Quasiparticle Mean Free Path in $YBa_2Cu_3O_7$ Measured by the Thermal Hall Conductivity

K. Krishana, J. M. Harris, and N. P. Ong
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544
(Received 8 June 1995)

Asymmetric scattering of quasiparticles by pinned vortices in $YBa_2Cu_3O_7$ produces an unusually large thermal Hall conductivity κ_{xy} in the mixed state. In high fields, the κ_{xy} vs field profile displays strong curvature, which allows the zero-field mean free path l_0 of the quasiparticles to be determined. We find that l_0 undergoes a remarkable increase from 90 Å near 90 K to 2500 Å at 15 K. The scattering rate changes by a factor of 3 between 90 and 75 K. We discuss implications for the scattering mechanism. From κ_{xy} and l_0, we obtain a new estimate of the thermal conductivity of in-plane quasiparticles.

PACS numbers: 74.60.Ge, 72.15.Gd, 74.72.Bk

When a quasiparticle in a type II superconductor is incident on a pinned vortex line, the "handedness" of the superfluid velocity around the vortex core leads to asymmetric scattering. The amplitude for scattering to the right is different from that to the left. The asymmetry produces a transverse quasiparticle (QP) current that changes sign with the field **B**. The transverse cross section $\sigma_\perp$ corresponding to the asymmetric scattering was calculated by Cleary in 1968 [1]. This effect, relatively unexplored, provides an interesting way to probe QP dynamics. In $YBa_2Cu_3O_7$ (YBCO), the QP conductivity in zero field has been measured at microwave frequencies [2,3]. In strong magnetic fields, the QP current has been shown to be important in the flux-flow Hall effect [4,5], as well as in the transmission of THz radiation [6]. However, in an experiment designed to isolate the asymmetric scattering of the QP, it is desirable to avoid electrical means of excitation and detection altogether, because induced vortex motion leads to large electric fields that dominate the QP signal. These complications may be avoided by sensing thermally the deflected QP current in a weak thermal gradient.

We have observed directly the asymmetric scattering of quasiparticles in YBCO by the detection of an in-plane transverse thermal current that changes sign with the field (applied normal to the CuO_2 planes) at temperatures as low as 15 K. The transverse current is equivalent to a thermal Hall conductivity κ_{xy} (also called the "Righi-Leduc" effect). Since no electrical current is injected into the sample, the vortices remain pinned in the thermal gradient, except possibly above the "melting line" close to T_c. Even then, their contribution to κ_{xy} may be shown to be very small compared to that of the QP current. Moreover, since phonons are scattered symmetrically by the vortices, the asymmetric scattering provides a selective "filter" that allows us to observe the QP current without the phonon background. (Studies of the longitudinal conductivity of YBCO κ_{xx} in intense fields suggest that the phonon contribution is substantial [7].) We show that κ_{xy} provides a measurement of the zero-field mean free path of in-plane quasiparticles. It also provides a new estimate of the thermal conductivity $k^{e,\mathrm{pl}}$ associated these excitations [8,9]. Our κ_{xy} is a qualitatively different effect from the oscillatory transverse thermal gradient observed by Yu *et al.* when **B** is rotated within the *ab* plane [10].

Detailed measurements were taken on two twinned crystals. A thin-film resistor glued to one edge generates a thermal current $\mathbf{J}^Q \| \mathbf{x}$. The thermal gradients $-\partial_x T \| \mathbf{x}$ and $-\partial_y T \| \mathbf{y}$ are measured simultaneously with thermocouples as a function of field ($\mathbf{B} \| \mathbf{c}$) (this gives the thermal resistivities W_{xx} and W_{yx} vs B). Below 45 K, the increasing remanence distorts the Hall signal significantly. To compensate for the trapped flux, we swept the field from -14 to $+14$ T and back to -14 T, and determined the Hall signal by averaging between the two scans. The conductivity tensor $\kappa_{ij}(B)$ is obtained by inverting W_{ij}.

