Decoherence, Continuous Observation, and Quantum Computing: A Cavity QED Model

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We use the theory of continuous measurement to analyze the effects of decoherence on a realistic model of a quantum computer based on cavity QED. We show how decoherence affects the computation, and methods to prevent it.

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The preparation and coherent manipulation of N-atom entangled states is fundamental to realizing a quantum computer [1] is the basis of tests of quantum mechanics vs local realists' theories [2], and promises a novel atomic spectroscopy with resolution better than the standard quantum limit [3]. In a *quantum computer* (QC), information is stored in a quantum register composed of N two-level systems representing the quantum bits (qubits), and the general state of the QC is an (entangled) linear superposition of their states. QCs can perform certain classes of computations exponentially faster than any classical machine [1,4]. Such a device should be able to perform arbitrary unitary operations on the quantum register, which can be decomposed into a sequence of steps involving the conditional dynamics of a few qubits (quantum gates) [1]. Given that a QC allows one to perform any desired operation on the qubits, building a QC is equivalent to building an N-atom quantum state synthesizer.

As yet few practical proposals for the realization of quantum gates [5] or a QC have been made [6]. First we need to identify a physical mechanism to entangle the state of the atoms and perform conditional dynamics on the qubits in a controlled way. In practice, the central obstacle is the fragility of macroscopic atomic superpositions with respect to decoherence by coupling to an environment [7]. It is thus of crucial importance to understand the effects of decoherence on a given computation, and to develop methods to compensate for and suppress quantum noise in realistic systems. In this Letter we analyze decoherence within a realistic quantum optical model, and how its effects can be remedied by continuously monitoring the decay channels of the QC (in the sense of continuous measurement theory [8]), and by astute design of the quantum gate. We emphasize that in contrast to Ref. [7], where the general scaling of the errors with the number of qubits was studied, we will focus on small systems with a few qubits, relevant for the experimental realization of a first generation of QC experiments.

Our scheme for the QC is based on a set of N atoms representing the qubits communicating via their interaction with a single quantized mode of a high-Q optical cavity (Fig. 1). We assume that the atoms are fixed inside the cavity at distances apart much larger than the wavelength of the cavity mode and interacting individually with laser beams, which allows for sequences of operations between any two qubits and thus the implementation of a whole quantum network. The qubits are stored in Zeeman ground state levels of the trapped atoms. From Ramsey spectroscopy [9] we infer that decoherence of these levels is negligible in the present context. Quantum gates are implemented by coupling atoms to individual lasers and entangling them by exchange of a cavity photon. Sources of decoherence are thus the spontaneous emission from the excited state of the atoms, and cavity decay, *during the gate operation*. This is in contrast to the models studied in Ref. [7] where the decay and decoherence of qubits due to coupling to an environment was studied. Spontaneous emission can be significantly avoided by performing this process as an adiabatic passage via a dark state of the strongly coupled N-atom + cavity system [10]. Cavity decay is minimized by having a photon present only during the gate operation.

Damping in quantum optical systems is typically described within the framework of master equations, deriving an equation for the density matrix of the system (the QC) by tracing over the degrees of freedom of the environment. This corresponds to a situation where the decay of the system is not observed (*a priori dynamics*) [8]. In contrast, we will study a situation where the QC is continuously monitored to detect the photon decays according to Fig. 1 (*a posteriori dynamics*) [8]. This gives a time evolution of the system conditional to the observed sequence of photon counts. In the case of unit efficiency

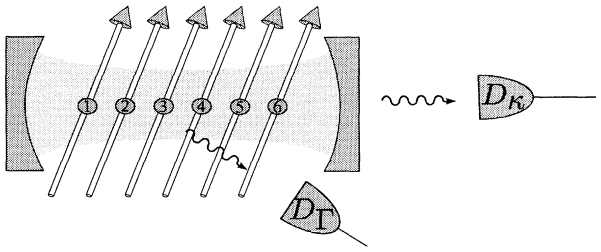

FIG. 1. Schematic representation setup. The arrows indicate laser beams interacting with individual atoms. D_κ and D_Γ detect cavity photons and spontaneously emitted photons, respectively.

