Assessing Big-Bang Nucleosynthesis

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Systematic uncertainties in the light-element abundances and their evolution complicate a rigorous statistical assessment. However, using Bayesian methods we show that the following statement is robust: The predicted and measured abundances are consistent with 95% credibility only if the baryon-to-photon ratio is between 2×10^{-10} and 6.5×10^{-10} and the number of light neutrino species is less than 3.9. Our analysis suggests that the ^{4}He abundance may have been systematically underestimated.

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Big-bang nucleosynthesis occurred seconds after the bang and offers the earliest test of the standard cosmology. Comparison of the predicted and measured light-element abundances has progressed dramatically beginning 30 years ago with the evidence for a significant primeval abundance of ^{4}He that could be explained by the big bang [1] to the present where the abundances of D, ^{3}He, ^{4}He, and ^{7}Li are all used to test the big bang.

The predictions of big-bang nucleosynthesis depend upon the baryon-to-photon ratio ($\equiv \eta$) and the number of light ($\lesssim 1$ MeV) particle species, often quantified as the equivalent number of massless neutrino species ($\equiv N_\nu$). For a decade it has been argued that the abundances of all four light elements can be accounted for provided η is between 2.5×10^{-10} and 6×10^{-10} and $N_\nu < 3.1$–4 [2–4]. The "consistency interval" provides the best determination of the baryon density and is key to the case for nonbaryonic dark matter. The limit to N_ν provides a crucial hurdle for theories that aspire to unify the fundamental forces.

These conclusions are not based upon a rigorous statistical analysis. Because the dominant uncertainties in the light-element abundances are systematic previous work focused on "concordance intervals." Here we apply two standard techniques, goodness of fit and Bayesian likelihood, and identify the conclusions that are insensitive to systematic error.

We begin with a brief overview. The predictions of standard big-bang nucleosynthesis are shown in Fig. 1. The theoretical uncertainties are statistical, arising from imprecise knowledge of the neutron lifetime and certain nuclear cross sections. Because of 10 Gyr or so of "chemical evolution" since the big bang (nuclear reactions in stars and elsewhere that modify the light-element abundances), determining primeval abundances is not simple and the dominant uncertainties are systematic.

Stars make additional ^{4}He; stars also make metals (elements heavier than ^{4}He). The primeval ^{4}He abundance has been inferred by correlating the ^{4}He abundance in metal-poor, extragalactic H II (ionized hydrogen) clouds with a metal indicator (C, N, or O) and extrapolating to zero metallicity: $Y_P = 0.232 \pm 0.003(\text{stat}) \pm 0.005(\text{syst})$ [5]. The range $Y_P = 0.221$–0.243 allows for 2σ statistical + 1σ systematic uncertainty and is consistent with the big-bang prediction provided $\eta \simeq (0.8$–$4) \times 10^{-10}$ [2]. Others have argued that the systematic uncertainty is a factor of 2 or even 3 larger [6]; taking $Y_P \simeq 0.21$–0.25 increases the concordance range significantly, $\eta \simeq (0.6$–$10) \times 10^{-10}$, reflecting the logarithmic dependence of big-bang ^{4}He production upon η [2].

There is a strong case that the ^{7}Li abundance measured in metal-poor, old pop II halo stars, $^7\text{Li/H} = (1.5 \pm 0.3) \times 10^{-10}$, reflects the big-bang abundance [7]. However, it is possible that even in these stars the ^{7}Li

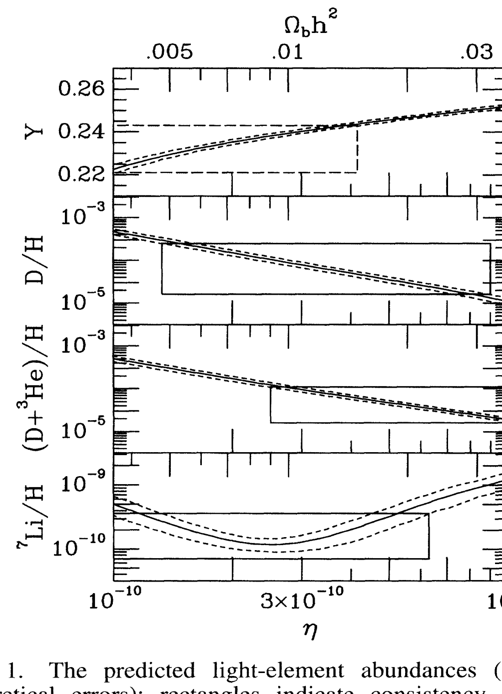

FIG. 1. The predicted light-element abundances (with 2σ theoretical errors); rectangles indicate consistency intervals, which all overlap for $\eta = (2.5 - 6) \times 10^{-10}$ (from [2]).

