Robust Acoustic Time Reversal with High-Order Multiple Scattering

Arnaud Derode, Philippe Roux, and Mathias Fink

Laboratoire Ondes et Acoustique, URA CNRS 1503, Université Paris VII Denis Diderot-Ecole Supérieure de Physique et de Chimie Industrielles de la Ville de Paris, 10 Rue Vauquelin, 75005 Paris, France

(Received 16 May 1995; revised manuscript received 28 April 1995)

We report the first experiments showing the reversibility of transient acoustic waves through high-order multiple scattering by means of an acoustic time-reversal mirror. A point source generates a pulse which scatters through 2000 steel rods immersed in water. The time-reversed waves are found to converge to their source and recover their original wave form, despite the high order of multiple scattering involved and the usual sensitivity to initial conditions of time-reversal processes. Surprisingly, the observed resolution was one-sixth of the theoretical limit for the mirror's aperture.

PACS numbers: 43.20.+g, 42.25.–p

Irreversibility of time is one among several long-time problems that still obsess many physicists. Although classical and quantum mechanics equations are reversible at a microscopic scale, our everyday observations show that macroscopic phenomena are irreversible. Physicists know, from statistical mechanics, that one source of this apparent irreversibility originates from the extraordinarily large number of particles that comprise macroscopic systems.

In theory, or in a thought experiment, it is possible to "freeze" a large number of particles and reverse their velocities: Imposing these new initial conditions would make the system travel back to its initial state. However, to actually perform such an experiment, one would need to resolve the positions and velocities of each particle with essentially infinite accuracy. In classical mechanics, the evolution of a system is highly sensitive to its initial conditions; this is the reason why the slightest error in the measurements makes such time-reversal experiments impossible. It would require an infinite amount of information and is thus impossible with any real experimental device.

In wave physics, however, the amount of information required to describe a wave field unambiguously is limited and depends on the shortest wavelength of the field. Thus it is possible and easier to make time-reversal experiments with waves rather than with particles; among all kinds of possible waves, acoustic waves turn out to be very appropriate when studying reversibility of transient disordered phenomena, as we will see. One could think of using optical waves instead; and indeed, in the past years, optical techniques such as phase conjugation have been successfully applied to acoustic waves [1]. But phase conjugation is basically a monochromatic technique; in order to extend this concept to broad-band signals and transient phenomena, one has to use a time-reversal system, i.e., a device that can record a wave form $f(t)$, time reverse it, and retransmit $f(-t)$ inside the medium. This is easily realizable for acoustic waves, since, unlike optical detectors, piezoelectric transducers can follow the real-time variations of an ultrasonic signal and can also be used as transmitters to create a given signal.

Consider an acoustic source located inside an inhomogeneous lossless medium. This source transmits a pulse that propagates through the inhomogeneities. An ideal time-reversal experiment requires perfect measurement of the Green function over a closed surface surrounding the source. In practice, however, the time-reversal operation can be performed only over a restricted aperture. The resulting time-reversal system is termed a finite size time-reversal mirror (TRM) [2]; such a device is diffraction limited and therefore has a limited resolution.

An ultrasonic TRM is an array of N wide-band piezoelectric transducers that perform a fine spatial sampling of the acoustic field. The N signals can be recorded, placed in a memory, time reversed, and retransmitted through the medium by the same transducer array. Previous studies showed that the TRM optimally focuses a wave in a homogeneous medium as well as through an aberrating layer [3].

What we report here is an acoustic equivalent of the thought experiment we mentioned above, i.e., making a macroscopic disordered system go back to its initial state. We start with a coherent transient pulse, let it propagate through a disordered, highly scattering medium, then record the scattered field, time reverse it, and see if it travels back to recover its original shape. Precisely we are interested here in inhomogeneous media with high-order multiple scattering. For instance, we will consider random sets of up to 2000 steel rods (diameter 0.8 mm, average spacing 2.5 mm). When a pulsed wave traverses such a medium, it undergoes many scattering events (up to 180 in our sample as we will see) before reaching the TRM.