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detection, this is described by a pure state wave function for the system evolving according to a non-Hermitian Hamiltonian which includes dissipative terms. Observation of a photon corresponds to a quantum jump [8], which is an incoherent step in the time evolution of the QC. We emphasize that *by observation of "no decay" we gain information about the system, which is reflected in some "unwanted" dynamics of the state vector.*

We are interested in performing conditional dynamics by transforming a target qubit conditional to the value of a control qubit according to [1,5]

$$|\epsilon_1\rangle|\epsilon_2\rangle \longrightarrow \delta_{\epsilon_1 0}|\epsilon_1\rangle|\epsilon_2\rangle + \delta_{\epsilon_1 1}|\epsilon_1\rangle\hat{U}|\epsilon_2\rangle \quad (\epsilon_j = 0,1), \quad (1)$$

where $\hat{U}$ is a unitarity transformation on the second qubit. Our scheme to implement (1) is based on the following three steps: (step 1) we map the *two* qubits $|\epsilon_1\rangle|\epsilon_2\rangle$ to a *single* four level system $|\epsilon_1\,\epsilon_2\rangle = |0 \equiv 0\,0\rangle, \ldots, |3 \equiv 1\,1\rangle$ in atom 2 while leaving atom 1 in state $|0\rangle$, $|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |0\rangle|\epsilon_1\,\epsilon_2\rangle$. This mapping is an isomorphism $\mathcal{H}_2 \otimes \mathcal{H}_2 \rightarrow \mathcal{H}_1 \otimes \mathcal{H}_4$, where $\mathcal{H}_i$ denotes the i-dimensional atomic Hilbert space. (step 2) We transform the states of the target atom 2 according to

$$|\epsilon_1\,\epsilon_2\rangle \longrightarrow \delta_{\epsilon_1 0}|\epsilon_1\,\epsilon_2\rangle + \delta_{\epsilon_1 1}\sum_{\epsilon_2'}|\epsilon_1\,\epsilon_2'\rangle\langle\epsilon_2'|\hat{U}|\epsilon_2\rangle. \quad (2)$$

It is clearly comparatively easy to manipulate single atoms, which is the advantage of transferring the information of both atoms to a single atom. (step 3) Finally, the inverse of step one is performed. We note that if in the second step we interchange $|2 \equiv 1\,0\rangle \leftrightarrow |3 \equiv 1\,1\rangle$, the total operation corresponds to a controlled-NOT gate $|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$ [1,5].

We now demonstrate that Zeeman coherence between ground state levels can be transferred between two atoms by employing adiabatic passage via a dark state of the two-atom + cavity system. This will be the basis to implement Eq. (1). Consider the Λ configurations shown in Fig. 2(a) [10]. The transitions $|b\rangle_j \rightarrow |c\rangle_j$ of both atoms ($j = 1,2$) are strongly coupled to the *same* quantized cavity mode with coupling strength g. The transitions $|a\rangle_j \rightarrow |c\rangle_j$ are coupled to separate classical coherent driving fields with frequency ω_L and Rabi frequencies Ω_j. The interaction part of the Hamiltonian is given by ($\hbar = 1$).

$$H_I = \sum_{j=1,2}\left(\frac{\Omega_j(t)e^{-i\omega_L t}}{2}|c\rangle_{jj}\langle a| + g/2|c\rangle_{jj}\langle b|\hat{b}\right) + \text{H.c.},$$

where $\hat{b}$ is the annihilation operator of a cavity photon. This Hamiltonian has dark eigenstates, i.e., superpositions of Zeeman ground states which by quantum interference are decoupled from the excited states. We are interested in dark states which contain the vacuum state of the cavity $|0\rangle_c$. Assuming Raman resonance conditions between the two ground states [10] we find

$$|D_0\rangle = |b,b,0\rangle \equiv |b\rangle_1|b\rangle_2|0\rangle_c,$$
$$|D_1\rangle \propto \Omega_1 g|b,a,0\rangle + \Omega_2 g|a,b,0\rangle - \Omega_1\Omega_2|b,b,1\rangle. \quad (3)$$

