Nonspreading Electronic Wave Packets and Conductance Fluctuations

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We find strongly localized wave packets tracing *without dispersion* the classical dynamics of a Rydberg electron in a circularly polarized microwave field using a full 3D quantum calculation. These wave packets are single eigenstates of the atom dressed by the microwave field and are very stable against ionization. Their widths fluctuate with microwave amplitude and/or frequency realizing an optical analog of the conductance fluctuations in mesoscopic systems. Similar behavior is observed for linear microwave polarization. We show that such behavior is expected when classical nonlinear resonances are important in an atomic system.

The construction of long-living strongly localized wave packets in atoms or molecules is very attractive for both experimental and theoretical physicists [1]. Such a wave packet, closely following a classical trajectory, is an interesting object in studies of classical-quantum correspondence. With the notable exception of the harmonic oscillator, wave packets, as nonstationary solutions of a time-dependent Schrödinger equation, i.e., coherent superpositions of stationary eigenstates, typically disperse in time.

The recent surprising discovery [2–4] of nonspreading wave packets in atoms dressed by a monochromatic field of either linear (LP) [2] or circular (CP) [3,4] polarization is of a potentially big importance for future experimental and theoretical studies in this domain. The nondispersive character of the motion is not due to a harmoniclike structure of the Hamiltonian, but rather due to a nonlinear coupling between the atom and the driving field that locks the electronic motion to the driving frequency. In the dressed atom picture [5], a single eigenstate of the full "dressed" Hamiltonian represents the wave packet [2,4] explaining immediately why no spreading of the motion is possible. The wave packet is localized in a stable 1:1 classical resonance island.

This behavior immediately brings into mind the solitons—and indeed such a wave packet may be viewed as a solitonic solution of the nonlinear but integrable set of equations describing the motion of eigenvalues as a function of driving field parameters (the frequency ω or amplitude F) [6].

The stationary character of the wave packet is, strictly speaking, approximate only. Firstly, the states in the presence of the driving field may slowly ionize, although keeping the same shape (see below). Secondly, other effects, such as, e.g., spontaneous emission, may lead to their eventual spreading and disappearance. It seems at first glance that ionization should be a primary mechanism limiting the lifetime of the wave packet, since the field

amplitude required for sufficient coupling between the atom and the driving field is comparable with that needed to ionize the atom from Rydberg states. Already first studies of the lifetime [2,4] revealed that it can be exceptionally long, reaching 10^6 Kepler periods for wave packets built from states of principal quantum number (around $n_0 = 60$). Since the atomic states building the CP wave packets are of large angular momentum (these are mainly the circular atomic states), their spontaneous emission lifetime is also extremely large.

The CP wave packet has been studied, up till now, in the simplified two-dimensional hydrogen atom model, where the electronic motion has been restricted to the polarization plane [3,4]. Its existence in a realistic 3D model has been conjectured only. We report below a full three-dimensional quantum calculation of the wave packet for realistic, experimentally accessible values of parameters. In fact, a whole family of wave packets, with quite unusual shapes, has been found.

Our second aim has been an accurate determination of the wave packet lifetime (or rather its inverse—the width) for both CP and LP cases as a function of the parameters of the problem. We explicitly show that the wave packets exist in quite a broad range of parameter values; on the other hand, their width is a very sensitive function of F, ω, or n_0. We explain the origin of the width's fluctuations and point out their similarity to conductance fluctuations in mesoscopic devices [7].

