Unified Theory of Stimulated Raman Scattering and Two-Plasmon Decay in Inhomogeneous Plasmas: High Frequency Hybrid Instability

Bedros B. Afeyan and Edward A. Williams
Lawrence Livermore National Laboratory, Livermore, California 94551
(Received 8 September 1995)

The theories of stimulated Raman scattering (SRS) and two-plasmon decay ($2\omega_{pe}$) in inhomogeneous plasmas have been unified by adopting a powerful new variational approach. We call this technique the minimum pump strength principle (MPSP), and the unified process, the high frequency hybrid instability (HFHI). The growth rate and frequency shift of the most unstable mode of HFHI are shown to match smoothly onto the two known limiting cases: $2\omega_{pe}$ at large k, and SRS backscattering as k goes to zero. The effectiveness of this method has allowed the generalization of the density inhomogeneity treated from a constant gradient to any positive-integer power law. Potential experimental signatures of HFHI modes are described.

PACS numbers: 52.35.Mw, 52.35.Nx

The theory of three-wave parametric instabilities [1] in inhomogeneous plasmas has been heavily dependent on eikonal approximations [2], *ab initio* local expansions [3], and the WKB quantization condition [4]. While very significant results have been obtained to date [5,6], many problems remain unsolved because these techniques are not sufficient to tackle them. In particular, whenever the instability occurs near the turning points of the waves, the eikonal approximation ceases to be valid, and, whenever the description of the problem is not reducible to a second order ordinary differential equation, WKB quantization conditions are not readily available. *Ab initio* local expansion techniques are not generally applicable because they rely on the reducibility of the problem to one of a very few available canonical models, typically the harmonic oscillator, whose eigenvalue condition is already known. In this Letter, we address the theory of high frequency parametric instabilities that occur near the quarter critical density of a laser-produced plasma, using a variational approach that circumvents these difficulties [7]. This same technique should prove useful in tackling other outstanding problems in the theory of parametric instabilities in inhomogeneous plasmas, especially in the presence of more than one perfect phase matching point, in multiple dimensions, and in the presence of magnetic fields and other anisotropies. The technique is based on the observation that the most unstable mode, or ground state, in the bound spectrum of a given parametric instability will require the *minimum* amount of pump power to be excited, hence the name minimum pump strength principle (MPSP).

In this Letter, we solve for the most unstable high frequency mode at the quarter critical density when the k vector of one of the two daughter waves goes to zero (or is very near its turning point). When the polarization of that daughter wave is assumed to be strictly electrostatic (longitudinal), with the pump being electromagnetic (transverse) and the large k daughter wave electrostatic as well, the instability is called two-plasmon decay ($2\omega_{pe}$) [8,9]. If, instead, it is strictly electromagnetic we have stimulated Raman scattering (SRS) [10–12]. To find the most unstable mode for a given k, we must treat a hybrid wave, partly electrostatic and partly electromagnetic. We call the resulting process the high frequency hybrid instability (HFHI). A singularity exists in $2\omega_{pe}$ theory as $k_\perp/k_0$, the component of the daughter wave k vectors perpendicular to the density inhomogeneity direction divided by that of the pump wave, becomes small. The stabilizing contribution to the growth rate due to inhomogeneity scales as $k_\perp^{-1}$, implying infinite stability of the $k_\perp = 0$ mode [8,9]. In contrast, SRS sidescattering has a finite threshold and positive growth rate for densities below quarter critical, and as $k_\perp/k_0 \rightarrow 0$, it reaches the finite backscattering limit and occurs at the quarter critical density [12]. By adopting a hybrid or mixed polarization wave description [13], SRS and $2\omega_{pe}$ are shown to be but two limits of a single instability that differs significantly from $2\omega_{pe}$ when $k_\perp/k_0 = O(v_0/c)^{1/2}$ or smaller, where v_0 is the oscillatory speed of electrons in the field of the pump and c is the speed of light. These modes will have lower thresholds and larger growth rates than the most unstable $2\omega_{pe}$ modes whenever $v_{\rm th}^2/c^2 > O(v_0/c)^{1/2}$, where $v_{\rm th}$ is the thermal speed of the electron fluid. These conditions imply that HFHI is an important instability in laser fusion scenarios, such as direct drive, where high temperature, quarter critical density plasma is illuminated by UV laser light.

