Relation between the Scissors Mode and the Interacting Boson Model Deformation

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An interacting boson model sum rule for the strength of exciting the magnetic dipole scissors mode is used to relate this strength to the quadrupole deformation, resulting in a parameter-free description of the $M1$ transition strength over the entire range of nuclei from Nd to W studied experimentally. Also, the experimentally established quadratic dependence on the quadrupole ground state deformation is quantitatively reproduced for nuclei up to Yb, but systematic deviations are predicted for heavier nuclei.

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The general success of the interacting boson model (IBM) in describing the collective properties of the low-energy spectra in heavy nuclei is well established [1]. The introduction of the proton-neutron degree of freedom (called IBM-2) has led to the prediction of a new class of states whose wave functions have mixed symmetry with respect to the interchange of protons and neutrons [2]. Among those, the transition to the lowest 1^+ state, now called "scissors mode" because of the peculiar motion of protons against neutrons responsible for it [3], has played a major role since its experimental discovery about a decade ago [4]. One of the first successes of the IBM-2 was the quantitative prediction of its $M1$ ground state transition strength (see [4,5] and references therein).

The properties of the $M1$ scissors mode are well established now over a wide range of rare-earth nuclei and several remarkable features have been established. The total strength is proportional to the square of the ground state (g.s.) quadrupole deformation [6]. This behavior has been (more or less) successfully reproduced in a variety of calculations (see [7] for references). Furthermore, such a deformation dependence implies a close correlation to the $E2$ strength of the transition to the first excited state, the latter being directly related to the experimental quadrupole moment [8]. In fact, when the $M1$ strength is plotted versus the factor $P = N_p N_n/(N_p + N_n)$, which counts the number of interactions between valence protons (N_p) and neutrons (N_n) and represents a measure of the deformation driving proton-neutron interaction [9], a sharp transition from vibrational to rotational nuclei is obtained at $P \simeq 4 - 5$ followed by a saturation for large P at a value $B(M1) \approx 3\mu_N^2$. Attempts to understand the correlation between magnetic dipole strength and quadrupole collectivity within the IBM-2 have been based on sum rule approaches [10,11].

Recently, a phenomenological parameter-free sum rule inspired by the two-rotor model (although valid in a general context) has been formulated [12] and shown to correspond to the IBM-2 sum rule of Ref. [10] in the classical limit [13]. It is able to describe the experimentally found low-lying $M1$ strength for transitional and well-deformed even-even nuclei ranging from Nd to Yb [7]. Here, we aim at a similar systematic description of the $M1$ scissors mode in the framework of IBM-2. The sum rule of Ref. [10] relates the $M1$ strength to the average number of quadrupole bosons in the g.s. Using the intrinsic state [14,15] we can express the average number of quadrupole bosons by the IBM deformation parameter β, which in turn can be related to geometrical definitions of deformation. As a result, a sum rule for the $M1$ scissors mode strength is derived which, besides an obvious dependence on the boson g factors, is parameter free.

Before beginning the discussion, we briefly summarize the experimental database on the scissors mode strength. With the exception of ^{154}Gd, where results are taken from (e,e') scattering [16], all quoted transition strengths for the $M1$ scissors mode are derived from (γ,γ') experiments. Experimental results have been reported for ^{146,148,150}Nd [17], ^{148,150,152,154}Sm [18], ^{156,158,160}Gd [19,20], ^{160,162,164}Dy [21], ^{166,168,170}Er [22], ^{172,174,176}Yb [23], and ^{182,184,186}W [24]. The prominent transitions in ^{164}Dy have been shown to contain an appreciable spin part [25] which has been removed assuming constructive interference with the orbital $M1$ strength. A preliminary result for ^{178}Hf is also available [26].

To determine the total $B(M1)$ strength of the scissors mode the transitions to all 1^+ states were summed in the excitation energy range $E_x = 2.5$–4 MeV. In cases where the parity of a dipole state is not uniquely determined all transitions with a ratio $B(1 \to 2_1^+)/B(1 \to 0_1^+) \le 1$ were assumed to have $M1$ character. Here, $B(1 \to 0_1^+)$, $B(1 \to 2_1^+)$ denote the reduced transition probabilities for the decay to the ground and first excited state, respectively. The given error bars are a conservative estimate obtained by summing up the errors of the individual transitions.

