Possible Enhancement of Magnetic Dipole Transitions between Gamow-Teller and Isobaric Analog States

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A new decay scheme between Gamow-Teller (GT) resonances and isobaric analog states (IAS) by magnetic dipole transitions is studied. The sum rule of $M1$ transitions between IAS and GT states is found to be significantly enhanced compared to the non-energy-weighted sum rule of the parent state. Calculated enhancement factors can be as large as ~ 2.5 for ^{48}Sc and ^{90}Nb, and 1.5 for ^{208}Bi. Transition strengths between specific states are calculated in the Tamm-Dancoff approximation. The interest of measuring $M1$ transitions between IAS and GT states to obtain information on the spin-isospin response in finite nuclei is stressed.

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Spin-isospin excitations in nuclei have been extensively studied by charge-exchange reactions during the last two decades [1,2]. Experimentally, the collective Gamow-Teller (GT) resonances are well established above the isobaric analog states (IAS) in many nuclei over a wide region of the mass table. It was also found [2] that a substantial part of the model-independent sum rule strength is missing in the energy region where they are expected to be observed by theoretical predictions [3]. The effects of the coupling to two-particle–two-hole (2p-2h) states [4] and/or to the Δ-hole (Δ-h) excitations [5] have been claimed as the main cause of the missing strength. Recent theoretical studies [6] suggest the importance of the 2p-2h states and the redistribution of the transition strengths in the continuum spectra just beyond the collective GT states. It is an open essential question how one can make a decisive measurement of these missing strengths in the continuum.

In this Letter, we would like to address possible measurements of the magnetic dipole ($M1$) transition between GT resonances and IAS. As will be shown below, this transition occurs with a sizable enhancement factor and could provide additional information on the quenching problem of GT states.

Let us consider the sum rule of $M1$ transitions between IAS and GT states:

$$m_0(\mathrm{IAS} \rightarrow \mathrm{GT}) = \sum_{\mathrm{GT}} |\langle \mathrm{GT}|\hat{O}(M1)\,|\mathrm{IAS}\rangle|^2 = \langle \mathrm{IAS}|\hat{O}^2|\mathrm{IAS}\rangle = \frac{1}{2T}\langle \pi|T_+\hat{O}^2T_-|\pi\rangle, \qquad (1)$$

where $|\pi\rangle$ is the parent state $|T = T_z, T_z\rangle$, and the IAS is written as

$$|\mathrm{IAS}\rangle = \frac{1}{\sqrt{2T}}T_-|\pi\rangle, \qquad (2)$$

with $T_\pm = t_x \pm it_y = \frac{1}{2}(\tau_x \pm i\tau_y)$. The $M1$ transition operator is expressed as

$$\hat{O}(M1) = \sqrt{\frac{3}{4\pi}}\sum_i(g_{s_i}\vec{s}_i + g_{\ell_i}\vec{\ell}_i) = \sqrt{\frac{3}{4\pi}}\sum_i(g_s^{IS}\vec{s}_i + g_s^{IV}\vec{s}_i\tau_{z_i} + g_\ell^{IS}\vec{\ell}_i + g_\ell^{IV}\vec{\ell}\tau_{z_i}), \qquad (3)$$

where $g_s^{IS} = (g_s^n + g_s^p)/2$, $g_s^{IV} = (g_s^n - g_s^p)/2$ ($g_s^n = -3.826$, $g_s^p = 5.586$), $g_\ell^{IS} = 0.5$, and $g_\ell^{IV} = -0.5$. Using $[T_+, T_-] = 2T_z$ and $T_+|\pi\rangle = 0$, it can be shown that

$$\langle \pi|T_+\hat{O}^2T_-|\pi\rangle = \langle \pi[T_+,[\hat{O}^2, T_-]]\,|\pi\rangle + (N - Z)S_0, \qquad (4)$$

where the non-energy-weighted sum rule of $M1$ transitions S_0 in the parent nucleus is defined as

$$S_0 = \langle \pi|\hat{O}^2|\pi\rangle. \qquad (5)$$

We end up with the following formula after calculating the double commutator:

$$m_0(\mathrm{IAS} \rightarrow \mathrm{GT}) = \frac{1}{N - Z}\langle \pi|\{\hat{O}_+\hat{O}_- - 4\hat{O}\hat{O}_0\}\,|\pi\rangle + S_0, \qquad (6)$$

