Novel Edge Magnetoplasmons in a Two-Dimensional Sheet of $^4He^+$ Ions

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We report the experimental observation of novel low-frequency edge magnetoplasma waves in circular pools of positive ions trapped below the surface of superfluid ^{4}He. The modes were detected through a nonlinear coupling with an axisymmetric plasma mode of the pool, when both types of modes were driven simultaneously. The observed frequencies of the new modes and their dependence on magnetic field and pool radius allow us to identify them with those predicted to exist by Nazin and Shikin [Sov. Phys. JETP **67**, 288 (1988)] when account is taken of the correct density profile at the edge of the pool.

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Pools of ions trapped below the surface of superfluid ^{4}He at a low temperature are ideal for the study of the different collective excitations of a bounded two-dimensional classical cold plasma [1,2]. Depending on the temperature, such pools can exist in either a fluid phase or a crystal phase, the two phases being possibly separated by a hexatic phase [2]. In the absence of a magnetic field there are two independent collective modes involving motion in the plane of the pool: longitudinal plasma modes and transverse shear or viscous modes. Pools of electrons trapped above the surface of the helium can also be used for these studies [3], although their behavior in the crystal phase is influenced much more strongly by ripplon interactions [4]. In bounded pools resonant modes can exist for only a discrete set of wave numbers that are determined by the boundary conditions at the edge of the pool; for a given wave number the frequency of the transverse mode is generally much less than that of the longitudinal mode. The application of a vertical magnetic field B modifies all the modes through the action of the Lorentz force and introduces some coupling between them [5–8].

In this Letter we report the experimental observation of a new type of magnetoplasma mode in a circular ion pool. It is an unconventional edge mode of a type predicted to exist by Nazin and Shikin [9], in the context of the electron system. A typical observed spectrum that includes several of the new modes is shown in Fig. 1. We shall first outline the relevant background theory and then discuss the new experimental results in more detail.

The electric field required to trap the ions is provided by two electrodes parallel to the plane of the ion pool and spaced $\pm d$ above and below the pool. In the simplest theoretical approach to the calculation of the mode frequencies in a bounded pool trapped in this way, the equilibrium density profile is taken to be spatially constant with a step function to zero at the edge of the pool. Modes are assumed to propagate as in the unbounded pool, the edge of the pool being taken into account by imposing appropriate boundary conditions. These boundary conditions for a circular pool are assumed to be the vanishing of both the radial displacement and the shear stress at the pool edge. In reality the edge of the pool is not abrupt; the density falls to zero over a distance of order d, although the pool still has a well-defined edge, at which the radial component of the density gradient becomes infinite. Provided that all wave numbers associated with the mode considered are small compared with $1/d$, the mode frequencies can still be calculated to a good approximation by replacing the actual density profile by one having an abrupt edge and by using effective boundary conditions, which do not differ very much from those we have mentioned; the calculation is rather easy for small wave numbers because the relation between the perturbed density n_1 in the pool and the resulting perturbation ϕ in the electrostatic potential is the local (the relevant range in the general nonlocal relation being d). Detailed experimental studies have yielded results that are consistent with this theoretical approach, although accurate agreement between theory and experiment can be achieved only if the detailed form of the density profile at the edge of the pool is taken into account [5,6,8].

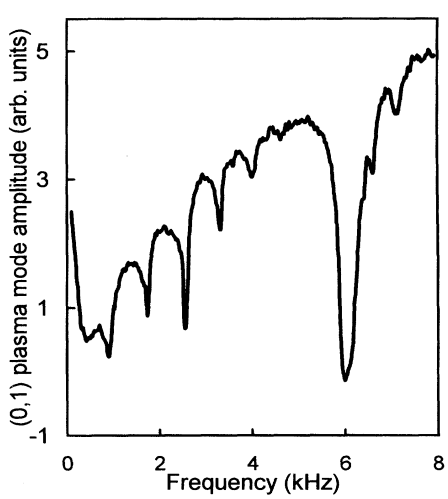

FIG. 1. A typical observed spectrum showing several peaks due to the new type of edge magnetoplasma mode. $n_0 = 8.8 \times 10^{10}$ m^{-2}, $R = 12.34$ mm; trapping depth below surface of helium: 60 nm, $T = 60$ mK, $B = 1.2$ T.