The Hall thermal conductivity in single-crystal YBCO is holelike in sign and anomalously large. In Fig. 1 we display traces of κ_{xy} vs the field B at fixed T. Above T_c, the thermal Hall conductivity in the normal state κ^n_{xy} is weak and linear in B up to 14 T. As the temperature falls below T_c, the weak-field Hall slope $p \equiv \lim_{B\to 0} k_{xy}/B$ increases very rapidly. Moreover, κ_{xy} develops a negative curvature that is increasingly prominent with decreasing T. The curvature occurs because scattering from the vortices reduces the QP transport lifetime τ (in addition to generating the transverse current). This scattering rate $\Gamma_{v,\mathrm{tr}} = v(\mathbf{k})\sigma_{\mathrm{tr}}|B|/\phi_0$ adds to the inelastic scattering rate Γ_{in} already present in zero field, where $\mathbf{v}(\mathbf{k}) = \partial E_{\mathbf{k}}/\hbar\partial\mathbf{k}$ is the QP velocity, σ_{tr} the transport cross section[1], and ϕ_0 the flux quantum (the QP energy $E_{\mathbf{k}}$ equals $\sqrt{[\varepsilon_{\mathbf{k}}^2 + \Delta(\mathbf{k})^2]}$ where $\varepsilon_{\mathbf{k}}$ is the normal-state energy and $\Delta(\mathbf{k})$ the superconducting gap). The asymmetric scattering rate $\Gamma_{v,a}$ is similarly related to the transverse cross section $\sigma_\perp$, viz. $\Gamma_{v,a} = v(\mathbf{k})\sigma_\perp|B|/\phi_0$. The Boltzmann equation may be solved by a variational method to give the QP thermal conductivity [11]

$$\kappa^e(B) = N(T)/[\langle\Gamma_{\mathrm{in}}\rangle + \langle v_n|\varepsilon/E|\sigma_u\rangle|B|/\phi_0], \tag{1}$$

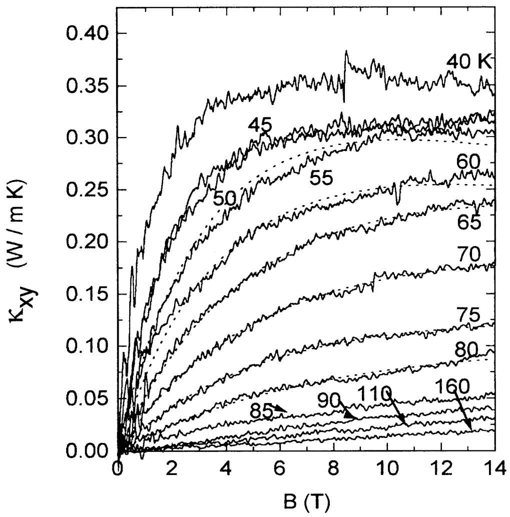

FIG. 1. The field dependence of the in-plane thermal Hall conductivity κ_{xy} measured in a twinned crystal of YBCO (size $1.35 \times 1.45 \times 0.04$ mm^3, $T_c = 91.2$ K) with $\mathbf{B}\|\mathbf{c}$. A second crystal with $T_c = 91.8$ K was also studied. The "holelike" Hall current results from asymmetric scattering of QP from pinned vortices. Curves above T_c represent the normal-state thermal Hall conductivity κ^n_{xy}. Fits to the expression $\kappa_{xy} = pB/[1 + \alpha|B|/\phi_0]^2$ at each T determine the two parameters p and α (broken lines). With a heater power of ~1 mW, the typical value of $\Delta T(\|\mathbf{x})$ is 1 K. The thermocouples (chromel-constantan) were calibrated in field using a piece of nylon (at 14 T the correction is 3% at 20 K).