defining

$$\hat{O}_\pm \equiv \pm[\hat{O}, T_\pm] = \sqrt{\frac{3}{4\pi}}\sum_i(g_{s_i}^{IV}\vec{s}_i + g_{\ell_i}^{IV}\vec{\ell}_i)\tau_{\pm_i} \qquad (7)$$

and

$$\hat{O}_0 \equiv \sqrt{\frac{3}{4\pi}} \sum_i (g_{s_i}^{IV} \vec{s}_i + g_{\ell_i}^{IV} \vec{\ell}_i)\tau_{z_i} . \qquad (8)$$

Note that the result (6) corresponds to the neglect of the $\langle\pi|O_- O_+|\pi\rangle$ contribution, which is justified in $N \geq Z$ nuclei. The enhancement factor κ for the sum rule is defined as

$$m_0(\text{IAS} \rightarrow \text{GT}) = S_0(1 + \kappa), \qquad (9)$$

with

$$\kappa = \frac{1}{S_0(N - Z)} \langle\pi|\{\hat{O}_+\hat{O}_- - 4\hat{O}\hat{O}_0\}|\pi\rangle . \qquad (10)$$

Using standard formulas for the $M1$ transition operator $\hat{O}_\mu(M1)$ [Eq. (3)]

$$\sum_{M\mu} |\langle (j'j^{-1})1^+M|\hat{O}_\mu|\pi\rangle|^2 = \frac{3}{4\pi}(2j+1) \times \begin{cases} \frac{j+1}{j}(\frac{1}{2}g_s + \ell g_\ell)^2 & \text{for } j = j' = \ell + \frac{1}{2} \\ \frac{j}{j+1}([\ell+1]g_\ell - \frac{1}{2}g_s)^2 & \text{for } j = j' = \ell - \frac{1}{2} \\ \frac{j-\frac{1}{2}}{2j}(g_s - g_\ell)^2 & \text{for } j = j' + 1 \\ \frac{j+\frac{3}{2}}{2j+2}(g_s - g_\ell)^2 & \text{for } j = j' - 1 \end{cases} \qquad (11)$$

the enhancement factors can be easily calculated for closed-subshell parent nuclei. In the above equation g_s and g_l can be also a combination of g_s^{IV} and g_l^{IV} or g_s^{IS} and g_l^{IS}. The operator $\hat{O}_-$ induces charge-exchange excitations to the GT states and it can excite not only the $j_> \rightarrow j_<$ but also $j_> \rightarrow j_>$, $j_< \rightarrow j_<$, and $j_< \rightarrow j_>$ ($j_> \equiv \ell + \frac{1}{2}$ and $j_< \equiv \ell - \frac{1}{2}$) configurations with the isospin factor $[\tau_+, \tau_-] = 4\tau_z$. On the other hand, the operator $\hat{O}_0$ excites only $j_> \rightarrow j_<$ configurations like the $M1$ transitions in the parent nucleus, and $\hat{O} \approx \hat{O}_0$ because of the isovector dominance of the $M1$ transition. In the case of ^{48}Sc (^{90}Nb), there are contributions from the p-h configurations $f_{7/2} \rightarrow f_{7/2}, f_{5/2}$ ($g_{9/2} \rightarrow g_{9/2}, g_{7/2}$) for $\hat{O}_-$, and only $f_{7/2} \rightarrow f_{5/2}$ ($g_{9/2} \rightarrow g_{7/2}$) for $\hat{O}$ and $\hat{O}_0$. In the case of ^{208}Bi, the p-h configurations $i_{13/2} \rightarrow i_{11/2}$ and $h_{11/2} \rightarrow h_{9/2}$ contribute to $\hat{O}$ and $\hat{O}_0$ while 12 configurations contribute to $\hat{O}_-$. As is seen in the first line of Eq. (11), the excitations of $j_> \rightarrow j_>$ configurations will dominate the sum rule because of the geometrical factor. Thus, the quantity $\langle\pi|\hat{O}_+\hat{O}_- - 4\hat{O}\hat{O}_0|\pi\rangle$ in Eq. (10) becomes much larger than $4S_0$, so that the values of κ are generally much larger than $4/(N - Z)$. The calculated values are shown in Table I. We obtain large enhancement factors κ for ^{48}Sc and ^{90}Nb and a moderate enhancement factor κ for ^{208}Bi. In the latter case κ is smaller than in ^{48}Sc and ^{90}Nb because of the large $N - Z$ value in the denominator, although there are substantial contributions to the sum rule from 12 configurations.