We have unified the fluid theories of these two instabilities and solved for the most unstable mode's properties by recasting the equations describing the instability in the form of a variational principle for the square of the coupling constant, which is proportional to the intensity of the pump wave. Starting with functionals of operators that have nonzero expectation values in the limit of "infinite pump strength," by which we mean the pump is then so strong that inhomogeneous effects are subdominant, we additionally retain dominant contributions strictly due to

 0031-9007/95/75(23)/4218(4)$06.00

inhomogeneity. Any functional that is discarded in this process can be evaluated *ex post facto* and is shown to be ignorable, or kept throughout the calculation and shown to yield no significant change to the ground state eigenvalue. We adopt a Euclidean pseudometric [14], as defined in Eq. (19), and not the usual Hermitian one because the approximation scheme described above leads to complex symmetric operators and not self-adjoint ones. The expectation value of a complex symmetric operator can be evaluated with a self-dual trial function. The form of this trial function, in the case of a power law density profile, is inspired by the form of the wave function in k space in the infinite pump strength limit, where it approaches a delta function at the complex k satisfying the Bers-Briggs pole pinching criterion [15]. For finite pump strengths, then, the wave function is taken to be a localized trial function, which we have, in fact, chosen to be a Gaussian, centered at the perfect phase matching point in space, where the local plasma frequency is $\omega_p(0)$, and centered at some k_H in Fourier space, with a complex width parameter α, which constitute the central variational parameters to be chosen by minimizing the pump strength functional. This procedure yields the complex frequency in the form of an infinite pump strength contribution plus a piece due to inhomogeneity, whose imaginary part is proportional to the wave function width in k space raised to the power $-2/(n+1)$, for a z^n profile. In its development phase, this method has already been applied to the Rosenbluth model equations [2], the Liu, Rosenbluth, and White sidescattering model [3], and to SRS and $2\omega_{pe}$ separately, where many of the simplification procedures were tested [7]. Ultimately, of course, any approximate analytic method such as this must be tested against numerical calculations to ascertain its accuracy. Our confidence is based on the fact that our HFHI results match onto both SRS and $2\omega_{pe}$ limits so well, these limits having been carefully studied numerically [9,12]. Details of these calculations will be published separately [16].

The coupled wave equations and the operators that describe the HFHI are [7,16]

$$\mathcal{L}_1\Psi_1 = \tilde{v}_0\mathcal{M}_{31}\Psi_3\,, \tag{1}$$

$$\mathcal{L}_2\Psi_2 = \tilde{v}_0\mathcal{M}_{32}\Psi_3\,, \tag{2}$$

$$\mathcal{L}_3\Psi_3 = \tilde{v}_0^*\mathcal{M}_{13}\Psi_1 + \tilde{v}_0^*\mathcal{M}_{23}\Psi_2\,, \tag{3}$$

$$\mathcal{L}_1 = (d_{t_1} - V)\nabla^2, \tag{4}$$

$$\mathcal{L}_2 = (d_{\ell_1} - V)\nabla^2, \tag{5}$$

$$\mathcal{L}_3 = (\overline{d}_{\ell_2} - V)\overline{\nabla}^2, \tag{6}$$

$$\mathcal{M}_{13} = -\overline{\nabla}^2/\sqrt{\epsilon}\,, \tag{7}$$

$$\mathcal{M}_{31} = [\nabla^2 - (\hat{\mathbf{u}}_0\cdot\nabla)^2]\overline{\nabla}^2/\sqrt{\epsilon}\,, \tag{8}$$

$$\mathcal{M}_{23} = \mathcal{M}_{32} = -i(\hat{\mathbf{u}}_0\cdot\nabla)[\overline{\nabla}^2 - \nabla^2]/\sqrt{\epsilon}\,, \tag{9}$$

$$d_{t_1} = [\omega_1^2 - \omega_p^2(0)(1 - i\nu_1/\omega_1) + \nabla^2], \tag{10}$$

$$d_{\ell_1} = [\omega_1^2(1 + i\nu_1/\omega_1) - \omega_p^2(0) + v_{\rm E}^2\nabla^2], \tag{11}$$

$$\overline{d}_{\ell_2} = [\omega_2^2(1 - i\nu_2/\omega_2) - \omega_p^2(0) + v_{\rm E}^2\overline{\nabla}^2]. \tag{12}$$