Our quantum mechanical calculation is based on a diagonalization of a large matrix obtained by expressing the Floquet Hamiltonian of the problem (for both LP and CP cases) in the Sturmian basis [2,8] and using the complex rotation technique. This yields (quasi)energies, widths (ionization rate induced by the microwave field), and eigenfunctions of the Floquet states [9]. The technique for the LP case has been described before [9]; the details of the calculations for the CP 3D calculations will be presented elsewhere [8] and are quite analogous to the
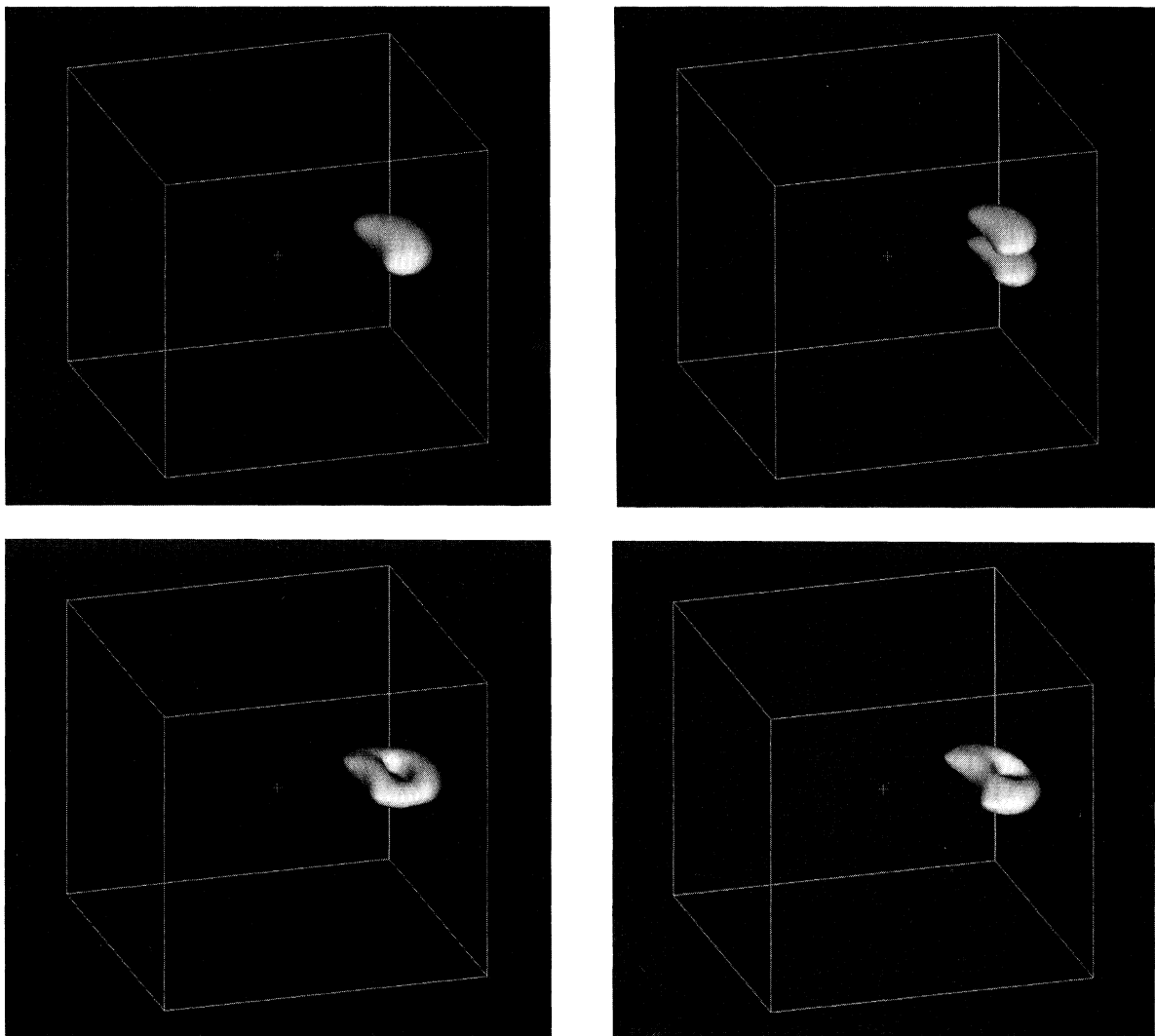
FIG. 1.   Visualization of 3D wave-packet states represented as isovalue plots at 30% maximum value in the frame rotating with the microwave field for $n_0 = 60$ ($\omega = n_0^{-3}$, $F_0 = 0.044\,42$ in atomic units). The cross indicates the position of the nucleus. The cube edge is 10 000 Bohr radii. In the laboratory frame the wave packets rotate around the nucleus without deformation or dispersion. Top-left—ground state wave packet $(0,0,0)$; top-right, bottom-left, and bottom-right plots represent $(0,0,1)$, $(1,0,0)$, and $(0,1,0)$ wave packets, respectively.

