ac-Field-Controlled Anderson Localization in Disordered Semiconductor Superlattices

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An ac field, tuned exactly to resonance with the Stark ladder in an ideal tight binding lattice under strong dc bias, counteracts Wannier-Stark localization and leads to the emergence of extended Floquet states. If there is random disorder, these states localize. The localization lengths depend nonmonotonically on the ac field amplitude and become essentially zero at certain parameters. This effect is of possible relevance for characterizing the quality of superlattice samples, and for performing experiments on Anderson localization in systems with well-defined disorder.

A major motivation for the introduction of semiconductor superlattices by Esaki and Tsu [1] was the possibility of observing Bloch oscillations in such effectively one-dimensional periodic structures. The time of one Bloch oscillation is proportional to the inverse lattice period, and can therefore become shorter than the typical dephasing times in superlattices with a spatial period of the order of 100 Å.

For realizing Bloch oscillations, superlattices of high quality are essential. However, a certain degree of disorder in these artificial lattices is inevitable. It is well known that in one spatial dimension even arbitrarily weak disorder leads to localization of all electronic eigenstates [2]. In short superlattices of high quality, consisting of perhaps 100 periods, the localization lengths will exceed the length of the whole sample, so that localization effects will be negligible under normal conditions.

In this Letter we demonstrate that the degree of localization in one-dimensional disordered tight-binding lattices can be controlled by external homogeneous ac fields, even to such an extent that for certain field parameters all electronic eigenstates are entirely localized at individual sites. This effect has at least two practical applications. First, it can be exploited to characterize the quality of superlattice samples. Second, and from a more fundamental point of view, it opens up a new possibility for the experimental investigation of Anderson localization [3–5]: the strength of an ac field may be used to manipulate the localization lengths in intentionally disordered superlattices.

We consider a single-band tight binding model in the presence of both an ac and a dc field:

$$H(t) = H_0 + H_{\text{int}}(t) + H_{\text{random}}, \qquad (1)$$

where the Hamiltonian H_0 describes an energy band of width Δ in an ideal, unperturbed lattice with lattice spacing d,

$$H_0 = -\frac{\Delta}{4} \sum_\ell (|\ell + 1\rangle\langle\ell| + |\ell\rangle\langle\ell + 1|); \qquad (2)$$

$|\ell\rangle$ denotes a Wannier state at the ℓth site. Next,

$$H_{\text{int}}(t) = ed[F_{\text{st}} + F_L \cos(\omega t)] \sum_\ell |\ell\rangle\ell\langle\ell| \qquad (3)$$

describes the interaction with a homogeneous static field of strength F_{st} and an oscillating field of strength F_L and frequency ω, polarized along the lattice direction.

$$H_{\text{random}} = \sum_\ell \nu_\ell |\ell\rangle\langle\ell| \qquad (4)$$

introduces site diagonal disorder [3]. The random energies ν_ℓ are distributed according to a certain probability density $\rho(\nu)$; we use a system of units with $\hbar = 1$.

In the absence of disorder, $\nu_\ell = 0$ for all sites ℓ, the solutions to the time-dependent Schrödinger equation $i\partial_t \psi(t) = H(t)\psi(t)$ are given by the so-called "accelerated Bloch states," or Houston states [6]:

$$\psi_k(t) = \frac{1}{\sqrt{N}} \sum_\ell |\ell\rangle \exp\left(- iq_k(t)\ell d \right.$$
$$\left. - i \int_0^t d\tau\, E(q_k(\tau)) \right), \qquad (5)$$

where $q_k(t) = k - eA(t)$; $A(t) = -F_{\text{st}}t - (F_L/\omega) \times \sin(\omega t)$ is the vector potential of the electric field, and $E(k) = -(\Delta/2)\cos(kd)$ is the energy dispersion of the unperturbed band. N denotes the number of lattice sites; finite size effects are neglected.