Here, $\overline{\nabla} = (\nabla - i\mathbf{k}_0)$, $\hat{\mathbf{u}}_0$ is the unit vector along the pump electric field, which is assumed to be along the x axis, and $V(\mathbf{x}) = \omega_p^2(\mathbf{x}) - \omega_p^2(0)$, which for power law profiles is $V(z) = \varepsilon_{nL}^n z^n$, where $\varepsilon_{nL}^n = \omega_p^2(0)/(\omega_0 L_n/c)^n$, and L_n is the density scale length. All frequencies in these equations are normalized to ω_0, the pump wave frequency, and all wave vectors are normalized to ω_0/c. The phenomenological damping terms are ν_1 and ν_2. The perfect phase matching point is at $z = 0$, where the dielectric constant is $\epsilon = 1 - \omega_p^2(0)$. The coupling coefficient, $\tilde{v}_0$, is related to the quiver velocity of the electrons in the uniform electric field of the pump, v_0, by $\tilde{v}_0 = (v_0/c)\omega_p(0)\sqrt{\epsilon}/2$, and $v_{\rm E}^2 = 3v_{\rm th}^2/c^2$. The correspondence between the dependent variables and the electromagnetic potentials is $\Psi_1 = A_1$, the component of the vector potential $\mathbf{A}_1$ of the hybrid wave along the electric field of the pump, $\Psi_2 = \phi_1$, and $\Psi_3 = \exp[i\mathbf{k}_0\cdot\mathbf{x}]\phi_2^*$. The equations that describe the $2\omega_{pe}$ and SRS instabilities are obtained by setting $\Psi_1 = 0$ for the former, and $\Psi_2 = 0$ for the latter. Combining the three wave equations by ignoring commutators between $\mathcal{L}$ and $\mathcal{M}$ operators, we obtain the following equation for Ψ_3, the amplitude of the strictly longitudinally polarized daughter wave:

$$\mathcal{L}_1\mathcal{L}_2\mathcal{L}_3\Psi_3 = |\tilde{v}_0|^2[\mathcal{M}_{13}\mathcal{M}_{31}\mathcal{L}_2 + \mathcal{M}_{23}\mathcal{M}_{32}\mathcal{L}_1]\Psi_3\,. \tag{13}$$

The MPSP for HFHI, obtained by taking the Euclidean pseudo inner product [14] between the dual function $\tilde{\Psi}_3$ and both sides of Eq. (13), is

$$|\tilde{v}_0|^2 = \frac{I_{\rm KE} + I_{{\rm PE}_2}}{I_{{\rm norm}_1} + I_{{\rm norm}_2}}\,, \tag{14}$$

where all commutators between V and the operators d_{l_1,t_1,l_2} are neglected and the latter are placed to the left of all V's, in anticipation of the evaluation of these functionals in k space. Also neglected are $I_{{\rm PE}_1}$ and $I_{{\rm PE}_3}$, which are the expectation values of the operators that include the terms $(d_{t_1}\overline{d}_{\ell_2} + d_{\ell_1}\overline{d}_{\ell_2} + d_{t_1}d_{\ell_1})V$ and V^3, respectively. By keeping them, it can be shown that they make negligible contributions to the ground state eigenvalue [7,16]. The four functionals in Eq. (14) then are

$$I_{{\rm norm}_1} = \langle\mathcal{M}_{13}\mathcal{M}_{31}d_{\ell_1}\rangle\,, \tag{15}$$

$$I_{{\rm norm}_2} = \langle\mathcal{M}_{23}\mathcal{M}_{32}d_{t_1}\rangle\,, \tag{16}$$

$$I_{\rm KE} = \langle\nabla^2\nabla^2\overline{\nabla}^2 d_{t_1}d_{\ell_1}\overline{d}_{\ell_2}\rangle\,, \tag{17}$$

$$I_{{\rm PE}_2} = \langle\nabla^2\nabla^2\overline{\nabla}^2(d_{t_1}d_{\ell_1}\overline{d}_{\ell_2})V^2\rangle\,, \tag{18}$$

where $\langle \mathcal{O}_i \rangle = (\tilde{\Psi}_3, \mathcal{O}_i \Psi_3)$, is the expectation value of the operator $\mathcal{O}_i$, defined not in terms of the Hermitian inner product but the Euclidean pseudo inner product between two complex functions given by

$$(\phi_1, \phi_2) = \int_V \phi_1(\mathbf{x})\phi_2(\mathbf{x})\, d\mathbf{x} = \int_{V_k} \phi_1(-\mathbf{k})\phi_2(\mathbf{k})\, d\mathbf{k}\,. \tag{19}$$