2D case discussed already [4]. In particular, a slight generalization to the full 3D Hamiltonian allows us to predict semiclassically the energy of the wave-packet dressed eigenstate (in the frame rotating with the field) based on the harmonic expansion around a stable fixed point

$$H_{\text{osc}} = E_{\text{eq.p.}} + \omega_+(a_+^\dagger a_+ + \tfrac{1}{2})$$
$$- \omega_-(a_-^\dagger a_- + \tfrac{1}{2}) + \omega_z(a_z^\dagger a_z + \tfrac{1}{2}), \quad (1)$$

where [4]

$$E_{\text{eq.p.}} = \frac{1 - 4q}{2q^{2/3}}\,\omega^{2/3} \quad (2)$$

and q is a dimensionless parameter describing the fixed

point position and its stability [3]. The $a_\pm^\dagger$ ($a_\pm$) are creation (annihilation) operators of the modes that entangle position and momentum variables in the polarization plane (assumed to be the x-y plane), while the harmonic binding in z direction is described by $a_z, a_z^\dagger$ operators. In 3D, the frequencies $\omega_\pm$ are the same as in the simplified 2D case [4], while $\omega_z = \omega\sqrt{q}$. Note the untypical minus sign for the ω_- term in Eq. (1) resulting from the L_z term in the original Hamiltonian. This does not affect the stability of the fixed point; the Hamiltonian is a binding one. Let us denote by (n_+, n_-, n_z) the wave packet corresponding to n_i excitation of the ith mode. The "ground state" wave packet will be denoted then by $(0,0,0)$, excited "2D" wave packets will correspond to $(n_+, n_-, 0)$,

while $n_z > 0$ will correspond to excitation in the direction perpendicular to the polarization plane.

The semiclassical prediction, Eq. (2), is essential for our diagonalizations since we can determine only a small number of eigenvalues around the predicted energy from matrices reaching a rank of 5×10^5 using the Lanczos algorithm. The method yields the corresponding eigenvectors (and wave functions [9]) at no additional cost. The exemplary wave packets are presented in Fig. 1 as 3D isovalue plots for squared modulus of the wave function. The top-left picture represents the $(0,0,0)$ wave packet and is an approximate Gaussian in radial, azimutal (in x-y plane), and z direction. It is a superposition of mainly circular states $|n, l = n - 1, m = l\rangle$ with n centered around $n_0 = \omega^{-1/3}$ (for $\omega n_0^3 = 1$ the Kepler frequency matches the driving microwave frequency leading to a 1:1 resonance). The frequency chosen for calculation yields $n_0 = 60$. The top-right picture represents the wave packet $(0,0,1)$ with a single excitation along the z axis—the genuine 3D wave packet. The electron becomes "split in half" since the wave function vanishes at $z = 0$. Bottom left [bottom right] plots represent the radially, $(1,0,0)$ [azimutally, $(0,1,0)$] excited wave packets taking the shape of deformed tori. They correspond to similar wave packets in 2D calculations presented by us before [4].