with $N(T) = (4/T)\sum_\mathbf{k} \varepsilon^2_\mathbf{k}(-\partial f^0_\mathbf{k}/\partial E_\mathbf{k})v_n(\mathbf{k})^2 \cos^2 \phi_\mathbf{k}$ [here $\mathbf{v}_n(\mathbf{k}) = \partial\varepsilon_\mathbf{k}/\hbar\partial\mathbf{k}$, and $\phi_\mathbf{k}$ is the angle between $\mathbf{v}$ and the x axis]. The brackets $\langle\cdots\rangle$ indicate an average over E and $\mathbf{k}$ around the Fermi surface FS [11]. Equation (1) may be seen to be equivalent to Eq. (5.41) of Kadanoff and Martin (KM) [12] by noting that $(\sigma_{\rm tr}|B|/\phi_0)^{-1}$ is the MFP associated with scattering from vortices. Cleary [1] has shown that $\sigma_{\rm tr}$ diverges as $|\varepsilon/E|^{-1}$ when $E \to \Delta^+$. Thus the product $v_F\sigma'_{\rm tr} \equiv \langle v_n|\varepsilon/E|\sigma_{\rm tr}\rangle$ is only weakly dependent on E (v_F is the average Fermi velocity). We will assume that $\sigma'_{\rm tr}$ is a T-independent constant and write the denominator in Eq. (1) as $\langle\Gamma_{\rm in}\rangle[1 + l_0\sigma'_{\rm tr}|B|/\phi_0]$, where the mean free path in *zero field* l_0 is defined as $v_F/\langle\Gamma_{\rm in}\rangle$. By noting that the fraction of the incident beam scattered into the transverse direction equals $\langle\Gamma_{v,a}\rangle/[\langle\Gamma_{\rm in}\rangle + \langle\Gamma_{v,\rm tr}\rangle]$, we obtain the Hall conductivity (here $v_F\sigma'_\perp \equiv \langle v_n|\varepsilon/E|\sigma_\perp\rangle$)

$$\kappa_{xy}(B) = N(T)l_0^2(\sigma'_\perp/v_F\phi_0)B/[1 + l_0\sigma'_{\rm tr}|B|\phi_0]^2. \tag{2}$$

Equation (2), written as $\kappa_{xy} = pB/[1 + \alpha|B|/\phi_0]^2$, provides rather close fits to the curves in Fig. 1 with only two adjustable parameters (p and α). Values of $p(T)$ from the fit are displayed in the main panel of Fig. 2. Starting at relatively small values above T_c, $p(T)$ rises abruptly at T_c (expanded scale) and eventually attains a peak value ~80 times larger then its value at T_c. This pronounced anomaly is reminiscent of the microwave conductivity $\sigma_1(\omega)$ [2,3]. We show next that the steep increase in $p(T)$ is related to the mean free path. The second fit parameter α, which measures the average QP lifetime ($\alpha = l_0\sigma'_{\rm tr}$), is shown in Fig. 3 (right scale). We remark that ϕ_0/α corresponds to the field scale that fixes the curvature of the traces in Fig. 1. Thus α is derived directly from the raw data between 15 and 85 K. To convert α to l_0, we need the transport cross section $\sigma'_{\rm tr}$. A previous analysis of the flux-flow Hall conductivity σ_{xy} [4] shows that $\sigma'_{\rm tr}$ varies weakly from 25 to 30 Å between 90 and 92 K. Taking $\sigma'_{\rm tr}$ to have the T-independent value 25 Å, we obtain the curve for l_0 in Fig. 3 (left vertical scale). In the Meissner state, l_0 undergoes a steep increase from ~90 Å at 92 K to 2500 Å at 15 K. This striking increase is responsible for the sharp upturn in p, as well as the large observed values of κ_{xy}.