The question is whether the above enhancement effects will be reflected, if one measures $M1$ transitions between IAS and collective GT states lying above the IAS energy. One obvious hindrance factor lies in the fact that only a fraction f_n of the total transition strength is brought by the above-mentioned collective GT states, the rest $1 - f_n$ being shared by GT states which are noncollective, or at lower energies. In order to evaluate transition strengths between IAS and specific states we have used the Tamm-Dancoff approximation (TDA) in a 1p-1h space since this model describes reasonably well the energy of the GT resonance and it fulfills the $3(N - Z)$ GT sum rule. Another hindrance factor $f_{1\text{p}1\text{h}}$ also appears since the sum rule (1) gets large enhancement from the $(2\text{p-}2\text{h})_{1^+}$ space (IAS is a 1p-1h state with respect to the 0p-0h parent state $|\pi\rangle$), while the calculated GT states are obtained within the $(1\text{p-}1\text{h})_{1^+}$ space.

We can safely assume that the GT states of the TDA model have $T_<$ isospin as they exhaust most of the GT strength [7]. The transition matrix element can be expressed as

$$\begin{aligned} \langle \text{GT}(T_<)|\hat{O}|\text{IAS}\rangle &= \frac{1}{\sqrt{2T}} \langle \text{GT}(T_<)|\hat{O}T_-|\pi\rangle \\ &= \frac{1}{\sqrt{2T}} \langle \text{GT}(T_<)|[\hat{O}, T_-] + T_-\hat{O}|\pi\rangle \\ &= -\frac{1}{\sqrt{2T}} \langle \text{GT}(T_<)|\hat{O}_-|\pi\rangle , \end{aligned} \qquad (12)$$

TABLE I. Calculated enhancement factors κ [Eq. (10)] of $M1$ transitions with respect to the non-energy-weighted sum rule S_0 in parent nuclei for ^{48}Ca, ^{90}Zr, and ^{208}Pb. S_0 is given in units of μ_N^2. Quenching factors $f_{1\text{p}1\text{h}}$ [Eq. (15)], and f_n for the collective GT states, are also shown.

π	$S_0(\mu_N^2)$	κ	$1 + \kappa$	$f_{1\text{p}1\text{h}}$	f_n	$f_{1\text{p}1\text{h}} f_n (1 + \kappa) S_0$
^{48}Ca	11.98	1.58	2.58	0.83	0.47	12.0
^{90}Zr	15.53	1.47	2.47	0.78	0.40	11.9
^{208}Pb	49.96	0.46	1.46	0.31	0.47	10.7

