Instabilities in Close Neutron Star Binaries

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We report on a new analysis of instabilities in close neutron star binaries based upon (3 + 1) dimensional general relativistic numerical hydrodynamics calculations. When a realistic equation of state is employed, orbit calculations for two $1.45M_{\odot}$ neutron stars reveal surprising evidence that general relativistic effects may cause otherwise stable stars to individually collapse prior to merging. Also, the strong fields cause the last stable orbit to occur at a larger separation distance and lower frequency than post-Newtonian estimates.

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Coalescing neutron stars are currently of interest for a number of reasons. Several neutron star binaries are known to exist in the Galaxy (e.g., PSR 1913 + 16, PSR 2303 + 46, PSR 2127 + 11C, PSR 1534 + 11 [1]) whose orbits are observed to decay on a time scale of $(1 - 3) \times 10^8$ yr. It has been recognized for some time [2,3] that the final orbits of such systems may produce detectable gravitational radiation. This possibility has recently received renewed interest with the development of next generation gravity-wave detectors [4] such as cryogenic bars or the Caltech-MIT LIGO detector and its European counterparts. An event rate due to binary neutron star coalescence out to 200 Mpc could be $\gtrsim 3/\mathrm{yr}$ [5]. It has also been proposed that such events could account for the rate and energetics of observed gamma-ray bursts [6].

For much of the evolution of a neutron star binary, the system should be amenable to a point source description using post-Newtonian techniques [7,8]. However, as the stars approach one another the gravitational fields become quite strong and hydrodynamic effects could become significant. Indeed, it is expected that the wave forms could become quite complex as the stars approach their final orbit. This complexity, however, may be sensitive to various physical properties of the coalescing system [3] such as the neutron star equation of state. Hence, careful modeling including nonlinear effects of strong gravitational fields and a realistic neutron star equation of state is needed as a foundation for extraction of the information contained in the detected gravity waves.

To this end, in this Letter we report on the first application of a fully relativistic method using a realistic neutron star equation of state to near final orbit calculations for two neutron stars with a gravitational mass of $1.45M_{\odot}$ each. We find that the nonlinear strong gravitational fields cause the last stable orbit to occur at a somewhat larger separation distance and lower frequency than that estimated using the $(\mathrm{post})^{5/2}$-Newtonian approximation [8]. We also find the surprising result that the strong fields may induce otherwise stable neutron stars to collapse into black holes many orbits before they actually merge. This finding could have a significant impact on observed properties of neutron star binaries as they approach coalescence.

Some preliminary discussion of the model employed here has been reported previously [9] and a detailed discussion of the method will appear in a forthcoming paper [10]. Here we present a brief sketch of some features relevant to the present discussion. We start with the slicing of spacetime into a one-parameter family of hypersurfaces separated by differential displacements in timelike coordinates as defined in the (3 + 1) formalism [11,12].

Utilizing Cartesian x, y, z isotropic coordinates, proper distance is expressed

$$ds^2 = -(\alpha^2 - \beta_i \beta^i) dt^2 + 2\beta_i dx^i dt + \phi^4 \delta_{ij} dx^i dx^j, \tag{1}$$

where the lapse function α describes the differential lapse of proper time between two hypersurfaces. The quantity β_i is the shift vector denoting the shift in spacelike coordinates between hypersurfaces. The curvature of the metric of the 3-geometry is described by a position dependent conformal factor ϕ^4 times a flat-space Kronecker delta which requires

$$2\alpha K_{ij} = (D_i \beta_j + D_j \beta_i - \tfrac{2}{3} \delta_{ij} D_k \beta^k), \tag{2}$$

where K_{ij} is the extrinsic curvature with zero trace [12] and D_i are covariant derivatives. This conformally flat condition on the metric is motivated both by the general observation that gravitational radiation in most systems studied so far is small [3,13], and the fact that conformal flatness simplifies the solution to the field equations. As a third condition, we take the coordinate system to be rotating in such a way as to minimize the matter motion in the coordinate grid.

The implementation of this method means that, given a distribution of mass and momentum on some manifold, we first solve the constraint equations of general relativity at each time for a fixed distribution of matter. We then evolve the hydrodynamic equations to the next time step.

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Thus, at each time slice we can obtain a solution to the relativistic field equations and information on the hydrodynamic evolution. Information on the generation of gravitational radiation can then be obtained from a multipole expansion [10,14].