All four functionals above are expectation values of operators that conserve parity. Therefore, and in an attempt to mimic the properties of the ground state eigenfunction of the instability, we choose as our trial and dual functions, a Gaussian in Fourier space centered at $k_z = k_{z_H}$ with a (complex) width $1/\sqrt{\alpha}$, and delta functions in the $k_\perp$ directions,

$$\Psi_3(\mathbf{k}) = \tilde{\Psi}_3(-\mathbf{k}) = \delta(k_x - k_{x_H})\delta(k_y - k_{y_H}) \exp[-\alpha(k_z - k_{z_H})^2]. \tag{20}$$

Evaluating the functionals in k space, the MPSP becomes

$$\left(\frac{k_-^2}{\epsilon}\right)|\tilde{v}_0|^2 = \frac{\{-D_{t_1}D_{\ell_1}(\overline{D}_{\ell_2} - C)[1 + \mu_2/(4\alpha)] + M_{\rm II}(n)\varepsilon_{nL}^{2n}\alpha^n[D_{t_1} + D_{\ell_1} - (\overline{D}_{\ell_2} - C)]\}}{\{RD_{\ell_1}[1 + \chi_2/(4\alpha)] + T(a_1)^2 D_{t_1}[1 + \rho_2/(4\alpha)]\}}, \tag{21}$$

where $k^2 = (k_{x_H}^2 + k_{y_H}^2 + k_{z_H}^2) = (k_\perp^2 + k_{z_H}^2)$, $k_-^2 = [(\sqrt{\epsilon} - k_{z_H})^2 + k_\perp^2]$, $C = [1 - 2\omega_p(0)]$, $a_1 = 1 - k_-^2/k^2$, $T = k_{x_H}^2/k^2$, $R = 1 - T$, D_{l_1,t_1,l_2} are the Fourier transforms of the three free propagators given in Eqs. (10), (11), and (12), $\overline{\nu}_i = \nu_i \omega_p(0)$, $M_{\rm II} = [(2m)!/(2^m m!)]$, and μ_2, χ_2, and ρ_2 are somewhat involved k-dependent coefficients [7,16]. By making $|\tilde{v}_0|^2$ stationary with respect to the variational parameters and inverting the resulting expression to obtain the complex frequency of the HFHI, we find [7,16]

$$\omega_1 = \frac{1}{2} - \frac{v_{\rm E}^2}{2}(k_-^2 - k^2) - \frac{i}{2}(\overline{\nu}_1 + \overline{\nu}_2) + i\gamma_0 + \Delta\,, \tag{22}$$

where γ_0, the infinite pump strength limit growth rate, is

$$\gamma_0 = \tilde{v}_0\left[\frac{(i\tilde{v}_0 - k_{x_H}^2)}{(i\tilde{v}_0 - k^2)}\right]. \tag{23}$$

The value of k_z about which the trial function is localized, namely, k_{z_H}, is found to be

$$k_{z_H}(k_{x_H}, k_{y_H}) = (i\gamma_0 - k_\perp^2)\left[\frac{1}{\sqrt{\epsilon}} + i\tilde{\beta}\sqrt{\frac{i\gamma_0 - k_\perp^2}{i\gamma_0 - k_{x_H}^2}}\right], \tag{24}$$

where $\tilde{\beta} = \sqrt{\epsilon}\, v_{\rm E}^2/\tilde{v}_0 = 2.75 T_{\rm keV}/(I_{14}^{1/2}\lambda_0)$, where I_{14} is the intensity of the pump wave in units of 10^{14} watts per square centimeter, λ_0, its wavelength in microns, and $T_{\rm keV}$ is the electron fluid temperature in kilo electron volts. The contribution to the growth rate due to inhomogeneity, when the instability occurs in the plane of incidence of the pump, is

$$\Delta = \tilde{v}_0[N_{\rm II}(n)\varepsilon_{nL}^{2n/(n+1)}\tilde{v}_0^{-(n+2)/(n+1)}] \times \left[\frac{e^{-i\pi/2}}{(1 + i\tau_x)^n}\right]^{1/(n+1)}. \tag{25}$$