Among the modes studied are the so-called *edge magnetoplasma waves,* which exist in a circular pool in sufficiently large magnetic fields, and in which the wave is localized to a strip at the edge of the pool. In essence the localized wave travels around the edge of the pool with a speed given in the limit $\delta \equiv s_0(0)/\omega_c \gg d$ by

$$s_0^2(0) = \frac{n_0 e^2 d}{2\varepsilon_0 m^*}, \tag{1}$$

where n_0 is the areal number density of charges in the pool in equilibrium, m^* is the effective mass of these charges, and ω_c is the cyclotron frequency eB/m^*. The localization means that the amplitude ϕ of the perturbation in the electrostatic potential associated with the wave falls off with the distance x from the edge of the pool as $\exp(-x/\delta)$. At sufficiently high magnetic fields δ must become comparable with or less than d. The correct density profile near the edge of the pool must then be taken into proper account, with a resulting decrease in the speed s_0. A number of detailed experimental and theoretical studies of these edge magnetoplasma oscillations have been published [5–8].

However, it has been pointed out by Nazin and Shikin [9] that this theoretical approach fails to reveal the existence of a *new set of edge modes, which depend for their existence on the fact that the density profile at the edge of the pool is not a step function.* The required calculations are quite complex because they must necessarily involve a recognition that the relation between n_1 and ϕ is generally nonlocal. For these new modes the dependence ϕ on x is more complicated than the simple exponential decay $\exp(-x/\delta)$. For the case of a small magnetic field it takes the form

$$\phi = \exp\left(-\frac{x}{\delta}\right) P_M^{(\alpha,\beta)}(\xi); \tag{2}$$

$P_M^{(\alpha,\beta)}(\xi)$ is the Jacobi polynomial; $\xi = 2\exp(-\mu x/d) - 1$, $\alpha = 0$, $\beta = 2d/\mu\delta$, and μ is a parameter that depends on the detailed form of the equilibrium density profile close to the edge of the pool and is approximately equal to π. (Note that there is a misprint in the value of β in Ref. [9].) The new modes form several families, depending on the value of the integer M. The family with $M = 0$ is simply the well-known set of (conventional) edge magnetoplasma waves that we have already described. The new modes are those with $M \geq 1$, and all involve one or more spatial oscillations in the dependence of ϕ on x within the region $x \lesssim d$ (they must therefore involve wave numbers of order $1/d$). The calculations of Nazin and Shikin relate to a straight edge belonging to a semi-infinite sheet of charge, for which ϕ varies with the direction y parallel to the edge as $\exp(iqy - i\omega t)$, and they lead to the speeds of propagation $s_M = \omega/q$ along the edge. The calculations can be applied to find the corresponding resonant modes of a circular pool of radius R, provided that $R \gg \delta$: We must simply take $q = n/R$, where n is an integer. Nazin and Shikin find that the speeds s_M have a curious dependence on the applied magnetic field B. This dependence is conveniently described by introducing the parameter $\gamma = d/\delta$, which is proportional to B. For the conventional edge magnetoplasma mode with $M = 0$, the speed simply decreases monotonically from the value given by Eq. (1) with increasing γ. However, for the mode with $M = 1$, the speed increases from zero at $\gamma = 0$, passes through a maximum at a value of γ slightly greater than 1, and then falls off with a further increase in γ. Plots of these speeds, derived from numerical work and valid for all values of γ, are given in [9]. Similar results for the case $\gamma \gg 1$ have been obtained by Aleiner and Glazman [10], and computer calculations have been performed by Xia and Quinn [11] for the unrealistic case when the density profile falls linearly to zero at the pool edge.

Our new experimental results relate to these new modes. The apparatus and the methods of excitation and detection of the modes were identical to those reported in connection with conventional edge magnetoplasma modes in [8]. In brief, positive ions (effective mass equal to about $30m_4$, where m_4 is the mass of a helium atom) were trapped below the surface of the helium midway between two electrodes separated by a distance $2d = 3.1$ mm; modes were excited by an oscillating electric field and detected indirectly by their effect, through a nonlinear coupling, on the frequency and linewidth of the fundamental axisymmetric $[(0,1)]$ magnetoplasma mode [8], which was simultaneously excited at a frequency of typically 150 kHz.

In the typical observed spectrum shown in Fig. 1, the detected signal is the amplitude of the in-phase response of the fundamental axisymmetric plasma mode driven at its resonant frequency of 157.710 kHz. The large peak at 6 kHz is the lowest conventional magnetoplasma mode. As we shall see, the other peaks may be identified with some of the new Nazin-Shikin modes.