Knowledge of p and l_0 at each temperature allows us to examine the temperature dependence of the interesting parameter $N(T)$ which measures the entropy current with the lifetime divided out. We express $N(T)$ as

$$N(T)/v_F = \kappa^e(0)/l_0 = (\phi_0/\sigma'_\perp)p(T)l_0^{-2} \tag{3}$$

[the first equality follows from Eq. (1) and the second from Eq. (2)]. Forming the product of $p(T)$ with l_0^{-2},

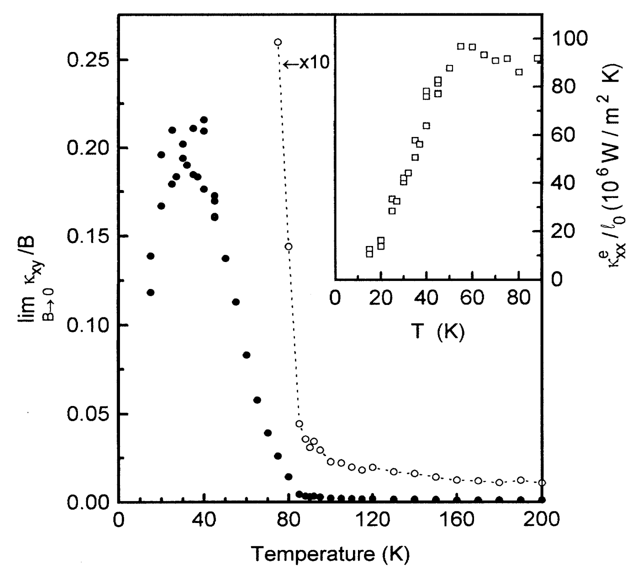

FIG. 2. (Main panel) The temperature dependence of the fit parameter $p \equiv \lim_{B\to 0}\kappa_{xy}/B$, which represents the weak-field Hall conductivity per tesla (solid symbols). The data above T_c represents the equivalent quantity in the normal state κ^n_{xy}/B. The expanded plot (open symbols) shows that p increases by an order of magnitude between 90 and 75 K. The inset plots the quantity $\kappa^{e,\rm pl}(0)/l_0 = N(T)/v_F$ (the numerical scale is fixed by assuming $\sigma'_\perp = 3.9$ Å). Note that $N(T)/v_F$ is weakly T dependent above 70 K.

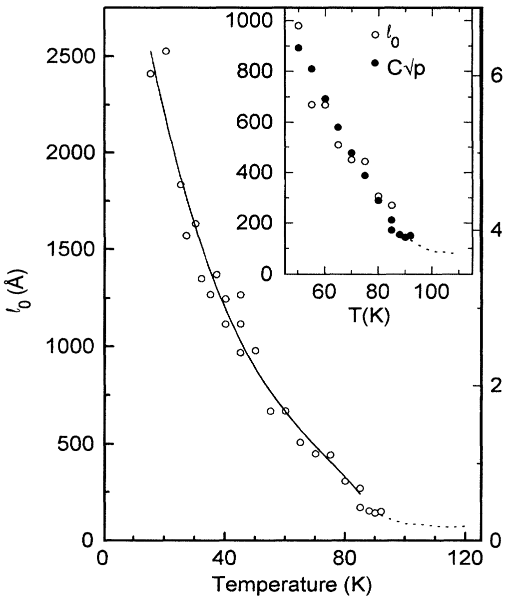

FIG. 3. (Main panel) Plot of the fit parameter α (right scale) which is the inverse of the field scale determining the curvature in Fig. 1. The zero-field mean free path l_0 (left scale) is calculated from $\alpha = l_0\sigma'_{\rm tr}$, using $\sigma'_{\rm tr} = 25$ Å. Broken lines above T_c represent the MFP estimated from the in-plane resistivity ρ_n. The inset compares the l_0 data derived from α (open symbols) with $C\sqrt{p(T)}$ (solid symbols). The scale factor C is chosen so that the two data sets agree in the interval 70–80 K.