Because the Hamiltonian (1) is periodic in time with period $T = 2\pi/\omega$, there should be a complete set of Floquet states, i.e., of T-periodic eigensolutions $u(t)$ to the equation

$$[H(t) - i\partial_t]u(t) = \varepsilon u(t). \qquad (6)$$

It was realized by Zak [7] that for vanishing disorder the construction of these Floquet states, and the calculation of their quasienergies ε, from the Houston states (5) becomes particularly transparent if $n\omega = eF_{\text{st}}d$ with

$n = 0, 1, 2, \ldots$, i.e., if the energy of n photons precisely matches the energy difference $eF_{st}d$ induced between adjacent sites by the static field. For such an "n photon-resonance," one finds Floquet states

$$u_k(t) = \psi_k(t) \exp[+i\varepsilon(k)t] \qquad (7)$$

and quasienergies

$$\varepsilon(k) = (-1)^n J_n\left(\frac{eF_L d}{\omega}\right) E(k), \qquad (8)$$

where J_n denotes the ordinary Bessel function of order n. These Floquet states are *extended* over all the lattice, and are characterized by the reciprocal lattice vector k as a good quantum number [7]. Remarkably, in the absence of disorder, even a weak resonant ac field fully counteracts Wannier-Stark localization [8–11] that would result from a strong static field alone.

In the absence of ac fields a single defect in an otherwise ideal lattice supports a localized energy eigenstate. Similarly, a single defect in a resonantly driven lattice gives rise to a localized Floquet state: If only ν_0 differs from zero in (4), the inverse exponential decay length L^{-1} of the Floquet state supported by the defect, measured in multiples of the lattice period d, is given approximately [12] by

$$L^{-1} = -2\ln\left(\sqrt{\frac{4\nu_0^2}{W^2} + 1} - \left|\frac{2\nu_0}{W}\right|\right), \qquad (9)$$

where $W = \Delta|J_n(eF_L d/\omega)|$ is the width of the quasienergy band (8). The approximation is valid as long as ω is not too small compared to Δ. Thus, L depends nonmonotonically on the ac field strength F_L, and even becomes zero when $eF_L d/\omega$ approaches a zero of the Bessel function J_n, for arbitrary defect strength.

Equation (9) looks exactly like the analogous equation for the decay rate of a time-independent impurity state in an energy band of width W. Thus, this equation indicates that a quasienergy band (8) plays, to some extent, the role of an ordinary energy band. If this were true even in the presence of random disorder, a most interesting possibility would emerge. It has been known since the pioneering work of Anderson [3] that electronic eigenstates in random lattices are strongly localized if the typical disorder strength $\bar{\nu}$ becomes comparable to the energy bandwidth. If the quasienergy bandwidth now takes over the role of the energy bandwidth in the presence of ac fields, then the degree of Anderson localization can be controlled by the ac field amplitude. Since it is possible to fabricate intentionally disordered semiconductor superlattices [13], and even to control the amount of lattice disorder during the growth process, experiments with intentionally disordered superlattices in far-infrared laser fields [14,15] could open up a novel access to the physics of localization phenomena [2–5]. The confirmation of the hypothesis that Anderson localization in one-dimensional lattices can be controlled by ac fields is the key result of the present Letter.

To quantify the degree of localization, we first employ the averaged inverse participation ratio P [16]. The Floquet states $u_m(t)$ for a disordered, finite lattice of N sites are expanded with respect to the Wannier states,

$$u_m(t) = \sum_{\ell=1}^{N} c_\ell^{(m)}(t)|\ell\rangle. \qquad (10)$$

Then P is defined as

$$P = \frac{1}{NT} \sum_{\ell,m=1}^{N} \int_0^T dt |c_\ell^{(m)}(t)|^4. \qquad (11)$$

(Actually, the time dependence of the localized states turns out to be weak, so that averaging over time becomes superfluous.) If *all* states are entirely localized at individual sites, P approaches unity, whereas it vanishes as $1/N$ if all states are extended, $|c_\ell^{(m)}| \approx 1/\sqrt{N}$ for all ℓ, m.