All the wave packets shown in Fig. 1 have long lifetimes exceeding 10^3 Kepler periods. Experimentally, they may be prepared by a fast microwave switching technique starting from atomic circular states as described in detail in [4] for the ground state wave packet, possibly combined with resonant transitions induced by an additional field to transfer the population among different (n_+, n_-, n_z) dressed states. We checked that the estimations for the switching time given in [4] for the 2D model are still valid for the real 3D atoms. Thus, the way is open for an experimental production of these wave packets. The interesting question, which remains to be answered, is how many different "wave packet" (i.e., strongly localized in a dynamical potential surrounding the 1:1 fixed point) states can be formed. We leave this problem for future consideration concentrating below on the width properties.

The range of microwave parameters allowing for formation of the wave packet in CP microwaves is related to the stability of the relevant fixed point (see [3] for details). From an experimental point of view it is of importance to know the dependence of the wave-packet energy and its width on the tunable parameters such as ω and F. The former is provided quite accurately by Eq. (1), the behavior of the width Γ is represented in Fig. 2 as a function of the microwave frequency ω or rather its "scaled" representation $\omega_0 = \omega n_0^3$. Note the strong fluctuations, the widths vary by 2 orders of magnitude under subtle changes of ω_0 in a quite erratic manner. As shown in the same figure, the behavior is qualitatively similar for both 2D and full 3D hydrogen atom model wave packets as
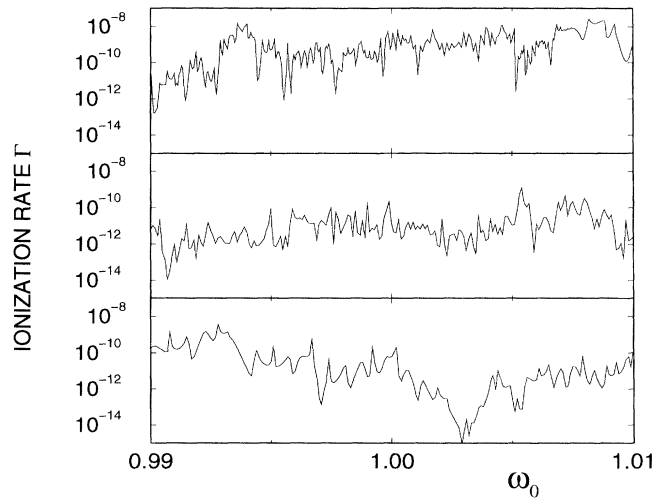
FIG. 2. Width of the "ground state" wave packet as a function of scaled frequency ω_0 for scaled microwave amplitude $F_0 = 0.044\,42$. Scaling performed with respect to n_0 corresponding to the center of the wave packet. Top—three-dimensional hydrogen atom in a circularly polarized microwave field ($n_0 = 40$); middle—same for a two-dimensional model ($n_0 = 70$); bottom—one-dimensional hydrogen atom in a linearly polarized microwave field ($n_0 = 30$). The existence of similar strong fluctuations shows the general character of these fluctuations induced by "chaos assisted tunneling" from nonlinear classical resonance.

well as for the corresponding wave packet obtained for a one-dimensional hydrogen atom illuminated by a linearly polarized microwave field—being thus a general phenomenon. The presence of strong fluctuations is quite unexpected, because the wave packets are strongly localized in a stable classical resonance island, thus their widths may be thought of as dominated by tunneling to the surrounding chaotic sea, a phenomenon insensitive to small changes of ω_0.