we observe that the plot of $N(T)/v_F$ rises rapidly with T and then saturates to a plateau value above 70 K (inset of Fig. 2). [The numerical scale for $N(T)/v_F$ depends on $\sigma'_\perp$, which we assume is a constant (for a spherical FS, $\sigma'_\perp = \pi/k_F$). We adopted the value $\sigma'_\perp \sim 3.9$ Å using the average Fermi wave vector $\langle k_F\rangle = 8.0 \times 10^9$ m^{-1} estimated from angle-resolved photoemission results.] To compare with low-T_c superconductors, we note that $N(T)$ is equivalent to the ratio κ_s/κ_n discussed by KM [12]. Close to T_c in high-purity samples of Sn and Pb [13], κ_s/κ_n decreases linearly with increasing reduced temperature $t = (1 - T/T_c)$. In contrast, our curve for $N(T)$ is only weakly T dependent above $\sim$ 70 K, which suggests that a large population of QP with long MFP persists deep into the Meissner state. The weak T dependence of $N(T)$ in this "plateau" region is noteworthy, when we recall that both $p(T)$ and l_0 vary strongly with temperature. This implies that, at the plateau, the T dependence of $p(T)$ must almost match that of l_0^2, i.e., $l_0 \sim \sqrt{p(T)}$ for 70 K $< T < T_c$ [see Eq. (3)]. Thus we have an alternate way to find l_0 (above 70 K), as well as a check for self-consistency. In the inset of Fig. 3, we compare l_0 obtained from α with $\sqrt{p(T)}$ scaled to agree in the interval 70–80 K. [Above 80 K, α is increasingly uncertain because κ_{xy} loses its curvature. The $\sqrt{p(T)}$ method provides a better way to extend the data for l_0 from 80 K to T_c.]

The temperature dependence of $N(T)$ and l_0 also gives the profile of the zero-field thermal conductivity associated with the in-plane quasiparticles $\kappa^{e,{\rm pl}}(0) = N(T)l_0/v_F$ (again, with our choice for $\sigma'_\perp$). The total conductivity $\kappa^{\rm tot}$ measured in our crystal displays the familiar peak near 40 K (main panel of Fig. 4). The issue whether the peak in $\kappa^{\rm tot}$ arises mostly from quasiparticles or from phonons is controversial [8,9]. The QP conductivity $\kappa^{e,{\rm pl}}(0)$ derived from the Hall current is shown in Fig. 4 as open circles. From a relatively small value in the normal state, $\kappa^{e,{\rm pl}}(0)$ rises to a peak comparable in size to that $\kappa^{\rm tot}$, implying that a large fraction of the anomaly is from QP excitations. Thus our data degree with Cohn *et al.* [9] who propose that the QP current remains small in the Meissner state. Given the tenfold increase in l_0 between 90 and 45 K, it seems compelling that a significant enhancement of the QP conductivity must occur below T_c. Our results are in better qualitative agreement with Yu *et al.* [8], although there are important quantitative differences. The ratio of the peak value of $\kappa^{e,{\rm pl}}(0)$ to its value above T_c is larger than that of Yu *et al.* ($\sim$6 vs 3.5). The temperature dependence of $1/l_0$ is incompatible with the power law $(T/T_c)^n$ proposed by Yu *et al.* ($n \sim$ 4–5), as discussed below.