For the numerical computations we employ a lattice of 101 sites and assume that the wave functions vanish at the chain ends. All following results, except Fig. 5, have been obtained for $n = 1$, i.e., $\omega = eF_{st}d$, and $\Delta/\omega = 1.0$.

First, we choose a square disorder distribution, $\rho(\nu) = 1/(2\nu_{max})$ for $|\nu| \le \nu_{max}$, and zero otherwise. Figure 1 shows the response of the disordered system to the resonant ac field, for various disorder strengths ν_{max}: When there is disorder, a certain minimal amplitude is necessary to destroy Wannier-Stark localization. In agreement with the criterion originally put forward by Anderson [3], the crossover from strongly to weakly localized states occurs when the quasienergy bandwidth of the *ideal* system has become comparable to the disorder strength, $2\bar{\nu}/\Delta \approx |J_1(eF_L d/\omega)|$, where $\bar{\nu} = \nu_{max}/\sqrt{3}$ is the standard deviation of ρ. Moreover, if the quasienergy
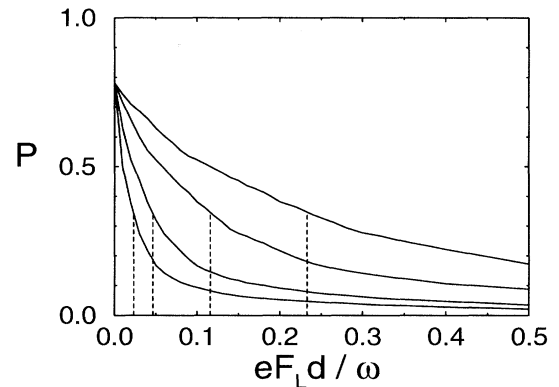

FIG. 1. Destruction of Wannier-Stark localization by a resonant ac field, $eF_{st}d = \omega$: inverse participation ratios (11) as functions of the scaled ac field strength. The lattice has 101 sites; $\Delta/\omega = 1.0$. The disorder distribution is $\rho(\nu) = 1/(2\nu_{max})$ for $|\nu| \le \nu_{max}$, with $\nu_{max}/\Delta = 0.10, 0.05, 0.02$, and 0.01 (top to bottom). Indicated are the field strengths where $2\bar{\nu}/\Delta = J_1(eF_L d/\omega)$, with $\bar{\nu} = \nu_{max}/\sqrt{3}$.
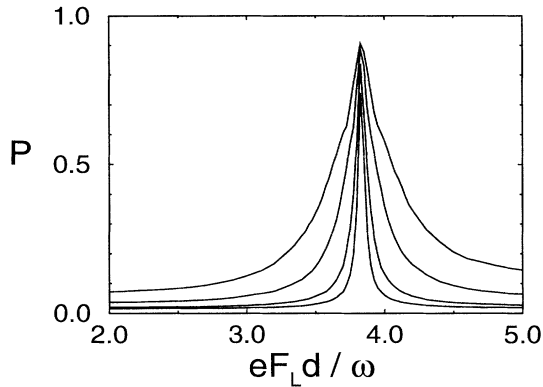
FIG. 2. Pronounced Anderson localization occurs when $eF_L d/\omega$ approaches 3.83171, the first positive zero of J_1. Other parameters are as in Fig. 1.

band plays a role analogous to that of the energy band in the static case, then systems with different disorder strengths, but identical ratios of disorder strength and quasienergy bandwidth must have the same value of P. This is clearly borne out by the numerical data [17].

When the field strength is increased further, such that $eF_L d/\omega$ approaches the first positive zero $j_{1,1} = 3.831\,71$ of J_1, the quasienergy bandwidth approaches zero again. This leads to the anticipated effect: Figure 2 shows the ac-field-induced pronounced Anderson localization near $j_{1,1}$. Since P almost reaches unity, the Floquet states become localized essentially at individual sites. The enhancement of the degree of localization even above the Wannier-Stark level ($F_L = 0$) occurs in the high-frequency domain, i.e., if ω is at least comparable to Δ.

