large area tunnel junctions [5,19]. These studies are therefore not directly applicable to our few channel junctions. It is nevertheless interesting to note that the renormalization of the charging energy derived in those works cannot explain the observed reduction of the charging energy in our experiments: the processes discussed in Refs. [5,19] only give a significant contribution when the conductance is much larger than $2e^2/h$. This clearly does not apply to our point-contact barriers.

In conclusion, we have performed and analyzed experiments aimed at understanding the role of charge fluctuations in the transport properties of quantum dots. We find that the dependence of the charging energy of a quantum dot on the conductance of the point-contact tunnel barriers can be well described using a scaling equation. It would be useful to verify the validity of the scaling equation for other power laws, which could be accomplished, e.g., by performing experiments in a high magnetic field, or by varying only one of the tunnel barriers.

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