Here, $\tau_x = (k_{x_H}^2/\tilde{v}_0)$, and $N_{\rm II}(n) = (1/8)(1 + 1/n)(4nM_{\rm II})^{1/(n+1)}$. Isolating its real and imaginary parts, we rewrite the frequency of the hybrid wave in the form $\omega_1 = \Delta\omega(n) + i\tilde{v}_0\Gamma_n$, where

$$\Delta\omega = [1 - v_{\rm E}^2(\epsilon + 2k_\perp^2)]/2 + \tilde{v}_0 C_{\rm inh}(n)[1 + \tau_x^2]^{-n/[2(n+1)]}\cos\{\pi/[2(n+1)] + [n/(n+1)]\tan^{-1}(\tau_x)\}, \tag{26}$$

$$\Gamma_n = 1 - \overline{\nu} - C_{\rm H}\tau_x - C_{\rm inh}(n)[1 + \tau_x^2]^{-n/[2(n+1)]}\sin\{\pi/[2(n+1)] + [n/(n+1)]\tan^{-1}(\tau_x)\}. \tag{27}$$

The three coefficients that define the relative strength of the damping, $k_\perp$-dependent homogeneous plasma limit growth rate, and the effects of inhomogeneity are $\overline{\nu} = (\overline{\nu}_1 + \overline{\nu}_2)/(2\tilde{v}_0)$, $C_{\rm H} = \sqrt{\epsilon}\, v_{\rm E}^2\tilde{\beta}/2 = 7 \times 10^{-3}T_{\rm keV}^2/I_{14}^{1/2}\lambda_0$, and $C_{\rm inh}(n) = N_{\rm II}(n)[\varepsilon_{nL}^{2n}/\tilde{v}_0]^{1/(n+1)}$, respectively. The inhomogeneity parameter is $\varepsilon_{nL}^n = [\omega_p^2(0)/\omega_0]/(\omega_0 L_n/c)^n$, where L_n is the density scale length, and the pump strength is given in terms of $\tilde{v}_0 = 1.85 \times 10^{-3}I_{14}^{1/2}\lambda_0$.

Equations (26) and (27), which are our principal results, are valid approximations for all τ_x $(= k_{x_H}^2/\tilde{v}_0)$. They reduce to the form of SRS backscattering from $n_c/4$ in the $\tau_x = 0$ limit, and approach pure $2\omega_{pe}$ asymptotically as $\tau_x \to \infty$ [7,16]. This is shown schematically in Fig. 1, where the inhomogeneous plasma contribution $(\Gamma_n^{\rm inh})$ to the normalized growth rate (Γ_n) is plotted for the hybrid instability, for $2\omega_{pe}$, and for SRS.

As long as SRS backscattering from $n_c/4$ is above threshold, i.e., $C_{\rm inh}(n) < (1 - \overline{\nu})/\sin\{\pi/[2(n+1)]\}$,

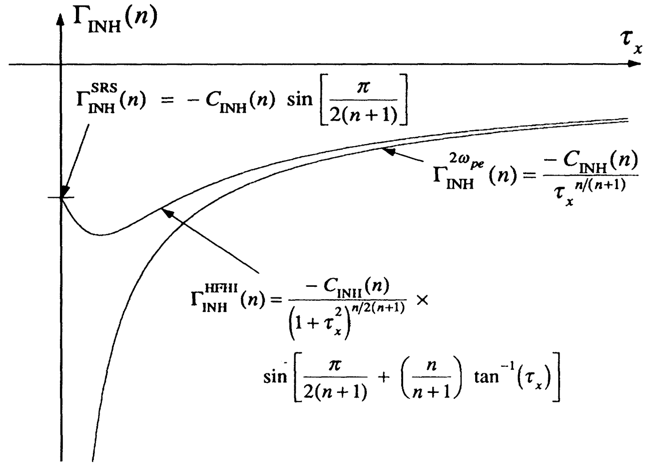

FIG. 1. Contribution to growth rate due to inhomogeneity for HFHI, SRS, and $2\omega_{pe}$ mode versus τ_x.