We start with the sum rule of Ref. [10]

$$\sum_f B(M1 : 0^+ \to 1_f^+) = \frac{9}{4\pi}(g_\pi - g_\nu)^2 \frac{P}{N-1}\langle 0|N_d|0\rangle, \tag{1}$$

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where $\langle 0|N_d|0\rangle$ is the expectation value of the d-boson number operator in the g.s. Here, N_π, N_ν are the proton and neutron boson numbers, g_π, g_ν are the corresponding g factors, $N = N_\pi + N_\nu$, and $P = 2N_\pi N_\nu/(N_\pi + N_\nu)$. Using this sum rule the average number of quadrupole bosons in the g.s., $\overline{N}_d = \langle 0|N_d|0\rangle/N$, is extracted from the measured orbital $B(M1)$ strengths using a considerably extended database compared to the one used in Ref. [10]. In Fig. 1 the quantity $\overline{N}_d$ is plotted as a function of the P factor. A new variable $P_{\rm rel}$ is introduced which distinguishes nuclei below ($P_{\rm rel} = P - P_{\rm max}$) and above ($P_{\rm rel} = P_{\rm max} - P$) midshell, where $P_{\rm max}$ is the value of P at midshell. In the rare-earth region one has $N_{\pi,\rm max} = 8$ and $N_{\nu,\rm max} = 11$. Thus, $P_{\rm max} = 2 \times 8 \times 11/(8 + 11) = 9.26$.

Both above and below midshell $\overline{N}_d$ is correlated with P, rising monotonically with P. However, a striking difference is visible when midshell is reached for either N_π or N_ν, or both. Below, one observes the transition from vibrational to rotational nuclei as P increases ($|P_{\rm rel}|$ decreases) followed by a saturation region roughly in accord with the prediction in the SU(3) limit, $\overline{N}_d = 2/3$. Above midshell, although $\overline{N}_d$ again increases with P ($|P_{\rm rel}|$ decreases), the average magnitude of $\overline{N}_d$ is reduced reflecting less deformed nuclei. This asymmetry can be traced back to the microscopic shell structure [11]. The $E2$ strength in these nuclei shows a similar behavior, indicating a reduction of the proton-neutron interaction energy due to Pauli blocking [27].

The correlation of $B(M1)$ and $B(E2)$ strength suggests a common origin in collective properties of the nucleus such as deformation. In the intrinsic state representation the average number of quadrupole bosons for large $\beta^2 N/(1 + \beta^2)$ is [14,28]

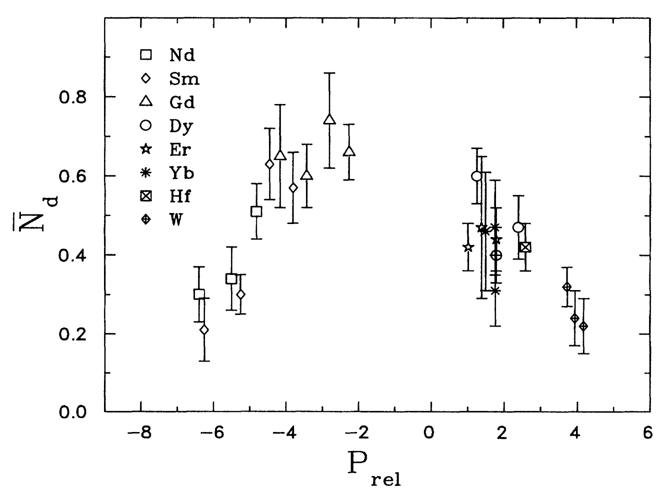

FIG. 1. Average number $\overline{N}_d$ of quadrupole bosons in the g.s. as a function of the P factor defined as $P_{\rm rel} = P - P_{\rm max}$ and $P_{\rm rel} = P_{\rm max} - P$ for nuclei below and above midshell, respectively.

$$\overline{N}_d \approx \frac{\beta^2}{1 + \beta^2} + O\left(\frac{1}{\beta^2 N/(1 + \beta^2)}\right). \quad (2)$$

The β values of the IBM can be related to the Bohr-Mottelson deformation parameter β^2 (defined, e.g., in Ref. [29]) by

$$\beta = \frac{\lambda}{\frac{2}{3}\sqrt{\pi}}\left(\frac{Z}{Z_{\rm val}}\right)\beta_2 . \quad (3)$$

