Determining the Phase of a Strong Scattering Amplitude from Its Momentum Dependence to Better Than 1°: The Example of Kaon Regeneration

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We quantitatively study dispersion relations giving the phase of a strong scattering amplitude from its momentum dependence. We consider C-odd contributions to neutral kaon scattering (regeneration) where accurate measurements of both amplitudes and phases are available. We find, somewhat surprisingly, that, even including many possibly corrupting effects, the uncertainty is well below 1°. This allows an accurate determination of the phase ϕ_{+-} in kaon decay to test CPT; conversely, assuming CPT symmetry, new phenomena can be limited.

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Dispersion integrals relate the phase of a strong scattering amplitude at one energy to its magnitude at all energies. It is of interest to determine the accuracy with which the phase can be deduced from the momentum dependence of the magnitude. To this end, we consider primarily the scattering of neutral kaons off nuclear targets. The individual K^0 and $\bar{K}^0$ strong amplitudes have several contributions, but the C-odd piece which we treat, with opposite sign between K^0 and $\bar{K}^0$, is simply behaved. The weak interaction mixes K^0 and $\bar{K}^0$, so the scattering of the antisymmetric K_L to the symmetric K_S gives the C-odd contribution free of systematic uncertainty; this is kaon regeneration. The K_S is identified by its common decay to $\pi\pi$. Furthermore, because the K_L also decays to $\pi\pi$ (violating CP), there is interference between regenerated K_S and transmitted K_L. The latter amplitude (η_{+-}) has been measured, allowing the determination of the phase of ρ, the C-odd scattering amplitude. Kaons also regenerate from the electric charge so that strong, weak, electromagnetic, and CP-violating interactions all come into play. That both the magnitude and phase of the scattering amplitude can be measured makes this system ideal for quantitative studies of dispersion relations.

Since dispersion integrals probe energies much higher than available, careful measurements can give sensitivity to new phenomena. For example, unexpected high-energy behavior in the difference between particle and antiparticle cross sections would show up at present energies in the phase of the difference amplitude. Or, neglecting such exotic possibilities, if one can determine ϕ_ρ from its momentum dependence, one can turn the argument around to accurately extract $\arg(\eta_{=-}) = \phi_{+-}$. We use new measurements [1] with a regenerator made of scintillator ($CH_{1.1}$) to ask how well one can determine the phase of an amplitude (in this case, the regeneration amplitude) from its local momentum dependence. To our knowledge, this is the first time this question has been comprehensively addressed.

The regeneration amplitude is a combination of scattering amplitudes and a geometrical term. One finds [2] $\phi_\rho = \phi + \phi_{\text{geom}} + \pi/2$, where $\phi = \arg(f - \bar{f})$ is the phase of the difference of nuclear scattering amplitudes for K and $\bar{K}$, and ϕ_{geom} is known.

At high energy, regeneration is dominated by the exchange of the ω Regge trajectory [3]. This by itself gives a pure power law $|(f - \bar{f})/k| \simeq p^{\alpha-1}$ (as a function of the laboratory kaon momentum, $p = \hbar k$). Analyticity of the scattering amplitudes gives the corresponding phase as $\phi = -(\pi/2)(1 + \alpha)$. Thus the momentum dependence of the modulus gives ϕ. This "phase-power" relation (PPR) follows via standard dispersion relations independent of Regge theory. Measurements [4] of high-energy regeneration show essentially perfect power laws implying only small corrections to the deduced phase.

If the modulus only approximates a power law, the corresponding phase is nearly constant and is given, on average, by the PPR. This behavior of analytic functions is well known in electrical network engineering [5]. The key ingredient is the requirement of analyticity.

One straightforward way to analyze regeneration data involves fitting the momentum dependence of $|(f - \bar{f})/k|$ to a single power law; the PPR then gives the phase. This procedure has been used [6] and is accurate. We now explicitly consider alterations in the relationship due to multiple elements in the regenerator, multiple trajectories exchanged, possible daughter trajectories, electromagnetic regeneration, nuclear screening (elastic and inelastic), low-energy structure, and the hypothetical Odderon. The error in the extracted phase reflects uncertainties in the magnitudes of the established effects.

We first consider hydrogen, where regeneration is due to ω and ρ exchange. The full expression for the amplitude is $(f - \bar{f})/k = \beta_\omega e^{i\phi_\omega} p^{\alpha_\omega - 1} - \beta_\rho e^{i\phi_\rho} p^{\alpha_\rho - 1}$. We take [7] α_ω (α_ρ) = 0.44 (0.575) and β_ω (β_ρ) = 11.5 (1.67). The phases $\phi_\rho = -141.8°$ and $\phi_\omega = -129.6°$ come from the PPR for each term. Over the range 20–160 GeV/c, the full expression follows a single power to within 1%; the phase varies from –123.7° to –120.3°. A single-power fit gives a phase of –122.3°.