It is important to appreciate that at each time slice a numerically valid solution to the field equations is obtained. The hydrodynamic variables respond to these fields. The only approximation is the neglect of an explicit coupling of the gravity waves, which contribute negligibly to the metric and stress energy tensor [10].

We reduce the solution of the equations for the field variables ϕ, α, and β^i to simple Poisson-like equations in flat space. We begin with the Hamiltonian constraint equation [12] which reduces to [10,13],

$$\nabla^2 \phi = -4\pi\rho_1 . \tag{3}$$

The source term is usually dominated [10] by the proper matter density ρ, but there are also contributions from the internal energy density E, pressure P, and extrinsic curvature. Thus we write

$$\rho_1 = \frac{\phi^5}{2}\left[\rho W^2 + E(\Gamma W^2 - \Gamma + 1) + \frac{1}{16\pi}K_{ij}K^{ij}\right], \tag{4}$$

where W is a generalized Lorentz contraction and $\Gamma = 1 + P/\rho\epsilon$ is an adiabatic index from the equation of state. Similarly, the lapse function is determined from

$$\nabla^2(\alpha\phi) = 4\pi\rho_2 , \tag{5}$$

$$\rho_2 = \frac{\alpha\phi^5}{2}\left\{\rho(3W^2 - 2) + E[3\Gamma(W^2 + 1) - 5] + \frac{7}{16\pi}K_{ij}K^{ij}\right\}. \tag{6}$$

We use the momentum constraints [12] to find the shift vector which reduces to

$$\nabla^2\beta^i = \frac{\partial}{\partial x^i}\left(\frac{1}{3}\nabla\cdot\beta\right) + 4\pi\rho_3^i , \tag{7}$$

$$\rho_3^i = [4\alpha\phi^4 S_i - 4\beta^i W^2(\rho + \Gamma E)] + \frac{1}{4\pi}\frac{\partial \ln(\alpha/\phi^6)}{\partial x^j}\left(\frac{\partial}{\partial x^j}\beta^i + \frac{\partial}{\partial x^i}\beta^j - \frac{2}{3}\delta_{ij}\frac{\partial}{\partial x^k}\beta^k\right), \tag{8}$$

where S_i is the covariant momentum density.

To solve for the fluid motions in curved spacetime it is convenient to use an Eulerian description [10,15] beginning with a perfect fluid stress-energy tensor,

$$T_{\mu\nu} = (\rho + E + P)U_\mu U_\nu + P g_{\mu\nu} . \tag{9}$$

Our routines for evolving the hydrodynamics have been well tested at the special and general relativistic levels [9,10,16]. A key part of the calculations presented here is the use a realistic neutron star equation of state. Specifically, we use the zero temperature, zero neutrino chemical potential equation of state from the supernova numerical model of Wilson and Mayle [17]. In its full temperature dependent form this equation of state gives a good reproduction of the neutrino signal and other observed properties from supernova SN 1987A. The maximum gravitational mass of an isolated neutron star with this equation of state is $1.55 M_\odot$. This limit roughly agrees with the upper limit of the smallest range of neutron star masses which overlaps all observational determinations.

The calculations reported here were performed on a three-dimensional Eulerian grid of effectively 10^6 zones. However, even with this many zones we have only about 15 zones in radius to represent each neutron star. As a test of this zoning, a hydrodynamic calculation was made of a single star using typical resolution in three dimensions. This calculation was compared with a one-dimensional spherical hydrodynamic calculation with fine zoning. For the same baryonic mass, $1.59 M_\odot$, the gravitational masses agreed to 2/3%, i.e., yielding a gravitational mass of $1.45 M_\odot$ and $1.46 M_\odot$ for the 3D and 1D calculations, respectively. This we take as indicative of the accuracy of the calculated gravitational binding energy of the binary system as well.

In this Letter we present calculations made at three selected values of the orbital angular momentum with no radiation damping of the orbits. The neutron stars were chosen to be of equal mass and corotating initially. The baryonic mass was selected so that in isolation each star has a gravitational mass of $1.45 M_\odot$. Although the calculations presented here ignore radiation damping, orbits with radiation damping should follow a sequence of quasiequilibrium configurations which closely match the equilibria computed here. We use a multipole expansion [10,14] to show that the radiation damping per orbit is small.