Here, $Z/Z_{\rm val}$ is the ratio of the total to the valence proton number. The parameter λ reflects the extent to which the transition to the first excited state saturates the $B(E2)$ sum rule

$$\lambda = \sqrt{\frac{\sum_i B(E2; 0^+ \to 2_i^+)}{B(E2; 0^+ \to 2_1^+)}} . \quad (4)$$

Equation (3) is inspired by a similar relationship within IBM-1 for the ratio of the total number of valence nucleons to the mass number [14]. However, since β_2 is determined from electromagnetic transitions only protons contribute. The scaling ratio $Z/Z_{\rm val}$ reflects the restriction of the IBM to valence nucleons compared to the total proton body which is considered in the geometrical definition of deformation. One should note that $Z_{\rm val}$ is *not* counted as a hole number above midshell.

In Fig. 2(a) the ratio R of experimental to theoretical $\overline{N}_d$ values calculated with Eqs. (2) and (3) is displayed as a function of the total boson number. The β_2 values

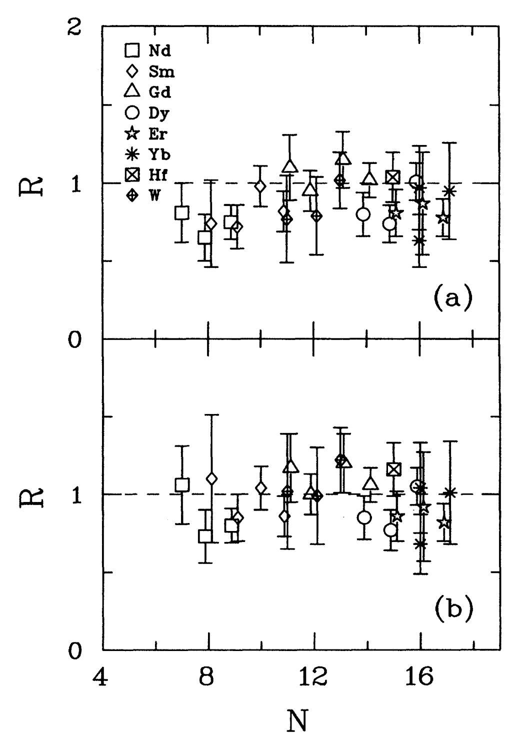

FIG. 2. (a) Ratio of experimental scissors mode strengths to IBM-2 predictions as a function of the total boson number N using Eq. (2), (b) using the $1/N$ corrected Eq. (5).

are taken from Ref. [29]. For simplicity, $\lambda = 1$ is used. Also the boson g factors are put to the microscopically suggested [30] free values $g_\pi = 1$ and $g_\nu = 0$.

The agreement is very encouraging, although the experimental values overall tend to be somewhat smaller than the prediction. The deviations are particularly pronounced for small boson numbers. Keeping in mind that Eq. (2) holds in the limit of large $\beta^2 N/(1+\beta^2)$ only, one should correct to the next order [31]

$$\overline{N}_d \approx \frac{\beta^2}{1+\beta^2}\left[1 + \frac{1}{N}\left(1 - \frac{1+\beta^2}{\beta^2}\right)\right] + O\left(\frac{1}{[\beta^2 N(1+\beta^2)]^2}\right). \tag{5}$$

The corresponding results are shown in Fig. 2(b). Including this correction, we attain *a satisfactory description of the M1 scissors mode strength for all nuclei where data are available.*

One can try to extend the approach presented here to the actinide region where a similar correlation of $B(M1)$ and $B(E2)$ is found [32]. However, data are vary scarce. For those nuclei, where experimental information is available, application of Eq. (5) yields $R = 0.76(7)$ for ^{232}Th [33], $R = 0.92(14)$ for ^{236}U [34], and $R = 0.78(6)$ for ^{238}U [33]. The somewhat low values for ^{232}Th and ^{238}U might be explained by the restriction in Ref. [33] to a comparison with strong $B(M1)$ transitions observed in (e, e') scattering, confined to a fairly small energy interval $E_x \simeq 2.0$–2.5 MeV. In the (γ, γ') study of ^{236}U good agreement with the sum rule prediction is obtained if one sums over the whole investigated energy range $E_x \simeq 1.7$–3.2 MeV. Thus, the few available data indicate an applicability of the sum rule in the actinide region.