Similar fluctuations may be observed by changing the microwave amplitude F for a fixed frequency and n_0 (not shown) as well as by fixing both "classical parameters" ω_0 and the scaled amplitude $F_0 = Fn^4$ and varying the principal quantum number (i.e., the effective size of $\hbar$) as shown in Fig. 3. The presence of the latter fluctuations proves the quantum origin of the fluctuations. An increase of n_0 (for n_0 sufficiently large) leads to a decrease of the *average* width in agreement with the tunneling picture. To understand the fluctuations, one has to consider the details of tunneling out of the stable island. The electron tunnels not directly to the continuum but rather via other higher excited states (or classically to the chaotic sea surrounding the island). Many quantum paths leak to the continuum, and the interference of them leads to a final value of the widths. The paths themselves are sensitive only to classical parameters ω_0, F, but the phases of their contributions depend strongly on the effective value
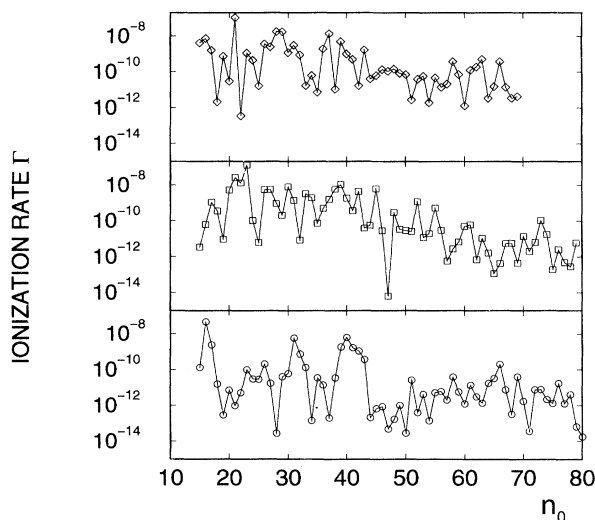
FIG. 3. Same as Fig. 2, but for fixed both $\omega_0 = 1$ and $F_0 = 0.044\,42$ and varying the principal quantum number n_0; i.e., the effective size of $\hbar$. The strong fluctuations in strictly similar classical situations prove the quantum origin of the fluctuations.

of $\hbar$, i.e., on n_0. In the chaotic sea, there are plenty of different uncorrelated paths with complicated shapes. The net effect of their interferencing contributions will be "quasirandom" fluctuations of the total ionization rate. In fact the tunneling process is "chaos assisted" as analyzed in detail by Tomsovic and Ullmo [10]. They discussed the tunneling between stable islands; here we observe such a chaos assisted tunneling from stable island to the scattering regime.

The fluctuations due to chaos assisted tunneling resemble the so-called conductance fluctuations observed in mesoscopic systems [7] resulting in a similar manner from quantum interference of different paths of the electron through the finite size sample. Thus, the fluctuations observed can be considered as an atomic analog of the conductance fluctuations (for other manifestation of such fluctuations in the ionization process, see [2,11,12]). However, the situation is here even simpler: The fluctuations are due to details of the transport from a single localized eigenstate of the system to the atomic continuum, not to the localization properties of the initial state.

The transport through the chaotic sea is clearly visible in the spectral picture. By a detailed analysis of the motion of resonances as a function of the parameters (ω, F), we have confirmed that the microscopic origin of rapid changes of widths may be traced back to avoided crossings (in the complex-energy–width plane) of the wave packets states with other Floquet states [2,12]. These other states (corresponding to very highly excited states of $n \approx 200$) may become quasidegenerate with the wave packet states due to the dressing by the microwave field. They have of course completely different localiza-

tion properties and, therefore, their quasienergies change with parameters, e.g., ω, differently than the energy of the wave packet states—that explains the multitude of avoided crossings observed on level motion plots. A more detailed study of this phenomenon will be presented elsewhere [13].

To summarize, we have shown that recently discovered nondispersing wave packets formed by placing H atom in a microwave field of either linear or circular polarization are extremely stable against the ionization over a broad range of parameters. Their width (lifetime) exhibits fine scale fluctuations—an atomic analog of conductance fluctuations. The physical mechanism under these fluctuations is the chaos assisted tunneling. We presented first a calculation of CP wave packets for a realistic model—a fully 3D hydrogen atom finding also the "excited state" wave packets exemplified in Fig. 1.