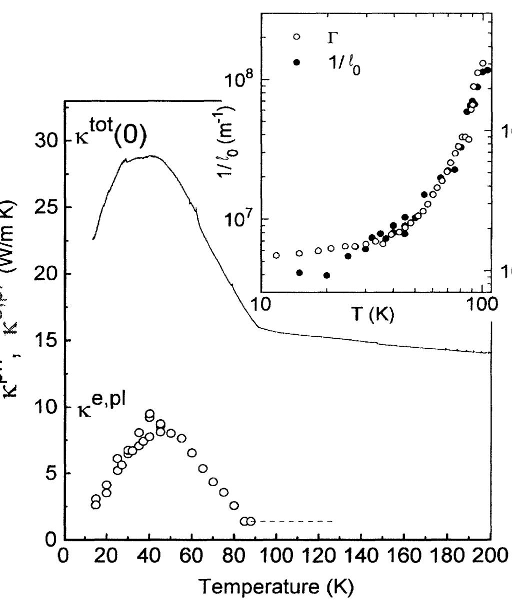

FIG. 4. (Main panel) The T dependence of the measured in-plane thermal conductivity $\kappa^{\rm tot}(0)$ (solid line) and the conductivity associated with the in-plane quasiparticles $\kappa^{e,{\rm pl}}$ (open circles), both in zero field. $\kappa^{e,{\rm pl}}$ is calculated from κ_{xy} in Fig. 1 and the values of l_0 in Fig. 3 assuming $\sigma'_\perp = 3.9$ Å. The inset plots l_0^{-1} vs T in log-log scale (closed symbols). Data on $\Gamma = 1/\tau$ from Ref. [3] on a crystal doped with 0.3% Zn are shown as open symbols (the plots suggest $v_F = 2 \times 10^5$ m/s).

While our results established the existence of a large electronic peak, we also obtain evidence of a large phonon current. The magnetoconductance data (κ^{tot} vs B) display a behavior qualitatively distinct from that of κ_{xy}. We report elsewhere [14] our analysis showing that the field variation of κ^{tot} is dominated by the scattering of phonons by vortices (see also Ref. [7]). Because both currents (QP and phonons) are so strongly field dependent, one cannot hope to disentangle the two using $\kappa^{\mathrm{tot}}(B)$ alone. In contrast, the Hall conductivity is more incisive since it senses the in-plane QP current only.

Since $\langle\Gamma_{\mathrm{in}}\rangle$ equals v_F/l_0, we may compare the T dependence of $1/l_0$ with that of the QP scattering rate $1/\tau$, a quantity of central interest in the superconducting state. Previously, Bonn *et al.* [2,3] used a two-fluid model to extract $1/\tau$ from the microwave surface resistance R_s. In the inset of Fig. 4, we compare our data plotted as $1/l_0$ (solid symbols) with that of Bonn *et al.* for a crystal lightly doped with Zn to 0.3% (open). There is rather good agreement overall except below 20 K (in an undoped, untwinned crystal in Ref. [3], the T dependence of $1/\tau$ is steeper). Equating our $\langle\Gamma_{\mathrm{in}}\rangle$ to $1/\tau$ gives an estimate for the average v_F equal to 2×10^5 m/s. An interesting feature of the two sets of data is the very steep decrease in scattering rate just below T_c (much steeper than estimated in Refs. [6,8]). Within the interval 90–75 K, our results show that $\Gamma_{\mathrm{in}} \sim 1/l_0$ decreases by ~ 3 (main panel of Fig. 2).

We discuss next the implications for carrier scattering, taking the gap function $\Delta(\mathbf{k})$ to be highly anisotropic, as in d-wave pairing. At low T, the QP population is restricted to the four nodes of $\Delta(\mathbf{k})$ on the FS. If the average phonon momentum $\hbar q$ is insufficient to span the distance between adjacent nodes, phonon scattering is severely restricted (we estimate $q \sim \pi/6a$ at 50 K). While this "final-state" argument may apply to the data below 50 K, it is irrelevant to the jump observed in Γ_{in} near T_c, where the thermal spread T is comparable to or larger than the maximum value of $\Delta(\mathbf{k})$. It is difficult to see how the final-state argument can apply to the steep change observed near T_c. The data are more consistent with models in which the dominant scattering mechanism above T_c is electronic in origin. The sharp drop in Γ_{in} suggests that the onset of superconductivity strongly affects the matrix element responsible for scattering in the normal state, most likely, by the opening of a gap Δ_{exc} in the spectrum of the electronic excitation responsible for the scattering. These measurements may provide sufficient accuracy to help distinguish between various models of electron scattering.