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abundance has been depleted by nuclear burning, perhaps by a factor of two (the presence of ^{6}Li in some of these stars, which is more fragile, provides an upper limit to depletion). Allowing for 2σ statistical uncertainty and for up to a factor of two depletion leads to the consistency interval $\eta = (1-6) \times 10^{-10}$ [2]. (Additional smaller, but important, systematic uncertainties arise from the modeling of stellar atmospheres and the possible enhancement of ^{7}Li by cosmic-ray production [2]. For simplicity we use depletion to illustrate the effects of all types of systematic uncertainty.)

The interpretation of D is particularly challenging because it is burned in virtually all astrophysical situations and its abundance has been accurately measured only in the solar vicinity. Because D is destroyed and not produced [8] and because its abundance is so sensitive to η (D/H $\propto \eta^{-1.7}$), a firm upper limit to η can be obtained by insisting that big-bang production account for the D observed locally, D/H $\gtrsim (1.6 \pm 0.1) \times 10^{-5}$ [9]. This leads to the two-decade-old bound $\eta \lesssim 9 \times 10^{-10}$, which is the linchpin in the argument that baryons cannot provide closure density [10]. Because D is readily destroyed, it is not possible to use D to obtain a lower bound to η. The sum of D and ^{3}He is more promising: D is first burned to ^{3}He, and ^{3}He is much more difficult to burn. On the assumption that the mean ^{3}He survival fraction is greater than 25%, the lower limit $\eta \gtrsim 2.5 \times 10^{-10}$ has been derived [4]. The D, ^{3}He concordance interval, $\eta \simeq (2.5 - 9) \times 10^{-10}$.

The overlap of the concordance intervals (see Fig. 1) for $\eta = (2.5 - 6) \times 10^{-10}$ is the basis for concluding that the light-element abundances are consistent with their big-bang predictions [2].

Systematic uncertainties dominate the light-element abundances—the primeval abundance of ^{4}He, and the chemical evolution of D and ^{3}He and of ^{7}Li. Systematic error is difficult to treat as it is usually poorly quantified. (If it were well quantified it would not be systematic error.) This is especially true for astronomical *observations*, where the observer has no control over the object being observed. There are at least three kinds of systematic error: (1) a definitive, but unknown, offset between what is measured and what is of interest; (2) a random source of error whose distribution is poorly known; and (3) an important unknown source of error. The first kind is best treated as an additional parameter in the likelihood function. The second kind is best treated by use of a distribution, or by several candidate distributions. The third kind is a nightmare.

The data themselves can clarify matters. Consider ^{7}Li; its measured abundance in old pop II stars is equal to the primeval abundance with a small statistical error and a larger systematic uncertainty due to depletion. This could be a systematic error of the first kind—if all stars reduce their ^{7}Li abundance by the same factor—or of the second kind—if the ^{7}Li abundance in different stars were reduced by different amounts. In the latter case, the measured ^{7}Li abundance should show a large dispersion—which it does not [7]. Thus, we treat depletion by considering two limiting possibilities: ^{7}Li/H $= (1.5 \pm 0.3) \times 10^{-10}$ (no depletion); and second, ^{7}Li/H $= (3.0 \pm 0.6) \times 10^{-10}$ (depletion by a factor of 2).

Several sources of systematic error for ^{4}He have been identified, and they can either reduce or increase the measured abundance [6]. If the same effect dominates in each measurement, use of an offset parameter in the ^{4}He abundance would be appropriate. On the other hand, if different effects dominate different measurements, enlarging the statistical error would be appropriate. We allow for both: The statistical error σ_Y is permitted to be larger than 0.003, and a possible offset in the ^{4}He abundance is considered, $Y_P = 0.232 + \Delta Y$.

Finally, there is the systematic uncertainty associated with the chemical evolution of D and ^{3}He. Based upon a recent study of the chemical evolution of D and ^{3}He [11] we consider three models that encompass the broadest range of possibilities: Model 0 is the plain, vanilla model; model 1 is characterized by extreme ^{3}He destruction (average ^{3}He survival factor of about 15%); and model 2 is characterized by minimal ^{3}He destruction. (Because the stars that destroy ^{3}He also make metals, it is not possible to destroy ^{3}He to an arbitrary degree without overproducing metals [11].) The likelihood functions for the three models are shown in Fig. 2.