The parameters considered here are not unrealistic for experiments with far-infrared radiation on semiconduc-

tor superlattices [14,15]. Assuming a scattering time $\tau = 10^{-12}$ s, and $\omega = 2$ meV, one has $\omega\tau \approx 3 > 1$, necessary for maintaining phase coherence. For this frequency, and a superlattice period $d = 100$ Å, an ac field strength $F_L = 10\,000$ V/cm already yields $eF_L d/\omega = 5$. The distinct advantage of working with artificial superlattices is that one may even predetermine the amount of disorder in the sample [13]. Thus, it is possible to realize somewhat exotic disorder distributions. As an example, we choose the singular distribution $\rho(\nu) = 1/\pi\nu_{max}\sqrt{1 - (\nu/\nu_{max})^2}$ for $|\nu| < \nu_{max}$, and zero otherwise. As an alternative measure for the degree of localization, we compute the standard deviations $\sigma^{(m)}$ of the distributions $p^{(m)}(\ell) = |c_\ell^{(m)}(3T/4)|^2$ of the Floquet states over the lattice sites, and plot in Fig. 3 the average value σ_{mean}, again as a function of the scaled ac field strength $eF_L d/\omega$. If all Floquet states were uniformly extended over all 101 sites, σ_{mean} would be 29.15. For weak disorder the numerical data almost reach this value between the zeros of J_1. But close to the zeros, there is again practically complete localization at individual sites. For strong disorder the states do not recover from their localization, since the quasienergy band does not become wide enough again.

Figure 4 shows the corresponding quasienergies for $\nu_{max}/\Delta = 0.2$. The peculiar clustering of eigenvalues into three groups is caused by the singular density $\rho(\nu)$; it is not found for the square distribution [17]. At the zeros of J_1 the quasienergy band of the disordered lattice has a width of the order of $2\nu_{max}$. This shows that Anderson localization in disordered lattices is quite different from the "dynamic localization" discovered by Dunlap and Kenkre [18] in ideal, ac-driven lattices. If one forms a wave packet that is initially localized at an arbitrary site of an ideal lattice, $H_{random} \equiv 0$, then the wave packet will remain localized at that site if $eF_{st}d = n\omega$ and, simultaneously, $eF_L d/\omega$ coincides with a zero of J_n:
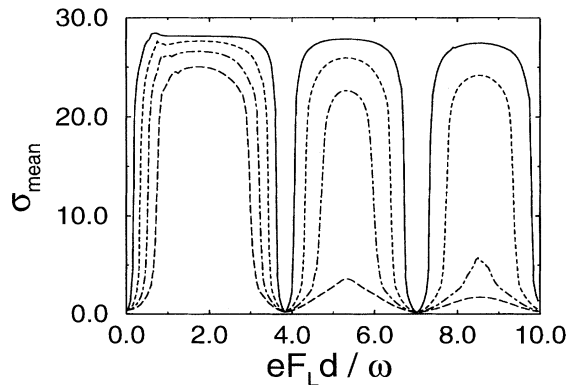

FIG. 3. Average width σ_{mean} of the spatial distribution of the Floquet states, in units of the lattice spacing. The disorder distribution is $\rho(\nu) = 1/\pi\nu_{max}\sqrt{1 - (\nu/\nu_{max})^2}$ for $|\nu| < \nu_{max}$, with $\nu_{max}/\Delta = 0.05$, 0.10, 0.15, and 0.20 (top to bottom). Note $\sigma_{mean} = 29.15$ for uniformly extended states.
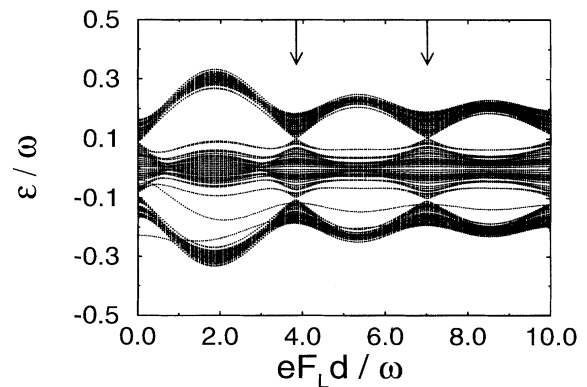