Finally we note that the large values of l_0 place strong constraints on calculations of the normal-state resistivity ρ_n in YBCO. Some calculations using conventional transport theory rely on a sizeable impurity-dominated resistivity ρ_{imp} to shift the calculated curve in order to match the observed linear-T profile. Within conventional theory, we may calculate the bound $\rho_{\mathrm{imp}} < 2\ \mu\Omega$ cm (taking $\rho_n = 50\ \mu\Omega$ cm at 95 K, and the impurity-scattering MFP $l_{\mathrm{imp}} > 2000$ Å). This bound is ten times smaller than the value $\rho_{\mathrm{imp}} \sim 20\ \mu\Omega$ cm apparently needed to make such schemes viable [15]. Thus our MFP data preclude these large-ρ_{imp} scenarios.

We acknowledge useful discussions with H. Fukuyama, Dima Khveshchenko, H. R. Ott, and T. M. Rice. This research is supported by the U.S. Office of Naval Research (Contract No. N00014-90-J-1013).

[1] Robert M. Clearly, Phys. Rev. **175**, 587 (1968).
[2] D. A. Bonn, P. Dosanjh, R. Liang, and W. N. Hardy, Phys. Rev. Lett. **68**, 2390 (1992); D. A. Bonn *et al.*, Phys. Rev. B **47**, 11 314 (1993); Martin C. Nuss, P. M. Mackiewich, M. L. O'Malley, E. H. Westerwick, and Peter B. Littlewood, Phys. Rev. Lett. **66**, 3305 (1991).
[3] D. A. Bonn, S. Kamal, Kuan Zhang, Ruixing Liang, D. J. Baar, E. Klein, and W. N. Hardy, Phys. Rev. B **50**, 4051 (1994).
[4] J. M. Harris, N. P. Ong, P. Matl, R. Gagnon, L. Taillefer, T. Kimura, and K. Kitazawa, Phys. Rev. B **51**, 12 053 (1995), and references therein.
[5] D. M. Ginsberg and J. T. Manson, Phys. Rev. B **51**, 515 (1995).
[6] S. Spielman *et al.*, Phys. Rev. Lett. **73**, 1537 (1994).
[7] R. A. Richardson, S. D. Peacor, C. Uher, and Franco Nori, J. Appl. Phys. **72**, 4788 (1992); S. D. Peacor, J. L. Cohn, and C. Uher, Phys. Rev. B **43**, 8721 (1991).
[8] R. C. Yu, M. B. Salamon, Jian Ping Lu, and W. C. Lee, Phys. Rev. Lett. **69**, 1431 (1992).
[9] J. L. Cohn, V. Z. Kresin, M. E. Reeves, and S. A. Wolf, Phys. Rev. Lett. **71**, 1657 (1993).
[10] F. Yu, M. B. Salamon, A. J. Leggett, W. C. Lee, and D. M. Ginsberg, Phys. Rev. Lett. **74**, 5136 (1995). The "Hall-like" transverse gradient observed by Yu *et al.* is unrelated to Hall scattering since it does not change sign with **B**. In their geometry ($\mathbf{B}\perp\mathbf{c}$), a Hall effect should produce a gradient $\|\mathbf{c}$.
[11] N. P. Ong (unpublished).
[12] Leo P. Kadanoff and Paul C. Martin, Phys. Rev. **124**, 670 (1961).
[13] For high-purity Pb, see M. H. Jericho, W. Odoni, and H. R. Ott, Phys. Rev. B **31**, 3124 (1985).
[14] K. Krishana, J. M. Harris, and N. P. Ong (to be published).
[15] A critical assessment of resistivity calculations is given by R. Hlubina and T. M. Rice (to be published).