To begin, consider the χ^2 test for goodness of fit. This technique is best suited when the errors are Gaussian and well determined and there are many degrees of freedom; neither apply here. Nonetheless, in Fig. 3 we show $\chi^2(\eta)$ for eight different assumptions about the systematic uncertainties: (1,5) $\sigma_Y = 0.003$, $\Delta Y = 0$; (2,6) $\sigma_Y = 0.01$, $\Delta Y = 0$; (3,7) $\sigma_Y = 0.003$, $\Delta Y = 0.01$; (4,8) $\sigma_Y = 0.01$, $\Delta Y = 0.01$. In (1)–(4) ^{7}Li/H $= (3 \pm 0.6) \times 10^{-10}$; and in (5)–(8) ^{7}Li/H $= (1.5 \pm 0.3) \times 10^{-10}$. For clarity, only the results for model 0 are shown, the results for models 1 and 2 are similar.

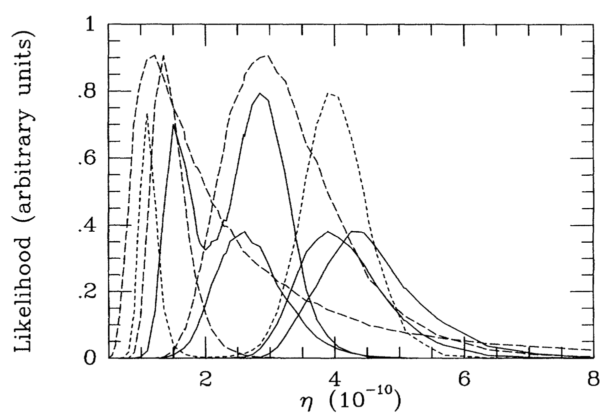

FIG. 2. Likelihood functions for D and ^{3}He (lower solid curves, from left to right: models 1, 0, and 2), ^{4}He (dotted curves, from left to right: $\sigma_Y = 0.01$, $\sigma_Y = 0.003$, and $\Delta Y = 0.01$), and ^{7}Li (broken = high ^{7}Li, solid = low ^{7}Li).

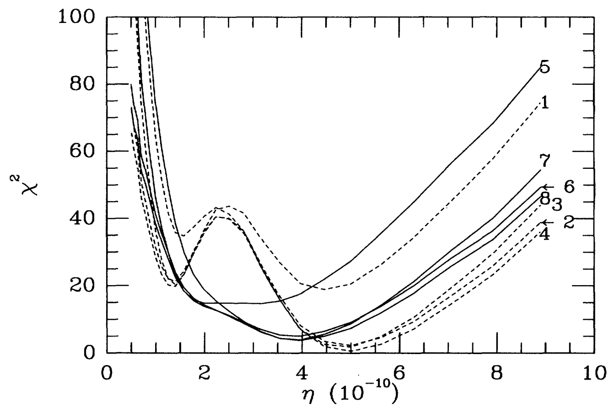

FIG. 3. $\chi^2(\eta)$ for eight different sets of assumptions about the systematic uncertainties.

Several conclusions can be drawn. First, goodness of fit depends sensitively upon assumptions made about the systematic errors, with the minimum χ^2 ranging from 18 to less than 1 (for $3 - 1 = 2$ degrees of freedom); it is smallest when a systematic shift in ^{4}He is allowed and/or σ_Y is increased. Second, in all cases the 99% confidence interval for η, defined by $\Delta\chi^2 = 6.6$, has a lower bound no lower than about 1.5×10^{-10} [in all but (5), no lower than 2.5×10^{-10}] and an upper bound no higher than about 6×10^{-10}.

Now we turn to the Bayesian approach to likelihood, where likelihood distributions are converted to probability distributions by multiplying by priors and integrating over some (or none) of the parameters [12]. We consider different chemical evolution possibilities for D and ^{3}He and for ^{7}Li and treat η, σ_Y, ΔY, or N_ν as parameters (sometimes with fixed values). Values of N_ν greater than 3 describe extensions of the standard model with additional light degrees of freedom. Likelihood is best suited to determining parameters of a theory or assessing the relative viability of two or more theories. While there are alternatives to the standard theory of nucleosynthesis (e.g., spatial variation in η or an unstable tau neutrino [13]), none is especially compelling or as successful, and we will focus on the determination of parameters of the standard theory.