The possibility of coherence transfer arises from the following behavior of the dark state $|D_1\rangle$: $|D_1\rangle \rightarrow |b,a,0\rangle$ for $\Omega_2/\Omega_1 \rightarrow 0$, and $|D_1\rangle \rightarrow |a,b,0\rangle$ for $\Omega_1/\Omega_2 \rightarrow 0$; that is, if we apply a "counterintuitive" pulse sequence where the pulse on atom 2 precedes the pulse on atom 1, i.e., leading from $\Omega_2/\Omega_1 \gg 1$ to $\Omega_1/\Omega_2 \gg 1$ [10] with g = const, an adiabatic transfer of the dark state between the two limiting cases of $|D_1\rangle$ may be achieved. This requires $gT, \Omega_j T \gg 1$ with T the laser pulse duration, as well as the conditions for the observation of vacuum Rabi splitting [11], $g, \Omega_j \gg \kappa, \Gamma$, with κ and Γ the cavity decay and atomic spontaneous emission rates, respectively [12]. In particular, we can transfer atomic coherence via a superposition of $|D_0\rangle$ and $|D_1\rangle$ according to $(A|a\rangle_1 + B|b\rangle_1)|b\rangle_2|0\rangle_c \rightarrow |b\rangle_1(A|a\rangle_2 + B|b\rangle_2)|0\rangle_c$, where A and B are arbitrary coefficients. Important features of this scheme are as follows: (i) The excited states are (in principle) never populated and thus the transfer is immune to spontaneous emission; (ii) the interaction can be resonant throughout the transfer, and thus there will be no phase shifts; (iii) the interaction times do not need to be adjusted very accurately as long as the adiabaticity condition is fulfilled; and (iv) cavity decay can occur only for the short time the intermediate state $|1\rangle_c$ is populated.

In order to perform quantum gates we require atoms with more degrees of freedom. In Fig. 2(b) we have plotted the internal structure of two of the atoms of the QC. To keep the method valid for a general atom, we have plotted only the internal levels of each atom that are relevant for the operations. The atoms thus consist of two hyperfine ground levels ($|a\rangle_j$ and $|b\rangle_j$) and one excited level ($|c\rangle_j$). Each of these levels is doubly degenerate (we denote by the subscripts $-$ and $+$ two different magnetic quantum numbers). The cavity mode is on resonance with each $|b\rangle \rightarrow |c\rangle$ transition with coupling strength g, whereas the $|a\rangle \rightarrow |c\rangle$ transitions of each atom are excited by a different laser beam, with Rabi frequency Ω_j. Note that each atom consists of two Λ systems behaving exactly in parallel, and therefore the above ideas can be used to transfer coherences between the atoms.

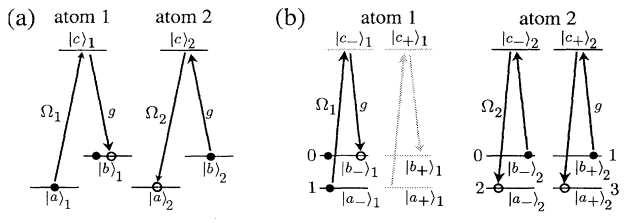

FIG. 2. (a) Transfer of atomic coherence between two Λ systems. (b) Transfer of two qubits to a single atom (four-level system).

We now show how a general unitary operation between two qubits can be performed. First assume that the qubits in atoms 1 and 2 are represented by levels $|b_-\rangle_1 \equiv |0\rangle$ and $|a_-\rangle_1 \equiv |1\rangle$ and levels $|b_-\rangle_2 \equiv |0\rangle$ and $|b_+\rangle_2 \equiv |1\rangle$, respectively [see Fig. 2(b)]. To implement step 1 we apply a counterintuitive pulse sequence which leaves the atom 1 in state $|0\rangle$, and atom 2 in one of the levels $|0 \equiv 0\,0\rangle, \ldots, |3 \equiv 1\,1\rangle$ [for the notation see Fig. 2(b)]. The conditional dynamics (step 2) are performed as a one-atom operation (Raman transition with two lasers), and, finally, the qubits are transferred back to the two atoms (step 3). In our QC model, the qubits are stored in the lower ground levels of each atom ($\{|1\rangle, |0\rangle\} = \{|a_-\rangle, |a_+\rangle\}$). Every time one has to implement conditional dynamics between two given atoms, the qubits are first transferred to the appropriate levels in each atom in order to apply the above method. At the end of the operation, the qubits are transferred back to the original levels. These steps prevent atoms extraneous to a given operation from being affected by the presence of a cavity photon. They will be in levels $|a\rangle_j$, which do not interact with the cavity mode.

The time evolution of the QC is described by the following master equation:

$$\dot{\hat{\rho}}(t) = -i[\hat{H}_{\rm eff}(t)\hat{\rho} - \hat{\rho}\hat{H}_{\rm eff}(t)^\dagger] + \sum_{j=1}^{N} J_{\Gamma j}\hat{\rho} + J_\kappa\hat{\rho}\,, \tag{4}$$

where, in the interaction picture and on resonance,

$$\hat{H}_{\rm eff} = -i\kappa\hat{b}^\dagger\hat{b} - i\frac{\Gamma}{2}\sum_{j=1}^{N}\sum_{k=-,+}|c_k\rangle_{jj}\langle c_k| + \sum_{j=1}^{N}\sum_{k=-,+}\left(\frac{\Omega_j(t)}{2}|c_k\rangle_{jj}\langle a_k| + \frac{g}{2}|c_k\rangle_{jj}\langle b_k|\hat{b} + \text{H.c.}\right) \tag{5}$$