$\tau_{x_{\max}}$, which defines the largest unstable $k_{x_H}^2$, is given by

$$\tau_{x_{\max}}(n) = \frac{1 - \overline{\nu} - C_{\rm inh}(n)\sin\{\pi/[2(n+1)]\}}{[C_{\rm H} + [n/(n+1)]C_{\rm inh}(n)\cos\{\pi/[2(n+1)]\}]}. \tag{28}$$

This defines a natural linewidth to the commonly observed Raman light with frequency very close to $\omega_0/2$ [17]. The interpretation is that whenever the ($k_\perp = 0$) pure SRS mode is unstable, there will be a range of mixed polarization modes around it which are unstable too. Using $\tau_{x_{\max}}$ and $\Delta\omega$ defined above, one may be able to determine *both* the scale length and the temperature of the plasma at the critical surface by measuring the frequency shift and the linewidth of the $\omega_0/2$ scattered light narrow feature in an ideal, short wavelength laser, near threshold experiment. (However, mixed polarization modes cannot dominate the spectrum of CO_2 laser-plasma experiments due to the long wavelength of the pump compared to UV lasers, which favors $2\omega_{pe}$ modes.) Thomson scattering measurements of the *hybrid wave's* electrostatic component would eliminate possible misinterpretations of the coexistence of SRS, $2\omega_{pe}$, and HFHI [17–20] by identifying their various and distinct $k_\perp$ dependences.

We acknowledge useful discussions with A. E. Chou, D. F. Dubois, T. W. Johnston, W. L. Kruer, C. S. Liu, and W. M. Nevins. This work was performed in part under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48.

[1] W. L. Kruer, in *The Physics of Laser Plasma Interactions,* Frontiers in Physics Series, Vol. 73 (Addison-Wesley Publishing Company, Reading, MA, 1988).
[2] M. N. Rosenbluth, Phys. Rev. Lett. **29**, 565 (1972).
[3] C. S. Liu, M. N. Rosenbluth, and R. B. White, Phys. Fluids **17**, 1211 (1974).
[4] G. Picard and T. W. Johnston, Phys. Rev. Lett. **51**, 574 (1983); Phys. Fluids **28**, 859 (1985).
[5] C. S. Liu, in *Advances in Plasma Physics,* edited by A. Simon and W. Thompson (J. Wiley & Sons, New York, 1976), Vol. 16.
[6] H. A. Baldis, E. M. Campbell, and W. L. Kruer, in *Handbook of Plasma Physics,* edited by A. Rubenchik and S. Witkowski (North-Holland, New York, 1991), Vol. 3.
[7] B. B. Afeyan, Ph.D. dissertation, University of Rochester, Rochester, New York, 1993.
[8] C. S. Liu and M. N. Rosenbluth, Phys. Fluids **19**, 967 (1976).
[9] A. Simon, R. W. Short, E. A. Williams, and T. Dewandre, Phys. Fluids **26**, 3107 (1983), and references therein.
[10] J. F. Drake and Y. C. Lee, Phys. Rev. Lett. **31**, 1197 (1973).
[11] M. A. Mostrom and A. N. Kaufman, Phys. Rev. Lett. **42**, 644 (1979).
[12] B. B. Afeyan and E. A. Williams, Phys. Fluids **28**, 3397 (1985).
[13] A. B. Langdon and B. F. Lasinski, Bull. Am. Phys. Soc. **19**, 920 (1974); Laser-Fusion Program Semiannual Report, Report No. UCRL-50021-73-1, January to June 1973, p. 187.
[14] F. W. Byron and R. W. Fuller, *Mathematics of Classical and Quantum Physics* (Addison-Wesley Publishing Company, Reading, MA, 1969), Vols. 1 and 2.
[15] R. J. Briggs, *Electron-Stream Interactions with Plasmas,* Research monograph Vol. 29 (M.I.T. Press, Cambridge, MA, 1964).
[16] B. B. Afeyan and E. A. Williams (to be published).
[17] W. Seka *et al.,* Phys. Fluids **28**, 2570 (1985).
[18] D. M. Villeneuve, J. E. Bernard, and H. A. Baldis, Phys. Fluids **30**, 3832 (1987).
[19] J. Meyer and Y. Zhu, Phys. Rev. Lett. **64**, 2651 (1990).
[20] D. F. Dubois, D. A. Russell, and H. A. Rose, Phys. Rev. Lett. **74**, 3983 (1995).