The contours of the marginal distribution $\mathcal{L}(\sigma_Y = 0.003, \Delta Y, N_\nu)$ are diagonal lines in the ΔY-N_ν plane because ΔY and N_ν are not independent parameters: The primary effect of an increase in N_ν is an increase in the predicted ^{4}He abundance ($\Delta Y_P \sim 0.01 \Delta N_\nu$). A likelihood function that is not compact must be treated with care, because no information about the parameters (here, N_ν and ΔY) can be inferred independently of prior knowledge.

For example, to set a limit to N_ν we convert the joint likelihood $\mathcal{L}(\sigma_Y = 0.003, \Delta Y, N_\nu)$ to a probability distribution for N_ν by multiplying by a flat prior (=0 for $N_\nu < 3$ or $|\Delta Y| > \delta Y$) and integrating over ΔY. The 95% credible limit depends upon δY as illustrated in

TABLE I. Limits to N_ν for models 0, 1, 2 and priors which are zero for $N_\nu < 3$ (first number) and <2 (second number). These limits are based upon the lower ^{7}Li abundance; corresponding limits for the higher ^{7}Li abundance are more stringent by $\Delta N_\nu \sim 0.1$.

ΔY	Model 0	Model 1	Model 2
0	3.1/2.5	3.2/2.8	3.1/2.5
0.005	3.2/2.6	3.3/2.9	3.2/2.6
0.010	3.3/2.9	3.5/3.1	3.3/2.9
0.015	3.5/3.2	3.7/3.4	3.4/3.2
0.020	3.7/3.5	3.9/3.8	3.7/3.5

Table I. The limit also depends upon the prior for N_ν; the limits for the flat prior, which is zero for $N_\nu < 2$ (corresponding to a massive, short-lived τ neutrino), are also shown in Table I.

In a recent paper the likelihood function $\mathcal{L}(N_\nu)$ obtained by integrating from $\Delta Y = -0.005$ to 0.005 was used in an attempt to assess the viability of the standard theory [14]. This likelihood is peaked at $N_\nu = 2.1$ with Gaussian $\sigma_{N_\nu} = 0.3$. On this basis it was claimed that the standard theory of nucleosynthesis is ruled out with 98.6% confidence. Equal weight was implicitly given to all values of N_ν (flat priors). The prior for $N_\nu = 3$ (standard model of particle physics) is certainly greater than that for $N_\nu < 3$ (e.g., massive, short-lived τ neutrino), and this, together with the dependence of $\mathcal{L}(N_\nu)$ upon the prior for ΔY (here $|\Delta Y| < 0.005$), casts strong doubt on the above assessment of the standard theory.

In Figs. 4 and 5 we show that 95% credible regions for the probability distributions $\mathcal{L}(\eta, \sigma_Y, \Delta Y = 0, N_\nu = 3)$ and $\mathcal{L}(\eta, \sigma_Y = 0.003, \Delta Y, N_\nu = 3)$ obtained assuming flat priors. Both figures suggest the same thing: The uncertainty in the primordial ^{4}He abundance has been underestimated. In the σ_Y-η plane $\sigma_Y = 0.003$ does not intersect the 95% credible contour, and in the ΔY-η plane $\Delta Y = 0$ does not intersect the 95% credible region (except for model 1, where they barely do). The 95% credible

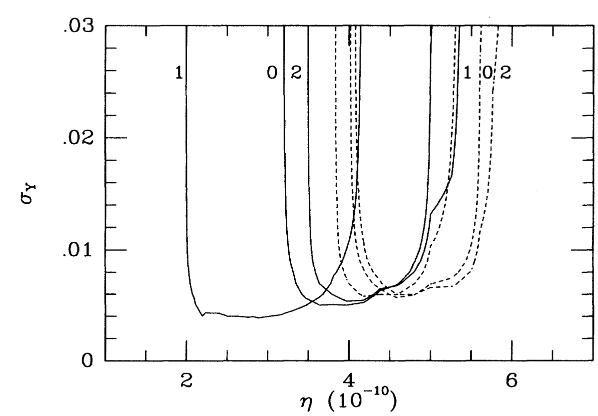

FIG. 4. The 95% credible contours of the probability distribution $\mathcal{L}(\eta, \sigma_Y, \Delta Y = 0, N_\nu = 3)$ (solid curves = low ^{7}Li, broken curves = high ^{7}Li).