Our observed spectra are generally rather complicated and contain a number of different types of modes. The conventional magnetoplasma modes are easily excited and identified, and the results of our own study of them were published in [8]. In the crystal phase we find modes in the range of frequencies 0.5 to 3 kHz that we have identified as shear modes, as we describe in [12]. Confirmation that we see both the conventional magnetoplasma modes and the shear modes comes from an observed characteristic dependence of the mode frequencies on applied magnetic field; in both cases the frequencies decrease with increasing field. The shear modes disappear as the temperature is raised through the melting temperature, and they are replaced by what are probably broad viscous modes. At the higher temperatures (near or above the melting temperature) the new modes shown in Fig. 1 appear, especially if the driving signal is rather large. The dependence of the frequencies of the new modes on magnetic field is shown for a typical case in Fig. 2. We see an increasing magnetic field, in marked contrast to the behavior of the lowest conventional magnetoplasma mode, which is also shown in

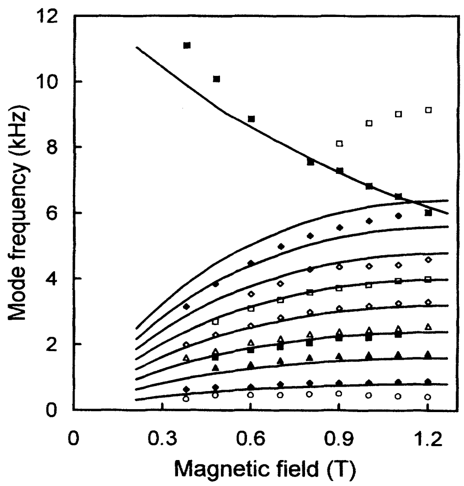

FIG. 2. Mode frequencies plotted against magnetic field, derived from spectra similar to those in Fig. 1. The set of eight solid lines in the lower half of the diagram are the predicted frequencies of the Nazin-Shikin modes with $M = 1$, $n = 1,2,3,4,5,6,7,8$. The solid line in the upper half of the diagram is the predicted frequency of the lowest conventional ($M = 0$) magnetoplasma mode. All predictions are strictly valid only in the limit $R \gg \delta$.

this figure. The shear modes tend to disappear in the presence of the large drive required to excite the new modes, and this disappearance may be associated with damage to the crystal [12]. We have not yet seen the new modes convincingly at low temperatures, where the shear modes are prominent, and we have not yet fully explored the conditions necessary to see one mode or the other.

For the results shown in Fig. 2 the value of γ varies between 0.34 at $B = 0.3$ T and 1.35 at $B = 1.2$ T. We see then that the conditions in our experiment are such that the speed s_1 should increase with increasing B, as is the case for the new modes that we observe. Furthermore, we note that the ratio δ/R varies between 0.34 and 0.085. Therefore, to a reasonable approximation the condition $R \gg \delta$ does hold, and we are justified in taking the resonant angular frequencies of the Nazin-Shikin modes to be given approximately by

$$\omega_{n,M} = \frac{n s_M}{R} \,. \tag{3}$$

The solid lines in Fig. 2 are plots of $\omega_{n,1}$ for integer values of n from 1 to 8. We see that there is quite good quantitative agreement with the data for our new modes. We conclude that the new modes are indeed the new type of edge magnetoplasma mode predicted to exist by Nazin and Shikin. We have checked that the mode frequencies vary in the expected way with pool radius and charge density.

Figure 2 includes our data on the frequency of the lowest conventional edge magnetoplasma wave, and we compare them with the predicted value $s_0(\gamma)/R$ derived from Nazin and Shikin. It should be noted that the field dependence of this predicted frequency arises from the fact that the strip of width δ into which the mode is localized is to a significant extent within the region of width d at the edge of the pool where the equilibrium ion density falls below n_0. We see that the calculations of Nazin and Shikin give a good account of this behavior, the increasing discrepancy as the magnetic field is reduced being due to the decreasing validity of the inequality $R \gg \delta$.

Figure 2 shows a few experimentally observed mode frequencies that are not described by the theory. Those lying within the band of $M = 1$ modes may correspond to Nazin-Shikin modes with $M \geq 2$, but the modes at higher frequencies remain a puzzle.

New types of low-frequency edge modes have been discovered recently in the two-dimensional *electron* system by Kirichek *et al.* [13]. The authors state that their results are not consistent with the calculations of Nazin and Shikin.

In summary, we have observed a new family of magnetoplasma modes in a two-dimensional pool of ions trapped below the surface of superfluid helium, and we have shown that they can be identified with a new type of edge magnetoplasma wave discovered theoretically by Nazin and Shikin.

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