In such conditions, can the time-reversal operation still work and refocus the wave field on its source? As surprising as it may sound, the answer is yes. This is indeed surprising because when such a high order of multiple scattering is involved, the slightest change could be expected to have a dramatic impact, and one might

 0031-9007/95/75(23)/4206(4)$06.00

suppose that the waves could not find their way back to the source. This supposed sensitivity to initial conditions, which leads to chaos, is well known for particles, and yet the TRM will be shown to be a very robust device that can reconstruct the initial pulse from high-order multiply scattered waves.

Another question then arises: How accurately can the TRM refocus the wave on a point source? For a homogeneous medium, the diffraction limit is roughly $\lambda z/a$, where a is the size of the TRM, z its distance to the source, and λ the average wavelength. Experimental results show that in the presence of high-order multiple scattering the resolution of the aperture can be increased to $\frac{1}{6}\ \lambda z/a$.

Reversibility of physical phenomena is a simple and beautiful idea but making it an experimental fact is far less evident. The aim of this Letter is not to prove reversibility, but to show that a (relatively) simple experimental process can take advantage of reversibility to focus a wave back on its source even in the presence of very high-order multiple scattering, with a greatly enhanced spatial resolution. This occurs, despite the great loss of information due to the finite angular aperture of the TRM (roughly 5°), not to mention the finite pitch of the array and the quantization effects.

The general principle of the experiments is presented in Fig. 1: An ultrasonic source S transmits a short pulse that propagates through one of the samples. The TRM receives a set of signals and records them into electronic memories. These signals are time reversed, then retransmitted by the mirror, thus creating an ultrasonic wave that propagates through the same sample. Then the pressure field is measured at the source S.

The mirror is a linear array of 96 transducers with central frequency 3.5 MHz (which corresponds to an average wavelength $\lambda \approx 0.43$ mm in water), bandpass 50%, and pitch 417 μm. On each channel, the signals are sampled at a 20 MHz rate, digitized on 8 bits, and recorded. The source S is a single transducer (size 387 μm, central frequency 3.5 MHz), and the sample is located between the source S and the mirror. After the time-reversal process, the same transducer S records the pressure field as a function of time; it can also be translated along the direction of the array (axis x in Fig. 1) in order to scan the pressure field, so S measures a two-dimensional signal $s(x,t)$. Such signals are represented by grey scale images known as B scans: time t is in the abscissa, position x in the ordinate, and each line on the picture represents the wave form received at a given position in grey scale levels (the bigger the amplitude, the brighter the image). From a B scan $s(x,t)$, the spatial resolution is measured by a directivity pattern $d(x)$: The maximum value for each line on the image is detected, which provides the directivity pattern $d(x) = \max_t\{s(x,t)\}$.

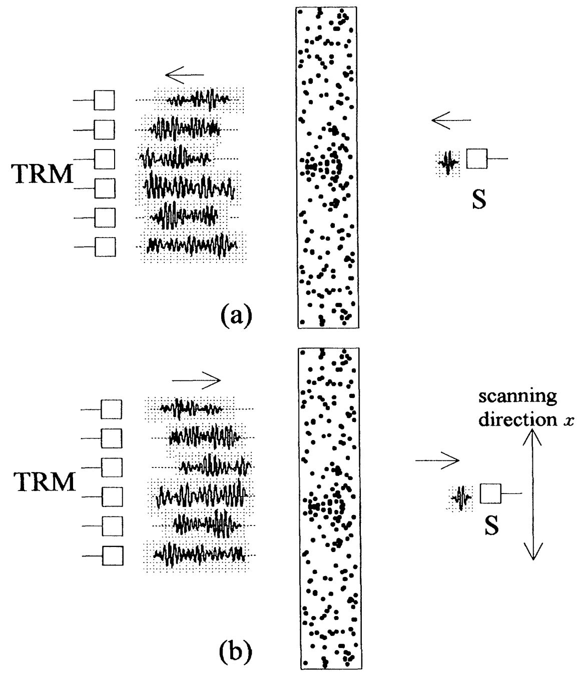

FIG. 1. Experimental setup. (a) First step: S sends a pulse though the sample, the transmitted wave is recorded by the TRM. (b) Second step: the multiply scattered signals have been time reversed, they are retransmitted by the TRM, and S records the reconstructed pressure field. The operation can be repeated as S is translated along x in order to scan the pressure field.