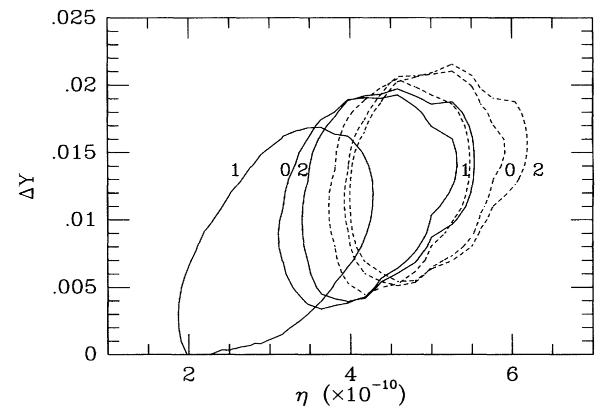

FIG. 5. Same as Fig. 4 for $\mathcal{L}(\eta, \sigma_Y = 0.003, \Delta Y, N_\nu = 3)$.

contour in the σ_Y-η plane becomes independent of σ_Y for $\sigma_Y \gtrsim 0.008$, with 95% credible interval $\eta \simeq (3 - 6.5) \times 10^{-10}$ (allowing both for the uncertainty in the depletion of ^{7}Li and in the chemical evolution of D and ^{3}He).

The 95% credible contours in the ΔY-η plane suggest that the primeval ^{4}He abundance has been systematically underestimated, by an amount $\Delta Y \approx +0.01$. (Though it should be noted that model 1 and the lower ^{7}Li abundance are just consistent with $\Delta Y = 0$ at 95% credibility.) Put another way, D, ^{3}He, and ^{7}Li are concordant and ^{4}He is the outlayer. (This can also be seen in Fig. 2.) When the likelihood function is marginalized with respect to ΔY, the 95% credible interval is $\eta \simeq (2 - 6.5) \times 10^{-10}$ (again, allowing for the uncertainty both in depletion of ^{7}Li and in the chemical evolution of D and ^{3}He).

While the sizes of the uncertainties are not sufficiently well determined to reliably assess goodness of fit, and the absence of a compelling alternative to the standard theory makes relative likelihood of limited use, our analysis does point to several important conclusions: (i) The predictions of the standard theory of primordial nucleosynthesis are only consistent with the extant observations with 95% credibility provided $\eta \simeq (2 - 6.5) \times 10^{-10}$; (ii) while one cannot exclude a problem with the standard theory (e.g., the τ neutrino could be massive and short lived), there is no credible evidence to support such; (iii) there is evidence that the primordial ^{4}He abundance has been systematically underestimated ($\Delta Y \approx +0.01$) or that the random errors have been underestimated ($\sigma_Y \approx 0.01$). Only for model 1 (extreme destruction of ^{3}He) are $\Delta Y = 0$ and $\sigma_Y = 0.003$ in the 95% credible region (cf. Figs. 4 and 5); and (iv) the limit to N_ν depends upon the uncertainty in the ^{4}He abundance (cf. Table I; taking $|\Delta Y| \leq 0.02$, which is four times the estimated systematic error, leads to the 95% credible limit $N_\nu < 3.9$.

This more rigorous analysis provides additional support for the conclusions reached previously about the concordance interval for η [2]. The limit to the number of neutrino species is less stringent than previously quoted bounds [2,3] because we considered a chemical-evolution model with extreme destruction of ^{3}He (which permits low values of η where ^{4}He production is lower) as well as a large systematic offset in the ^{4}He abundance.

There are two measurements that should clarify the chemical evolution of D, ^{3}He, and ^{7}Li and permit an even sharper test of big-bang nucleosynthesis. The first is a determination of the primeval D abundance by measuring D Ly-α absorption due to high-redshift hydrogen clouds. The second is a determination of the primeval ^{7}Li abundance by studying short period, tidally locked pop II halo binaries; depletion is believed to involve rotation-driven mixing and is minimized in these stars because they rotate slowly [15]. At the moment, there are conflicting measurements and upper limits for the primeval D abundance seen in high-redshift hydrogen clouds [16], and there is one study that indicates that the ^{7}Li abundance in short-period binaries is no higher (evidence against significant depletion) and another that finds weak evidence that the ^{7}Li abundance is higher [17].

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