is a non-Hermitian Hamiltonian including decay terms from spontaneous emission and cavity decay. The superoperator $J_{\Gamma j}$ describes the return of the electron to the atomic ground states after a spontaneous emission, and J_κ the corresponding term for the cavity decay (quantum jump) [10]. The time evolution of the QC conditioned to no photon decay occurring is given by the pure state wave function

$$|\Psi(t)\rangle = \hat{U}_{\rm eff}(t)\,|\Psi(t_0)\rangle/\|\hat{U}_{\rm eff}(t)\,|\Psi(t_0)\rangle\|\,, \tag{6}$$

where $\hat{U}_{\rm eff}(t) = \hat{T}\exp[-i\int_{t_0}^{t} dt\,\hat{H}_{\rm eff}(t)]$ with $\hat{T}$ the time-ordering operator, and $|\Psi(t_0)\rangle$ the initial state.

According to Eq. (6) the dynamics of the present model for step 1 in the quantum gate operation is to a very good approximation described by $\hat{U}_{\rm eff}$: $|\epsilon_1\rangle|\epsilon_2\rangle \to e^{-\lambda\epsilon_1}|0\rangle|\epsilon_1\,\epsilon_2\rangle$, where λ is a damping constant whose detailed dependence on the damping parameters, etc. [12] is not relevant for the following discussion. Note that only the component of the wave function with $\epsilon_1 = 1$ experiences damping, since this is the part undergoing transfer of coherence between the atoms via a cavity photon in the dark state $|D_1\rangle$. This illustrates that even in the case of selecting computer runs with no decay, damping will distort the dynamics of the QC. In the present example, for the complete gate operation, $\hat{U}_{\rm eff}$: $|\epsilon_1\rangle|\epsilon_2\rangle \to \delta_{\epsilon_1 0}|\epsilon_1\rangle|\epsilon_2\rangle + e^{-2\lambda}\delta_{\epsilon_1 1}|\epsilon_1\rangle\hat{U}|\epsilon_2\rangle$. This error due to damping can be avoided, however, by designing the conditional dynamics (or quantum gates) in such a way that the basis states are equally damped, so that the damping factorizes out in any linear combination, $\hat{U}_{\rm eff}$: $|\epsilon_1\rangle|\epsilon_2\rangle \to e^{-\lambda}[\delta_{\epsilon_1 0}|\epsilon_1\rangle|\epsilon_2\rangle + \delta_{\epsilon_1 1}|\epsilon_1\rangle\hat{U}|\epsilon_2\rangle]$, so that upon normalization of the wave function according to (6) the *effect of the damping drops out* in the subensemble with no decay. This can be accomplished by swapping $|0\,\epsilon_2\rangle \leftrightarrow |1\,\epsilon_2\rangle$ in the atom 2 before the transfer back (step 3), followed by a corresponding swap in both atoms after the transfer.

We have studied numerically the error accumulated by repeated application of a controlled-NOT gate to two atoms prepared in the product state $(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)/2$, for the different scenarios described above. In Fig. 3(a) we plot the probability $P_{\rm ex}$ of finding the system in the exact state (ideal gate) against the number of gate operations M when cavity loss is the dominant decay channel. As the figure shows, the runs corresponding to no photons detected (solid line) have a smaller error than the others, although it is still important. These results can be dramatically improved by designing the controlled-NOT gate with "equalized damping" [Fig. 3(b)]. If there is no detection of photons, there is no improvement compared to the case without error correction (dash-dotted line). However, if we detect all cavity photons (dashed line), this method prevails over the corresponding situation in Fig. 3(a). For this method, note that if all

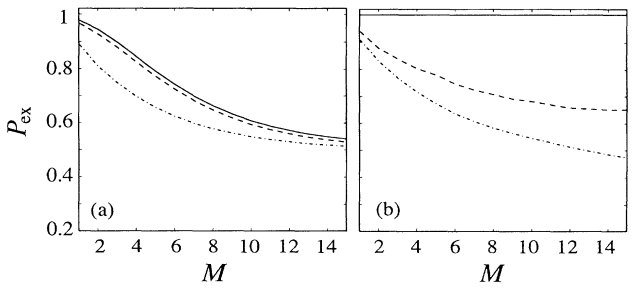

FIG. 3. Probability to measure the correct result $P_{\rm ex}$ after the application of M controlled-NOT gates. (a) Gate without equalized damping, (b) gate with equalized damping. Solid lines: subensemble conditional to no photon emission [Eq. (8)]; dashed line: subensemble conditional to no cavity photon emission, dash-dotted line: full master equation (6) (no photon detection). Parameters: $\Gamma = 0.02g$, $\kappa = 0.4g$, $\Omega_j^{\rm max} = 0.7g$, Gaussian laser pulse shape with a width $T = 6g^{-1}$, and pulse delay $\tau = 6g^{-1}$, for on-resonance conditions.