Recently, the total scissors mode strength in the Sm and Nd chains [6,17] was experimentally established to have a quadratic dependence on the g.s. deformation. In Fig. 3(a), all available experimental data are plotted as a function of β_2^2. A linear relation is clearly visible. The solid line is a least-squares fit (including the origin) which results in an average value

$$B(M1)^{\text{av}}_{\text{exp}} = 26.0(4)\beta_2^2[\mu_N^2]. \tag{6}$$

It may be noted that an unrestricted fit leads to an intercept value compatible with zero. The data on ^{178}Hf and ^{182,184,186}W have not been included in the fit for reasons which become clear below. The experimental numbers on the W isotopes are significantly smaller than the values predicted by Eq. (6). This implies that Eq. (5) has a wider range of applicability.

In Fig. 3(b) we plot the scissors $M1$ strength calculated with Eqs. (1), (3), and (5) as a function of β_2^2 for all stable even-even isotopes from Nd to W. The solid line is again the linear fit obtained for the restricted set of data shown in Fig. 3(a). Although some individual scattering is obtained, the phenomenologically established quadratic β_2 dependence is well reproduced for all nuclei in the Nd to Yb isotope chains. However, systematic deviations of the scissors mode strength from the quadratic deformation dependence are obtained starting from Hf and increasing towards the heavier W isotope chains. The available data on ^{182,184,186}W seem to confirm this interesting prediction, as suggested by Fig. 2(b), but the experimental uncertainties may be somewhat large at present.

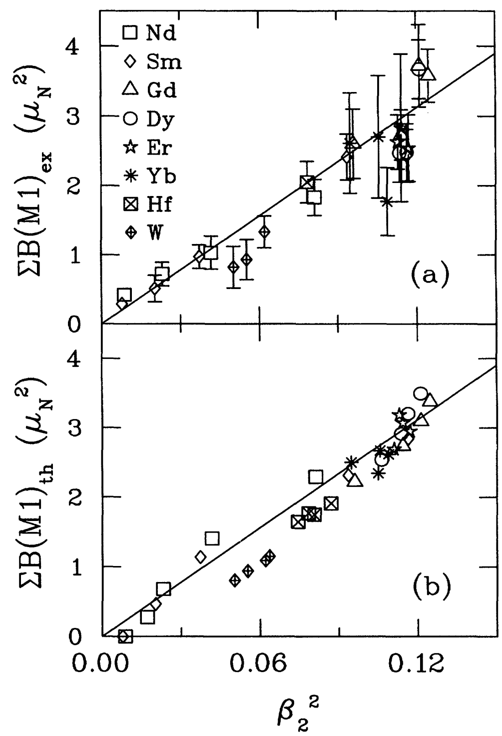

FIG. 3. (a) Experimental $M1$ scissors mode strengths as a function of the square of the quadrupole deformation parameter β_2. The straight line is a least-squares fit assuming intercept zero. (b) Prediction of the scissors mode strength for all even-even stable nuclei from Nd to W calculated with Eq. (5).

In the region of γ-soft nuclei the predicted $B(M1)$ values decrease very fast. However, the approximation given in Eq. (5) is not valid for the ground state of nonaxially symmetric nuclei. The $1/\beta^2 N/(1+\beta^2)$ corrections in Eq. (5) result from an angular momentum projection to the angular momentum zero state of the rotational band built on an axially symmetric intrinsic state. For nonaxially symmetric nuclei, there is more than one angular momentum zero state in a band. In this case, Eq. (5) gives the total $\overline{N}_d$ value for all angular momentum zero states to order $1/[\beta^2 N/(1+\beta^2)]^2$. The ground state is expected to have the largest share of quadrupole bosons, so this approximation may still be valid to within the present experimental error as long as $\beta^2 N/(1+\beta^2)$ is large. Furthermore, the intrinsic state formalism will be less valid for nuclei which are not strongly deformed; that is, for nuclei which have a shallow energy minimum as a function of β.

To summarize, a parameter-free approximate sum rule for the $M1$ scissors mode is derived within the IBM-

2. A relation between the deformation in the intrinsic IBM mode (which is due to valence nucleons only) and the geometrical definition of the Bohr-Mottelson model is given, which allows a calculation based solely on experimental information about quadrupole moments. The variation of the scissors mode strength can be described throughout the whole body of experimental data ranging from Nd to W. The few available experimental results indicate that the sum rule might also be applicable to the actinide region.