Initial conditions were obtained by placing two neutron stars on the grid with a rotational velocity sufficient to keep them in orbit and an initial "guess" density profile from a solution to the Tolman–Oppenheimer–Volkoff-like equation for two single neutron stars in isotropic coordinates. The field equations were then solved and the hydrodynamics evolved with viscous damping until equilibrium was achieved. We follow the time evolution of the system with constant angular momentum until it has settled down. As the stars settle down the damping is slowly removed. Once found, the equilibrium configuration for one angular momentum could be used as an initial condition for the next angular momentum.

Some parameters characterizing this binary at the final time calculated for various angular momenta are

TABLE I. Parameters characterizing the orbit calculations. Note that f denotes the gravity wave frequency.

J (cm^2)	2.2×10^{11}	2.3×10^{11}	2.7×10^{11}
$B.E.$ ($M_\odot$)	0.088	0.080	0.073
M_G ($M_\odot$)	1.416	1.420	1.423
d_P (km)	39.4	40.5	53.0
$\rho_{\max}$ (g cm^{-3})	1.87×10^{15}	2.42×10^{15}	1.80×10^{15}
$\alpha_{\max}$	0.440	0.379	0.463
$\phi^2_{\max}$	1.90	2.05	1.84
$h \cdot r$ (cm)	1.03×10^4	6.76×10^3	9.60×10^3
$\dot{E}$ ($M_\odot$ sec^{-1})	0.016	0.0040	0.0059
j (cm)	1.23	0.607	1.07
I^{22} (cm^3)	1.19×10^{18}	1.28×10^{18}	2.31×10^{18}
f (Hz)	410	310	265
Orbit	Unstable	Stable	Stable
Sars	Unstable	Unstable	Stable

summarized in Table I. The first calculation was made with an orbital angular momentum of 2.2×10^{11} cm^2. The stars settled down into what appeared at first as a stable orbit, but later (after about one complete orbit) the stars began to slowly spiral in. For this system the angular momentum was apparently not enough to support the orbit. The stars were followed to a proper separation distance $d_P = 9.4m$, where m is the total gravitational mass of the binary.

By the end of the calculation, the binding energy $B.E.$ was increasing and the separation d_P decreasing sufficiently rapidly that it could be concluded that no stable orbit would result. Even so, the stars were still quite far apart. The ratio of proper separation distance to the single-star radius was $d_P/r \gtrsim 4$. At this distance, the stars are still nearly spherical.

Although the stars were far apart, we note that the central density had increased significantly. By the last time calculated, the stars exceeded the critical density for support against collapse. For our equation of state the maximum proper matter density for a single stable neutron star is $\rho_{\text{crit}} = 1.7 \times 10^{15}$ g cm^{-3} corresponding to a maximum single neutron star mass of $1.55 M_\odot$. The maximum central matter density in the stars is given as $\rho_{\max}$ in Table I. Since the central density has continuously increased for the stars, it seems likely that neither the stars nor the orbit are stable for this angular momentum as summarized at the bottom of Table I.

We have estimated the amplitude h and power in gravity waves $\dot{E}$ based upon a multipole expansion [14] the leading term of which is the mass quadrupole moment (I^{22} in Table I). Note that the energy radiated in gravity waves per orbit is a negligible fraction of the binding energy of the binary. Based upon the rate of angular momentum loss, the calculations discussed here should span a time frame of $\Delta J/J \lesssim 4$ sec or $\lesssim$500 orbits.

The next calculation was made with an angular momentum of 2.3×10^{11} cm^2. The orbit now appeared stable (cf. Table I). However, after about one revolution the central densities were noticed to be rising. By the end of the calculation (after two revolutions) the central matter density had risen to the largest value of the orbits studied here, $\rho_{\max} = 2.4 \times 10^{15}$ g cm^{-3}. At the same time the lapse function decreased to $\alpha_{\min} = 0.38$ and the conformal factor increased to $\phi^2_{\max} = 2.0$. Thus, it appears that neutron stars of this mass range and the adopted equation of state may form black holes before their orbit becomes unstable to plunge. For this orbit the stars are at a proper separation distance of $d_p = 9.7m$, and still many orbits from merging. However, the nonlinearities in the gravitational field have pushed the stars over the critical density for collapse.