FIG. 4. Quasienergy spectrum for the same disorder distribution as used in Fig. 3, and $\nu_{max}/\Delta = 0.2$. The arrows indicate the first two positive zeros of J_1.
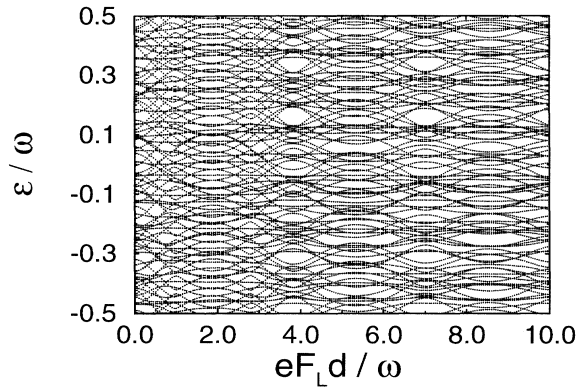
FIG. 5. Quasienergy spectrum for a nonresonant case, $eF_{st}d = 1.11\,\omega$. $\rho(\nu)$ is the same as in Fig. 4.

since the quasienergies of all the wave packet's components are then equal, all components acquire precisely the same phase factor during one cycle of the ac field, so that the wave function simply reproduces itself, apart from an overall phase factor [19]. However, the Floquet states (7) remain extended over all the lattice. Dynamic localization of wave packets in ideal tight binding lattices merely reflects the degeneracy of all quasienergies at the zeros of J_n. It has been shown recently that this effect persists even in the presence of Coulomb interactions [20].

In contrast, ac-field-induced strong Anderson localization in disordered lattices implies the localization *of the Floquet states themselves*, cf. Figs. 2 and 3, and there is *no* total degeneracy of the eigenvalues, cf. Fig. 4. Experimentally, there is a possible signature for ac-field-induced Anderson localization: If all the eigenstates are localized, the only mechanism enabling conductance will be variable range hopping. At the zeros of J_n, the conductance of a disordered superlattice should therefore decrease with decreasing temperature, whereas it should increase in between, where phonons impede transport via the (effectively) extended states. It is also of interest to note that the localization lengths are of the order $[\Delta J_n(eF_L d/\omega)/\nu_{max}]^2 d$, whereas the electron-phonon scattering mean free path is proportional to $\Delta J_n(eF_L d/\omega)$. This can always be chosen sufficiently small that the mean free path is much longer than the localization length, so that localization effects of static disorder are not washed out by phonon scattering.

We reemphasize the special role of *resonant* ac fields, $eF_{st}d = n\omega$ [7]. If this condition is not satisfied, the states remain Wannier-Stark localized; there are no "extended" states at all. Figure 5 shows the quasienergy spectrum for precisely the same realization of disorder

as employed in Fig. 4, but for $eF_{st}d = 1.11\,\omega$. (In the ideal, nonresonant system, the quasienergies are simply $\varepsilon_\ell = eF_{st}\ell d \bmod \omega$.) The corresponding values of σ_{mean} stay below 2.0 in the entire range of $eF_L d/\omega$.

Up to now, interest in the manipulation of Anderson localization by external fields remained restricted to magnetic fields [21]. The present results indicate that combined dc and ac electric fields also have pronounced and systematic effects on localization phenomena, and that these effects can be studied with the help of existing facilities [14,15] in disordered semiconductor superlattices—with the distinct advantage that even the sample-specific realizations of disorder are under experimental control.

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