With this approach the experimentally established quadratic dependence of the scissors mode strength on the quadrupole g.s. deformation can be reproduced for even-even nuclei in the Nd to Yb chains. Towards heavier nuclei (Hf, W) systematic deviations are predicted, although Eq. (5) may be less accurate for both these nuclei because $\beta^2 N/(1 + \beta^2)$ is decreasing. For γ-soft nuclei, Eq. (5) gives an upper limit on the average number of quadrupole bosons for large values of $\beta^2 N/(1 + \beta^2)$. The properties of the scissors mode in the Os-Pt region are still an open problem and experimental studies are clearly needed. Although challenging, the feasibility of such experiments was recently demonstrated for the ^{196}Pt(γ, γ') reaction at the S-DALINAC accelerator in Darmstadt within a Cologne/Darmstadt/Rossendorf collaboration using a EUROBALL cluster [35] module.

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[1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
[2] T. Otsuka, A. Arima, F. Iachello, and I. Talmi, Phys. Lett. **76B**, 139 (1978).
[3] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
[4] D. Bohle *et al.*, Phys. Lett. **137B**, 27 (1984).
[5] F. Iachello, Phys. Rev. Lett. **53**, 1427 (1984).
[6] W. Ziegler, C. Rangacharyulu, A. Richter, and C. Spieler, Phys. Rev. Lett. **65**, 2515 (1990).
[7] A. Richter, Prog. Part. Nucl. Phys. **34**, 261 (1995).
[8] C. Rangacharyulu *et al.*, Phys. Rev. C **43**, R949 (1991).
[9] R.F. Casten, Phys. Rev. Lett. **54**, 1991 (1985).
[10] J.N. Ginocchio, Phys. Lett. B **265**, 6 (1991).
[11] K. Heyde, C. De Coster, A. Richter, and H.J. Wörtche, Nucl. Phys. **A549**, 103 (1992).
[12] N. Lo Iudice and A. Richter, Phys. Lett. B **304**, 193 (1993).
[13] N. Lo Iudice, Prog. Part. Nucl. Phys. **34**, 309 (1995).
[14] J.N. Ginocchio and M.W. Kirson, Nucl. Phys. **A350**, 31 (1980).
[15] A. Leviatan and M.W. Kirson, Ann. Phys. (N.Y.) **201**, 13 (1990).
[16] U. Hartmann, D. Bohle, F. Humbert, and A. Richter, Nucl. Phys. **A499**, 93 (1989).
[17] J. Margraf *et al.*, Phys. Rev. C **47**, 1474 (1993).
[18] W. Ziegler *et al.*, Nucl. Phys. **A564**, 366 (1993).
[19] H.H. Pitz *et al.*, Nucl. Phys. **A492**, 411 (1989).
[20] H. Friedrichs *et al.*, Nucl. Phys. **A567**, 266 (1994).
[21] C. Wesselborg *et al.*, Phys. Lett. **B207**, 22 (1988).
[22] S. Lindenstruth, Ph.D. thesis, Universität Giessen, 1994.
[23] A. Zilges *et al.*, Nucl. Phys. **A507**, 399 (1990).
[24] R.-D. Herzberg *et al.*,Nucl. Phys. **A563**, 445 (1993).
[25] D. Frekers *et al.*, Phys. Lett. B **218**, 439 (1989).
[26] U. Kneissl (private communication).
[27] R.F. Casten, K. Heyde, and A. Wolf, Phys. Lett. B **208**, 33 (1988).
[28] A. Leviatan, Prog. Part. Nucl. Phys. **24**, 85 (1990).
[29] S. Raman *et al.*, At. Data Nucl. Data Tables **36**, 1 (1987).
[30] M. Sambataro, O. Scholten, A.E.L. Dieperink, and G. Piccito, Nucl. Phys. **A423**, 333 (1984).
[31] S. Kuyucak and S.C. Li, Phys. Lett. B **349**, 253 (1995).
[32] J. Margraf *et al.*, Phys. Rev. C **45**, R521 (1992).
[33] R.D. Heil *et al.*, Nucl. Phys. **A476**, 39 (1988).
[34] J. Margraf *et al.*, Phys. Rev. C **42**, 771 (1990).
[35] J. Eberth, Prog. Part. Nucl. Phys. **28**, 495 (1992).