jumps are detected we are close to the ideal limit $P_{ex} \simeq 1$, in the parameter range of Fig. 3 (solid line).

We now show how decoherence affects the final result in a nontrivial calculation. Consider a Fourier transform (FT) on $N = 5$ atoms, initially prepared in the periodic state $|\Psi_i\rangle \propto \sum_l |rl + k\rangle$ with period r and k a random offset, where in the present example $r = 3$. Finding the period r of $|\Psi_i\rangle$ by a FT is the crucial step of Shor's factorization algorithm, which allows factorization of large integers in polynomial time [4]. Here, $|y\rangle (y = 0, 1, \ldots, 2^N - 1)$ represents the state of the quantum register, which is expressed in terms of the states of the five qubits (using binary representation). The FT is defined as

$$\hat{FT}|y\rangle = \frac{1}{\sqrt{2^N}} \sum_{x=0}^{2^N-1} e^{2\pi i xy/2^N} |x\rangle,$$

and can be accomplished by applying a series of one- and two-bit gates [6,13]. The probability $P(y)$ of measuring the state (y) after the FT is plotted in Fig. 4, where the shaded bars represent the exact result $P(y) = |\langle y|\hat{FT}|\Psi_i\rangle|^2$. The period r can be read off from the separation of the peaks of the Fourier transform. The outlined graph in (a) corresponds to the full solution of the master equation (4) [8]. Although the main features of the Fourier transform are still clearly visible, we find a central peak at $y = 16$, which is a systematic error and arises due to unequal damping. For the actual parameters, the ratio of flawless realizations to realizations which undergo one or more quantum jumps is approximately 1:2. In Fig. 4(b) all erroneous realizations have been rejected, which results in a significant improvement of the result. The central peak, however, cannot be removed by this method. Further improvement is obtained by selecting the subensemble conditioned to not observing a photon, and using gates designed such that the damping is equalized [Fig. 4(c)]. In Fig. 4(d) we plot the population of the basis states where we keep only those realizations which underwent at least one quantum jump (incoherent background). Surprisingly, we find that the quantum computation is not destroyed and information can still be inferred from the result. The mean number of jumps per realization is 1.8. We have found that the effect of spontaneous photons from excited atomic states on the computation is much more destructive than cavity decay. This is because after a cavity photon emission the system is still in a dark state, which is not necessarily true for spontaneous emission. Thus the atomic excited states can be populated, which gives rise to further spontaneous photons.

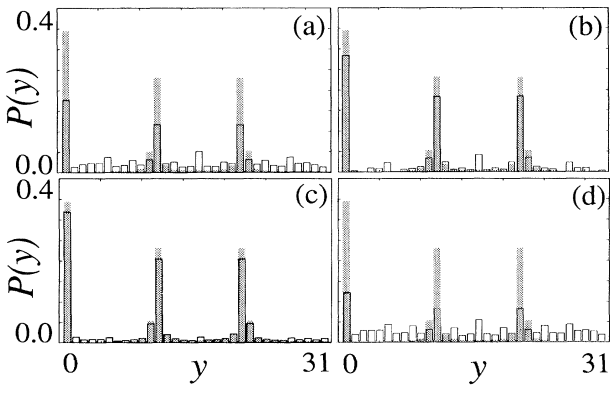

FIG. 4. Probability distributions $P(y)$ from a FT on $N = 5$ atoms. Shaded bars: exact result. Outlined bars: (a) decays not detected (master equation); (b) subensemble conditioned to not observing a decay; (c) as in (b) but with equalized damping; (d) incoherent background. Parameters see text and Fig. 2.

In view of recent experimental progress in optical cavity QED [11], and of trapping and cooling of atoms and ions, realization of a QC model as proposed in the present Letter is likely to be within reach of soon available technology. We expect that the schemes to minimize the effects of decoherence as developed in the present work are generally valid in quantum optical QC proposals, beyond the scope of the present model.

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