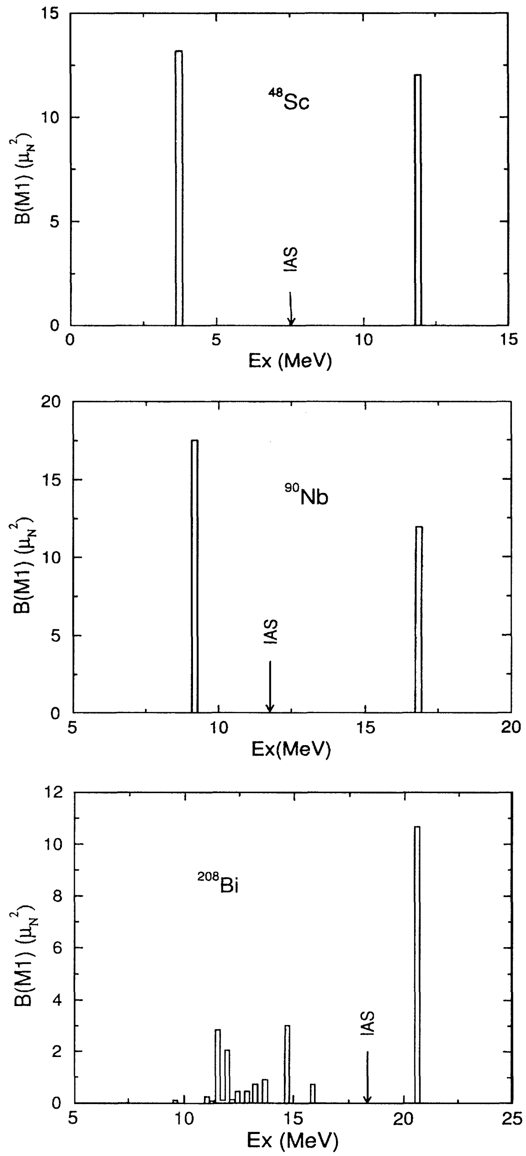

FIG. 1. Calculated $M1$ transition strengths (in units of μ_N^2) in ^{48}Sc, ^{90}Nb, and ^{208}Bi. The results are obtained by the TDA with a Skyrme interaction SGII. The arrows show the energies of IAS states calculated by the same model. The decay transition rate from GT states to the IAS can be obtained dividing the values shown by a factor of 3.

where Eqs. (2), (7), and $T_+|\mathrm{GT}(T_<)\rangle = 0$ are used. Expressing GT state vectors in terms of TDA amplitudes,

$$|n;\mathrm{GT}\rangle = \sum_{Aa} X_{Aa}^{(n)} [p_A^\dagger \tilde{n}_a]^{JM} |0\rangle , \qquad (13)$$

where $p_A^\dagger$ ($\tilde{n}_a$) is the proton-particle (neutron-hole) creation operator and $J = 1^+$, the transition strength from the IAS to a GT state, $B(n;M1)$, can be written as

$$B(n;M1) = \frac{1}{2T} \left| \sum_{Aa} X_{Aa}^{(n)*} \langle j_A || \hat{O}_- || j_a \rangle \right|^2 . \qquad (14)$$

Thus one can calculate the hindrance factor due to the restriction to 1p-1h space:

$$f_{1\mathrm{p}1\mathrm{h}} = \frac{\sum_n B(n;M1)}{S_0(1+\kappa)} , \qquad (15)$$

as well as the fraction $f_n \equiv B(n;M1)/\sum_{n'} B(n';M1)$ of $M1$ strength for each GT state. The results obtained with the Skyrme SGII interaction [8] are shown in the last three columns of Table I. Calculated energies E_n, fractions of $M1$ transition strength f_n, and $B(n;M1)$ values of the GT states are shown in Fig. 1 and Table II, as well as energies of the IAS obtained by TDA. For transition strengths from GT states to the IAS one simply has to divide by 3 the values of Table II.

In the case of ^{48}Sc (^{90}Nb), the sum of the $B(n;M1)$ to the two main GT states is $25.2\mu_N^2$ ($29.4\mu_N^2$), that is 81.5% (76.6%) of the sum rule value $S_0(1+\kappa) = 30.9\mu_N^2$ ($38.4\mu_N^2$). In these nuclei, the sum of the two $B(n;M1)$ values is really enhanced compared with S_0 by 2.10 (1.90) for $A = 48$ ($A = 90$), which corresponds to $\kappa = 1.10$ (0.90), and this enhancement is large enough to be measurable. Note that the energy of one of the GT states is lower than the IAS. Therefore, γ decays from the GT state to the IAS are about 50% (40%) of the enhancement for ^{48}Ca (^{90}Zr), and the other strengths will be measured as γ decays from the IAS to the GT states.

In the case of ^{208}Bi, the $M1$ strengths are much fragmented as shown in Fig. 1 and Table II and the sum of the $B(M1)$ values to nine GT states is $21.2\mu_N^2$, that is only 29.2% of the sum rule value $S_0(1+\kappa) = 72.9\mu_N^2$. We cannot see any enhancement of the $B(M1)$ values in ^{208}Bi compared to S_0 in ^{208}Pb [9]. However, the transition from the GT state at 20.6 MeV to the IAS has the strength $B(M1) = 10.7/3\mu_N^2$, which appears large enough for measurements. Most of the missing strength

TABLE II. Calculated energies, fraction of $M1$ transition strengths, and $B(M1)$ values of the GT ($T_<$) states obtained by TDA with the use of the SGII interaction. Calculated energies E_x of IAS obtained by TDA are also shown. All energies are referred to the parent ground state.