We used two types of samples. The first one (medium I) is a random set of parallel steel rods ($c_L = 5.9$ mm/μs, $c_T = 3.2$ mm/μs, $\rho = 7.85$) with diameter 0.8 mm and surface fraction 3.9%, the average spacing between two rods is 3.6 mm (there are roughly 900 rods in a 150×75 mm^2 area). The second one (medium II) contains glass spheres ($c_L = 5.7$ mm/μs, $c_T = 3.2$ mm/μs, $\rho = 2.3$, diameter 1.5 mm) randomly distributed in an agar-based gel, with a 5.5% volume fraction (i.e., there are 8200 spheres in a sample with dimensions $84 \times 112 \times 28$ mm^3).

The mean free path l is an essential parameter of wave propagation when multiple scattering takes place. l can be considered as a coherence decay length; indeed, when a wave undergoes multiple scattering as it traverses a slab of thickness L, the transmitted intensity can be divided into two terms: a coherent term which corresponds to the remainder of the incoming wave and an incoherent term which contains the multiscattered contribution to the transmitted field. The transmission coefficient for coherent intensity is given by $T(L) = e^{-L/l}$ [4,5]. When L is sufficiently larger than l, almost all the initial energy has been transferred to multiscattered waves; the transport of the energy is diffusive and can be described by a classical diffusion equation [6] which, unlike the propagation equation, is not invariant under time reversal.

Using short ultrasonic signals, it is possible to separate the coherent ("ballistic") part of the transmitted field, which arrives first on the array, from the incoherent multiscattered paths. Thus the transmission coefficient for coherent intensity $T(L)$ can be measured experimentally with wide-band transducers (central frequency 3.5 MHz) as a function of the thickness L. The slope of the curve on a logarithmic scale leads to an estimate value of the mean free path $l = 9 \pm 0.5$ mm for medium I. As to medium II, we could not observe the exponential decay of T since the sample thickness was fixed; the measurement of T for this thickness leads only to $l = 14$ mm.

In the following experiments, the thickness is fixed at $L = 45$ mm for medium I and 28 mm for medium II. In all cases, this distance is greater than the mean free path, which indicates a multiple scattering regime. This is indeed obvious when looking at the transmitted signal through medium I (Fig. 2): After the coherent ballistic wave front a very long signal appears (it lasts over 120 μs after the coherent wave-front arrival, which corresponds to multiple paths with lengths of up to 225 mm inside the sample (i.e., 25 times the mean-free path, or 60 times the average spacing between two rods). We will now observe the reversibility properties in such a medium.

It should be noted that a linear array is only a TRM for 2D wave fields, whereas neither medium II nor even medium I produce truly 2D fields. In the case of cylinders, the 2D approximation is reasonable even if S is not a perfect line source, since it is 12 mm in height, the same as the mirror. In the case of spheres, the 2D approximation is obviously not valid; however, considering that the diameter of the spheres is roughly 4 times the average wavelength, the incident wave is mainly scattered in the forward direction, which limits vertical out-of-plane spreading.

First, the directivity pattern of the TRM is measured in water. In the absence of any scattering medium, the array receives a spherical wave front coming from the source S, as is shown on Fig. 2(a). Then the time-reversed retransmitted wave converges towards the source; the signal received by S is represented in Fig. 3(d), and the corresponding directivity pattern is plotted in Fig. 4; it exhibits symmetrical sidelobes as predicted by diffraction theory, and its width at half maximum is $\Delta = 2.3$ mm (the theoretical value is $1.2\lambda z/a = 2.32$ mm, with $z = 180$ mm and $a = 96 \times 0.417 = 40$ mm).