The single-power fit yields the average phase to better that 1° even though it differs from that of either the ω or ρ alone, by more than 7°. This surprising behavior

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formally arises (for sufficiently well-behaved functions) from a so-called derivative analyticity relation between the phase and the local power law [8,9] $\phi = -\pi - \tan(\frac{\pi}{2} d/d\ln p)\ln|(f - \bar{f})/k|$, with the first term holding asymptotically [10]. Threshold effects, which we limit below, can spoil this relationship [10].

Consider next $CH_{1.1}$ and assume for now regeneration off (isoscalar) C comes only from ω exchange. The deviation induced by the ρ from a pure ω will be an order of magnitude smaller than for hydrogen. Uncertainties in the hydrogen parametrization have negligible effect.

Nuclear screening in carbon modifies the above assumption. Studies of K_L scattering on C and Pb performed at both high and low momenta are useful [1,4,11–13]. Data with heavier nuclei (e.g., Pb), where screening effects are dominant, help validate our procedures. We treat the real part and momentum dependence of the physical Pomeron amplitude by doing a full Glauber-Franco modeling [14] of nuclear screening. With this consistent procedure we study the accuracy of the PPR. The dominant effect is multiple elastic interactions in the nucleus. Our treatment of elastic screening closely follows [15]; a full description, including a treatment of p_t^2 dependences consistent with dispersion relations, is given elsewhere [16]. The input consists of KN cross sections and nuclear densities for the target nuclei. The cross sections are fit to theoretically motivated forms; this smooths the data and uses analyticity for real parts which agree with measurements. The exact functional forms are *not* important.

The four KN amplitudes are written as a sum of Regge terms F_i ($i = P, f, \omega, \rho$, and A_2) with the relative signs determined by charge parity and isospin. The Pomeron is parametrized as $F_P = a/p + b\pi \log p + i(c + b\log^2 p)$. (The a/p term adjusts the low-energy real parts. Its magnitude is not important to our conclusions.) An equally good fit uses $F_p = a/p + b\pi/2 + i(c + b\log p)$. Our conclusions are not sensitive to the choice of F_P parametrizations. Such parametrizations provide excellent fits to both the magnitude and phase of pp and $p\bar{p}$ scattering up to much higher energies [17].

We now display the formula for elastic screening in a revealing form [18]. Writing $F = (4\pi/k)f$ and $F_{KN} = (F_{Kp} + F_{Kn})/2$, etc., the first few terms of the Glauber series for the forward carbon amplitudes are

$$F_{KC} \pm \bar{F}_{KC} = 12(F_{KN} \pm F_{\bar{K}N})[1 + (i/24)I_2(F_{KN}^2 \pm F_{\bar{K}N}^2)/(F_{KN} \pm F_{\bar{K}N})\ldots]. \quad (1)$$

The coefficients I_n are proportional to the probability the kaon scatters n times within the nucleus; they depend on the nuclear density for which we use the harmonic-oscillator form [19].

The first term is a sum over nucleons. For the total cross section, the second term is dominated by an additional Pomeron exchange: $(F_{KC} + F_{\bar{K}C})/2 \simeq 12F_P[1 + i(I_2/24)F_P]$. For regeneration, $F_{KC} - F_{\bar{K}C} \simeq 24F_\omega[1 + i(I_2/12)F_P]$; the screening effect is twice as large. A purely imaginary, momentum-independent Pomeron would rescale the *magnitudes* of $F \pm \bar{F}$, leaving the PPR exact. A *physical* Pomeron has more complicated effects. In Table I we compare the data for carbon and lead to our full calculations, with and without elastic screening. The bulk of the observed screening is due to this easily calculated elastic effect; effects are also much larger in lead than in carbon.

To fully treat nuclear effects, we must include inelastic screening (IS) [20,21]: The incoming K is scattered into inelastic intermediate states K', which reform into a K at a subsequent scatter. At high energies, the effect is significant, and total cross-section data [13,22] provide clear evidence. Thresholds to reach high-mass intermediate states make IS unimportant at low energies. We use the treatment of Ref. [15] where C-even contributions are extracted by fitting total cross-section data [4]; these also impact regeneration, whereas the C-odd terms essentially affect only regeneration. The latter we vary, using the new measurement of carbon regeneration [1] as a guide, and the lead data [13] rescaled to the modern value of η_{+-}. The approximations used are more accurate for carbon where the multiple scattering series converges faster.