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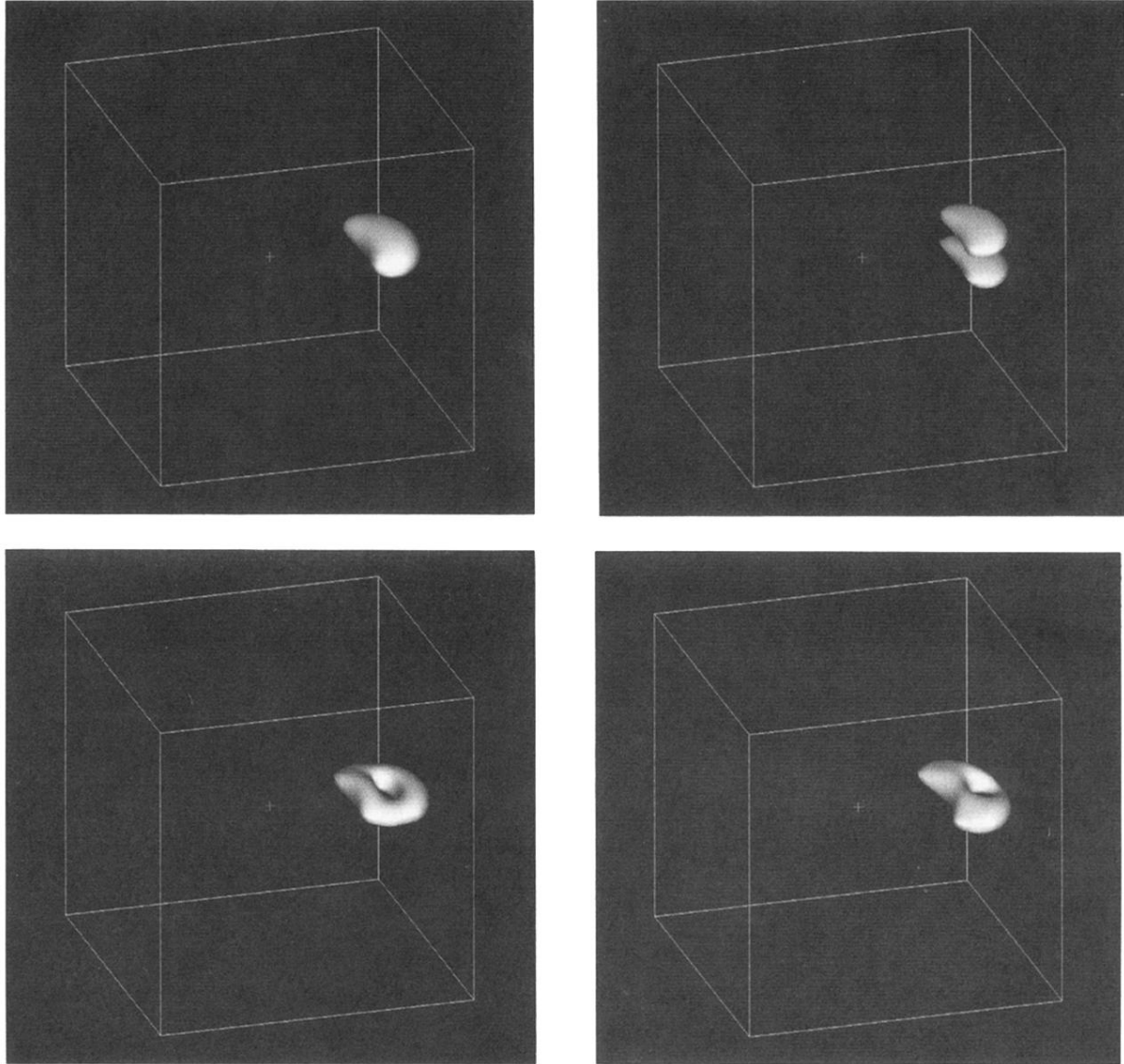
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