We now place medium I between the point source and the mirror. The signals received on the mirror are now much longer [Fig. 2(b)]. The first 80 μs of these 96 signals are recorded, time reversed, and retransmitted (the playback resolution is also 8 bits). The resulting acoustic wave traverses the sample. An amazing compression is observed, since the received signal lasts about 1 μs [Fig. 3(b)], against over 120 μs for the time-reversed multiply scattered signals.

The same experiment can be repeated with a different time window; this time, the TRM time reverses and retransmits the last 80 μs of the multiply scattered signal, which means that the time-reversal window begins 40 μs *after* the ballistic front. As is shown in Fig. 3(c), the initial pulse can still be reconstructed at the source S. In both cases, the time-reversed waves seem to have effectively gone backwards through the same paths and found their way back to the source. Moreover, the directivity pattern (Fig. 4) shows a finer resolution than in an homogeneous medium: Its width at half maximum is $\Delta_{\mathrm{I}} = 1.2$ mm, which is less than the resolution limit of a diffraction-limited aperture in a homogeneous medium.

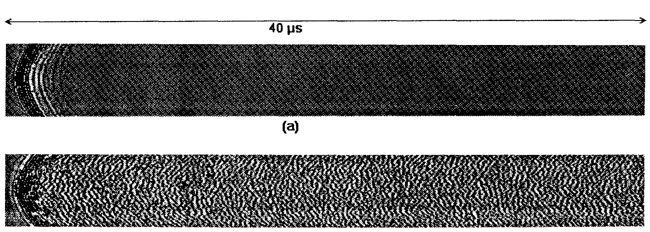

FIG. 2. B scans of the pressure field received on the TRM in homogeneous medium (a) and through medium I (b).

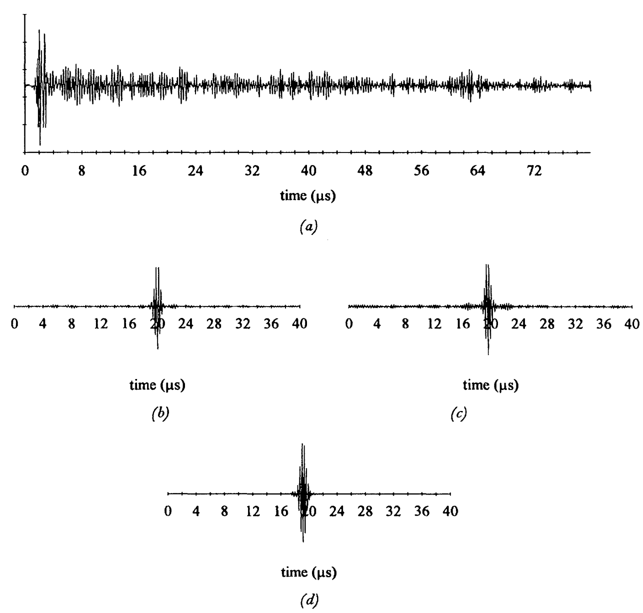

FIG. 3. (a) Wave form received on the TRM by element 46 through medium I. (b) Wave form received by S after time reversing the first 80 μs of the multiply scattered signals through medium I. (c) Wave form received by S after time reversing the last 80 μs of the multiply scattered signals through medium I. (d) Wave form received by S after time reversing the signals in the absence of any scattering medium. (The time origins have been chosen arbitrarily.)

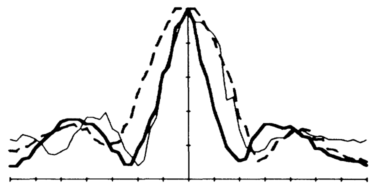

FIG. 4. Directivity pattern of the pressure field received by S in homogeneous medium (dashed line), through medium I (thick line), and through medium II (thin line).