TABLE I. Predictions and data for total cross sections and regeneration. The data are interpolations from Refs. [1,4,11–13]. Our models are based on Ref. [15] (see text) and do not make use of any regeneration data. $\Delta\sigma_{\rm tot}$ and $\Delta\phi$ are the changes in the total cross-section and regeneration phase for the noted energy ranges. The inelastic calculations give the range as the C-odd inelastic term is varied from the maximum considered (from factorization) to zero; only regeneration is sensitive to these terms.

Element	Model	$\phi_{\rm tot}$ (mb) 70 GeV/c	$\Delta\phi_{\rm tot}$ (mb) 30–150 GeV/c	$(f - \bar{f})/k$ (mb) 70 GeV/c	$\Delta\phi$ (deg) 5–70 GeV/c
C	No screening	232	12	1.70	−0.3
C	Elastic screening	194	9	1.17	2.5
C	Inelastic screening	182	3	1.08 → 1.21	5.7 → 1.4
C	Data	190(2)	3(10)	1.21(1)	1.5(0.8)
Pb	No screening	4024	203	32.3	−0.3
Pb	Elastic screening	2249	64	9.3	8.3
Pb	Inelastic screening	2042	−45	9.3 → 11.2	10.8 → 2.6
Pb	Data	2047(8)	−106(44)	9.5(1)	8.5(3.8)

Any momentum dependence of the measured difference $\phi - \phi_{+-}$ is due to ϕ. Our calculations are compared to the data in Table I. It is seen that the C-even inelastic screening term alone reproduces well the carbon phase change from 5 to 70 GeV/c and that inclusion of the C-odd term (using factorization—see Ref. [23]) leads to several σ disagreements. Examining the magnitude of carbon regeneration leads to the same conclusion: Only a small amount of the C-odd term is required. However, the lead data are better described with more C-odd contributions.

Using elastic screening, and adding on C-even and C-odd inelastic screening, in turn, gives three functional forms for the full amplitude. With these terms, there is a 1° to 3° change in phase across the high-energy range while $|(f - \bar{f})/k$ deviates by $\pm 0.3\%$ from a single power. We do three fits to the high-energy carbon data [1] using these functional forms, with the amplitude and power of the ω varying. (The hydrogen is easily corrected for.) To gauge the systematic error, we examine how each fit extrapolates through the low-energy data and compare the amplitude and power to extractions independent of nuclear screening.

Figure 1 shows the results of the fits; for our nominal fit we use only elastic screening, which adequately reflects the data over the full energy range. The ω intercept is 0.437(7), in excellent agreement with a determination from scattering [0.43(1) [24]] and with a linear Chew-Frautschi plot 0.436. The amplitude from this fit agrees well with data. The fit with the C-odd term clearly disagrees with the low-energy data; in addition, the ω

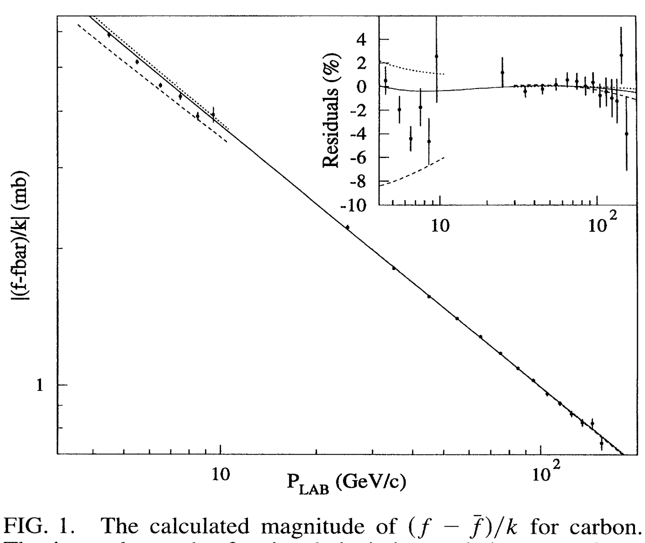

FIG. 1. The calculated magnitude of $(f - \bar{f})/k$ for carbon. The inset shows the fractional deviations relative to a single power $p^{-0.572}$. The solid curves are elastic screening only; the dotted curves include the C-even inelastics, fixed by the total cross-section data. The dashed curves show the effect of the maximum value of the C-odd inelastics considered. The data are from Ref. [15] and from a fit to the data of Ref. [5] (the errors are correlated due to common fit parameters).

intercept of 0.468(7) contradicts the other determinations. As the phase of the screening correction varies, so does its energy dependence; on average, these cancel via the PPR. The residual movement of ϕ with respect to our nominal fit is 0.17° (−0.24°) for the C-even (C-odd) fit. We take $\pm 0.25°$ as the systematic from nuclear screening.