π	IAS E_x (MeV)	E_n (MeV)	GT($T_<$) f_n (%)	$B(n;M1)$ (μ_N^2)
^{48}Ca	7.5	3.7	51.5	13.2
		11.9	47.0	12.0
^{90}Zr	12.0	9.2	58.6	17.5
		16.8	40.0	11.9
^{208}Pb	18.6	11.5	12.4	2.8
		12.0–13.7	20.6	4.7
		14.7	13.1	3.0
		20.6	46.6	10.7

is expected to be found in the 1^+ states built on 2p-2h configurations, much more for nuclei with larger $N - Z$.

The coupling to 2p-2h states always gives quenching in the cases of magnetic transitions and GT strengths in the parent states. It is not the case, however, for the transitions between IAS and GT states since the IAS is a 1p-1h state with respect to the parent state. Actually a small portion of the 2p-2h states is taken into account in the transition matrix element (12) by the isospin projection of GT states; the $T_<$ state is an antianalog state in the case of the neutron spin-flip excitation and needs some 2p-2h components on top of the 1p-1h component of the wave function in order to construct a good isospin. These 2p-2h states are not explicitly included in the TDA calculations, but the operator $\hat{O}_-$ takes care of the effects of 2p-2h states on the transition amplitude, which give some enhancement. Most of the 2p-2h components are in $T_>$ GT states, so that the transition from the $T_>$ states to IAS will be substantial and give a unique opportunity to observe experimentally the $T_>$ GT states which have never been found so far. Theoretically it would be necessary to perform a TDA or RPA calculation in 1p-1h + 2p-2h space with isospin projection to obtain more realistic strength distributions, especially in heavy nuclei such as ^{208}Bi [10].

The coupling to Δ-h excitation has been claimed a major part of the origin of missing strength of GT transitions [5]. It is interesting to notice that the orbital part of the transition matrix elements (11) plays an important role in the present calculations on which Δ-h excitation has no effect, while only the spin part exists in GT transition and will be affected by the Δ-h excitation. Thus the effect of the Δ-h excitation on the present case might be quite different from that on the GT transition, although it is still an open question whether the effect of Δ-h excitation is very important or not.

In summary, we studied the $M1$ transition strengths between the GT states and the IAS in ^{48}Sc, ^{90}Nb, and ^{208}Bi. Substantial enhancement of the sum rule values is found in all three nuclei compared with those of the parent nuclei. The TDA calculations are also performed to obtain specific $M1$ strength distributions in these nuclei and the results show large $M1$ transition strengths between several GT states and the IAS. Measurements of these $M1$ decays could be feasible [11] and quite interesting since they might add more detailed information on the spin-isospin response problem in medium and heavy nuclei.

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[1] R. R. Doering *et al.,* Phys. Rev. Lett. **35**, 1691 (1975); D. E. Bainum *et al.,* Phys. Rev. Lett. **44**, 1751 (1980); C. D. Goodman *et al.,* Phys. Rev. Lett. **44**, 1755 (1980).
[2] C. Gaarde, Nucl. Phys. **A396**, 127c (1983).
[3] K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Lett. **3**, 271 (1963).
[4] G. F. Bertsch and I. Hamamoto, Phys. Rev. C **26**, 1323 (1982).
[5] M. Ericson, Ann. Phys. (N.Y.) **63**, 562 (1971); G. E. Brown and M. Rho, Nucl. Phys. **A372**, 397 (1981); A. Bohr and B. R. Mottelson, Phys. Lett. **100B**, 10 (1981).
[6] F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992), and references therein; S. Kamerdziev, G. Tertychny, and J. Speth, Nucl. Phys. **A569**, 313c (1994).
[7] N. Auerbach, Phys. Rep. C **98**, 273 (1983).
[8] Nguyen Van Giai and H. Sagawa, Phys. Lett. **B106**, 379 (1981).
[9] R. M. Laszewski, R. Alarcon, D. S. Dale, and S. D. Hoblit, Phys. Rev. Lett. **61**, 1710 (1988); S. I. Hayakawa *et al.,* Phys. Rev. Lett. **49**, 1624 (1982); K. Wienhard *et al.,* Phys. Rev. Lett. **49**, 18 (1982).
[10] K. Takayanagi, K. Shimizu, and A. Arima, Nucl. Phys. **A481**, 313 (1988).
[11] M. Fujiwara and M. N. Harakeh (private communication).