This can be explained by the large lengths of multiple scattering paths involved. As a matter of fact, multiple scattering widens the acoustic beam, thus creating a halo that is much larger than in the homogeneous case. So it is a plausible explanation that when the time-reversed field is retransmitted by the array, the sample acts as a source with a larger angular diameter, which accounts for the enhanced resolution. Similar results are obtained with medium II. The observed resolution is $\Delta_{\mathrm{II}} = 1.5$ mm (Fig. 4), which is still finer than in a homogeneous medium, but the improvement in resolution is smaller than with medium I, probably due to the larger mean free path (and consequently the shorter and fewer multiple scattering paths), and also to the dimensional issues mentioned above. As a paradox, multiple scattering that altogether degrades the directivity of a wave as well as its temporal resolution can help to refocus this wave as long as the time-reversal process can make it travel back through all its past, towards its original source: An amazing time and space compression is observed, the accuracy of focusing being even better than the theoretical resolution for a diffraction-limited aperture.

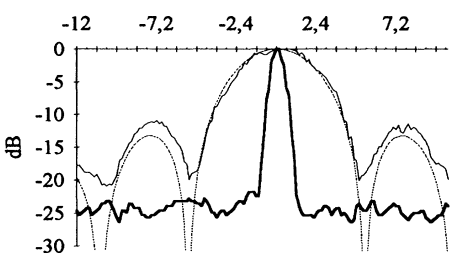

FIG. 5. Directivity patterns of the TRM through 2000 steel rods (thick line) and in water along (thin line). The theoretical sinc function is plotted in dashed lines.

The last experimental directivity pattern we present (Fig. 5) points out the influence of high-order scattering in this narrowing effect. It has been obtained through a set of 2000 steel rods (average spacing 2.5 mm, the overall thickness of the sample was $L = 75$ mm) by reversing a 235 μs time window beginning 10 μs after the ballistic front. Given the sample thickness, 245 μs correspond to paths of up to $\sim$440 mm, that is to say roughly 180 times the average spacing between two scatterers. Sixty-four transducers have been used ($a = 64 \times 0.417 = 26.7$ mm); the distance between the mirror and the source was $z = 33$ cm. The resolution we observed is 1.05 mm; that is to say, one-sixth of the theoretical limit in an homogeneous medium $1.2\lambda z/a = 6.38$ mm, and the average background level is roughly -24 db, in comparison to -13 db for the classical sinc function.

In conclusion, the experimental results we presented in this Letter clearly show two points. First, the acoustic time-reversal mirror is a very robust device that can focus a wave even in the presence of high-order scattering despite the finite size of the array and the eight bits quantization, whereas a chaotic behavior might have been expected. Second, in the presence of high-order scattering, the resolution of the aperture is greatly increased due to the long multiple paths involved. The aim of this Letter was to present experimental facts. Clearly, several parameters are involved in the quality of reversibility: the elastic and transport mean free paths, the size and duration of the time-reversal window, and the size of the mirror. We are currently working on a theoretical model to account for the influence of these parameters on the quality of reversibility.

The authors would like to thank Professor R. Maynard for his advice, as well as Professor J. H. Rose for his comments and his help in editing the manuscript.

[1] D. R. Jackson and D. R. Dowling, J. Acoust. Soc. Am. **91**, 3257 (1992).
[2] D. Cassereau and M. Fink, J. Acoust. Soc. Am. **94**, 2373 (1993).
[3] F. Wu, J.-L. Thomas, and M. Fink, IEEE Trans. Ultrason. Ferroelec. Freq. Contr. **39**, 567 (1992).
[4] A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Academic Press, New York, 1978), Vols. 1 and 2.
[5] D. Sornette, Acustica **67**, 251 (1989).
[6] E. Akkermans *et al.*, J. Phys. (Paris) **49**, 77 (1988).

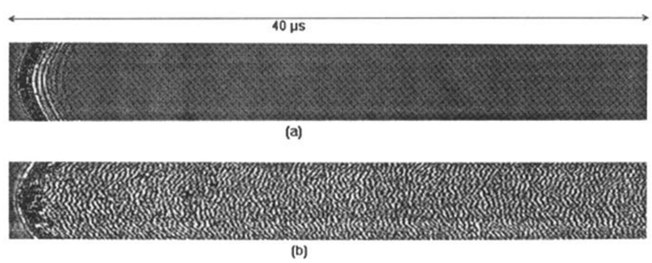

FIG. 2. B scans of the pressure field received on the TRM in homogeneous medium (a) and through medium I (b).