In the Regge picture, subleading (daughter) trajectories may occur. We consider a sub-leading trajectory with $\alpha = \alpha_\omega - 2$ as well as $\alpha = \alpha_\omega - 1$; in the latter case, the terms differ in phase by 90°. We fit the data of Refs. [1] and [12] simultaneously. The residual shifts in the predicted phase are $<0.1°$ with the second trajectory's amplitude statistically insignificant. Other fits give no significant evidence for a second trajectory of any α value.

We estimate the effects due to the detailed structure of the low-energy amplitude on the phase at our energies. The low-energy phase data [12] and the structure of the dispersion relation easily limit this to 0.2°.

For completeness, our calculations include electromagnetic regeneration which adds a constant real term (which will actually dominate regeneration, at sufficiently high energies). This gives a $(-0.10 \pm 0.05)°$ phase change; the error comes from uncertainty in the kaon mean-square charge radius [25].

The total error on the regeneration phase is then a combination in quadrature of effects summarized in the last four paragraphs: $\pm 0.35°$. The total shift in ϕ between a naive single-power fit and our nominal fit with the above effects was −0.04°.

We finally consider an Odderon contribution [26]. This amplitude $F_O = d + e(\ln p - i\pi/2) + f(\ln p - i\pi/2)^2$ is C-odd and contributes directly to $F - \bar{F}$. It can be limited by existing pp and $p\bar{p}$ data. Such analyses find *no evidence* for the Odderon, and typically limit $|F_O/F_P| \sim 1 \times 10^{-3}$ in the 100 GeV/c range [17]. Taking the Odderon parameters from the best fit of these authors, we find a shift of $-(0.2 \pm 0.6)°$ in the measured phase. Future regeneration data (assuming CPT symmetry) can better limit the Odderon contribution.

While this analysis was underway, we learned of a claim [27] now published [28] that the techniques for extracting the regeneration phase at high energy in [6] have large uncertainties. The authors utilize a phase-magnitude dispersion relation; the input parametrized by $p^{\alpha-1}$ behavior for $|(f - \bar{f})/k|$ in three momentum ranges. At high energy, and low energy when available, α is taken from published data. Above 150 GeV, the authors introduce a large, unphysical change (kink) in the power law, the dominant source of the claimed uncertainty. The notion appears to be that, since changes in the power occurred from low to high energy, the regeneration process must not be simple and such changes may occur again. The kinks are placed just above the momentum range of the experiment, having the greatest impact on the local phase. In addition, the errors for the three regions are set fully correlated, producing the largest discrepancy. Furthermore, the error on α for the energy range covered by the experiment is taken as *systematic* when it is already counted in the *statistical* error. In response, we point out the following. Regeneration was extensively studied in the low 1–10 GeV range [11,12,29,30] with a variety of nuclei. Regeneration for copper and lead (*not* carbon) was found to be steeper when the Fermilab data were reported 15 years ago [3,4]. But this effect and its A dependence were well understood, and theoretical work further clarified the issue [15,18,23]: The power law at *low* energies for heavy nuclei is, in fact, distorted. The distortion comes from changes in elastic screening from the very rapid drop in the $\bar{K}N$ cross sections over the 1–10 GeV range. For $\bar{p}p$, we now know that this decade shows *an order of magnitude* greater change than that for any of the next *five* higher decades. Even without comparable measurements with kaons, this gives high confidence that screening leads to no further breaks in the power—even with heavy nuclei—at any energy below $\sim 10^6$ GeV, let alone at 150 GeV as hypothesized in [28].

To summarize, we have considered the accuracy of extracting ϕ from the momentum dependence of the magnitude of regeneration using dispersion relations. We find that the deviations from the phase-power relation in the 20–160 GeV energy region are small and quantifiable, especially for low-Z nuclei. From this study of regeneration, a systematic error of 0.35° can be safely assigned. The major application of this technique relates to the determination of ϕ_{+-} as reported in a previous Letter. However, the technique is more general in that there is now the possibility that a departure from the phase-power relation of even less than a degree can point to new phenomena at much higher-energy scales, e.g., the Odderon.

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