A third calculation was made with the angular momentum increased to 2.7×10^{11} cm^2. The stars relaxed to a stable orbit at a proper separation of $d_p = 12.6m$. The stars also appeared to settle to a stable configuration. Although the central density was slightly above the critical density for nonrotating stars, in this case the corotating stars were stabilized by their angular momentum.

It is of interest to compare the present results with those obtained by a post-Newtonian treatment. Our intermediate orbit ($J = 2.3 \times 10^{11}$ cm^2) appears on the verge of the transition from steady inspiral to unstable plunge. Therefore, it is convenient to compare our results with the (post)$^{5/2}$-Newtonian analysis of [7,8] of the inner most stable circular orbit. We caution, however, that this comparison is ambiguous as parameters can have different meanings in the two formalisms.

In the post-Newtonian calculation of Ref. [8] the last stable circular orbit for equal-mass binaries occurs for a separation distance $d = 6.03m$ in harmonic coordinates (or $\sim 7m$ in Schwarzschild coordinates). The corresponding circular gravity wave frequency is $f \approx 1300$ Hz for $1.45 M_\odot$ stars. In the results reported here, however, the last stable orbit occurs at a proper distance of $d_P = 9.7m$. In the post-Newtonian regime, the gravity wave frequency scales as $f \sim (m/d)^{-3/2}$. Thus, if the strong-field results simply scaled as the post-Newtonian formulae, we would have expected a gravity wave frequency which was a factor of $\sim[(7/9.7)(1.42/1.45)]^{3/2}$ slower, i.e., $f \sim 770$ Hz. In fact we observe a gravity wave frequency which is about a factor of two slower than that. This slower frequency can perhaps be traced to the effect of the strong-field metric coefficients, i.e., α and ϕ^2 in Table I.

The combination of decreasing quadrupole moment and slowly increasing frequency also leads to a net decrease in the amplitude of the gravity wave signal ($h \cdot r \sim f^2 I^{22}$ in Table I) as the orbit decays from $J = 2.7 \times 10^{11}$ to 2.3×10^{11} cm^2. This is contrary to the increase expected from post-Newtonian analysis.

Even though the stars are much farther apart, the relativistic treatment gives a stronger gravitational binding energy for this system. Here we define binary binding energy ($B.E.$ in Table I) as the difference between the total gravitational mass of the binary ($2M_G$ in Table I) and the gravitational mass of two isolated stars. Whereas the post-Newtonian ratio of binding energy to reduced mass for the last stable orbit is $B.E./\mu = 0.037$, the numerical calculation described here gives $B.E./\mu = 0.11$. Much of this difference is probably due to increased binding energy of the individual stars. In our calculations there is sufficient numerical dissipation (i.e., viscosity) to accommodate the increased binding energy.

These calculations show two new results which to our knowledge have not been reported previously. One is that nonlinearities in the fully relativistic gravity of a neutron star binary imply fields so strong that the stars can become *individually* unstable to collapse into two black holes. When or whether this instability occurs is of course dependent upon the equation of state employed. For the equation of state adopted here, this collapse is observed to occur while the stars are still in a quasistable orbit implying that there could be many orbits from the onset of collapse to the time the stars actually merge.

This is an entirely new binary instability which, if correct, will have a significant impact on future studies of binary neutron star mergers and renders the two-black-hole coalescence problem much more important. The possibility of collapse to black holes many orbit periods before coalescence may also have observational consequences not only for gravity wave detectors, but in electromagnetic (radio, optical, x-ray, or γ-ray) bursts as well.

A second significant aspect of the present work is that the binary orbit becomes unstable due to nonlinear effects of gravity at a larger separation distance (a factor of ≈ 1.4) and lower frequency (by a factor of ~ 4) than that derived from (post)$^{5/2}$-Newtonian analysis. This lower frequency is important, since it places the coalescence frequency closer to the maximum sensitivity range of laser interferometer gravitational wave detectors such as LIGO [4]. Our estimate of the gravity wave amplitude near the final orbit is $h \approx 3.3 \times 10^{-23}$ at 100 Mpc.

From the above discussion it is clear that further studies are warranted, particularly a better determination of the last stable orbit and the approach to this orbit. Work along this line is currently in progress. There is also a need to study orbits at larger radii to make a connection to the post-Newtonian regime.

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