Oblique Confinement and Phase Transitions in Chern-Simons Gauge Theories

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We investigate nonperturbative features of a planar lattice Chern-Simons gauge theory modeling the physics of Josephson junction arrays. By identifying the relevant topological configurations and their interactions, we determine the phase structure of the model. Our results match observed phase transitions in Josephson junction arrays and suggest also the possibility of *oblique confining* ground states corresponding to purely planar quantum Hall regimes for either charges or vortices.

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Planar gauge fields play an important role in effective field theories describing the low-energy degrees of freedom of two-dimensional condensed matter systems [1]. When the discrete P and T symmetries are either explicitly or spontaneously broken, the dynamics of the gauge fields is usually governed by the topological Chern-Simons term. In particular, the theory with Lagrangian (we use units $c = 1$, $\hbar = 1$)

$$\mathcal{L} = \frac{\kappa}{\pi} A_\mu^{\text{e.m.}} \epsilon^{\mu\alpha\nu} \partial_\alpha B_\nu + \frac{\eta}{\pi} B_\mu \epsilon^{\mu\alpha\nu} \partial_\alpha B_\nu \qquad (1)$$

has been proposed [2] as the effective field theory describing the long distance physics of *chiral incompressible fluids*. Here, the conserved current $j^\mu \equiv (\kappa/\pi)\epsilon^{\mu\alpha\nu}\partial_\alpha B_\nu$ describes charged matter fluctuations above the ground state. The first term in (1) is the standard electromagnetic coupling, while the second term describes the kinetic term for matter fluctuations. This can be written as a nonlocal Hopf interaction; the introduction of the effective pseudovector gauge field B_μ allows us, however, to avoid nonlocal terms in the effective field theory [3]. The theory (1) describes a *quantum Hall regime* [4], as is easily recognized by integrating out the matter degrees of freedom and computing the current induced by a constant, uniform, electric field. This is a purely transverse Hall current with Hall conductivity given by $\sigma_H = \kappa^2/2\pi\eta$, from which one identifies κ as the charge unit and η as the filling fraction [5].

In this paper we consider an effective Abelian gauge theory describing the coupled fluctuations of *charges* and *vortices* in a purely planar system. The model is formulated in terms of a vector gauge field A_μ [not to be confused with the electromagnetic gauge field $A_\mu^{\text{e.m.}}$ in (1)] and a pseudovector gauge field B_μ. The charge and vortex fluctuations are described by the conserved currents $j^\mu = (\kappa/\pi)\epsilon^{\mu\alpha\nu}\partial_\alpha B_\nu$ and $\phi^\mu = (\kappa/\pi)\epsilon^{\mu\alpha\nu}\partial_\alpha A_\nu$, respectively. The dynamics of these fluctuations is governed by the Lagrangian

$$\mathcal{L} = -\frac{1}{2e^2} F_\mu F^\mu + \frac{\kappa}{\pi} A_\mu \epsilon^{\mu\alpha\nu}\partial_\alpha B_\nu - \frac{\eta}{\kappa e^2} F_\mu f^\mu - \frac{1}{2g^2} f_\mu f^\mu + \frac{\eta}{\pi} B_\mu \epsilon^{\mu\alpha\nu}\partial_\alpha B_\nu , \qquad (2)$$

where $F^\mu \equiv \epsilon^{\mu\alpha\nu}\partial_\alpha A_\nu$ and $f_\mu \equiv \epsilon^{\mu\alpha\nu}\partial_\alpha B_\nu$ are the dual field strengths associated with the two gauge fields.

In order to explain the physics of this model, let us first analyze the case $\eta = 0$. In this case, the only interaction between charges and vortices is encoded in the mixed Chern-Simons term. This represents both the Lorentz force exerted by the vortices on the charges and the Magnus force [6] exerted by the charges on the vortices. Charge-charge and vortex-vortex interactions are best exposed in the Coulomb gauge Hamiltonian derived from (2):

$$H = \int d^2\mathbf{x} \left\{ j^0 \left(\frac{e^2}{2} \frac{1}{-\nabla^2} + \frac{\pi^2}{2\kappa^2 g^2} \right) j^0 + \phi^0 \left(\frac{g^2}{2} \frac{1}{-\nabla^2} + \frac{\pi^2}{2\kappa^2 e^2} \right) \phi^0 \right\} + \int d^2\mathbf{x} \left\{ \frac{\pi^2}{2\kappa^2 g^2} \mathbf{j}_L^2 + \frac{\pi^2}{2\kappa^2 e^2} \phi_L^2 \right\}, \qquad (3)$$

where j_L^i and ϕ_L^i denote the longitudinal components of the charge and vortex current densities, respectively. The first two terms represent long-range Coulomb interactions between charges and vortices. The second two terms represent the kinetic terms for charge and vortex motion, respectively. For $\eta = 0$, our simple gauge theory encodes thus the essential physics of Josephson junction arrays [7] (in the limit where the junction capacitance dominates, $C \gg C_0$) upon identifying the two mass parameters e^2 and g^2 with the charging energy E_C and the Josephson coupling E_J as follows: $E_C = e^2/4$, $E_J = g^2/2\pi^2$. The only difference lies in the kinetic term for the vortices, which is absent in the Hamiltonian describing the arrays: It is the absence of this term that breaks the perfect self-duality of (2) (for $\eta = 0$) under the transformation $A_\mu \leftrightarrow B_\mu$, $e \leftrightarrow g$ in the model

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describing the arrays. By a simple diagonalization, it is easy to see that the propagating modes described by (2) (for $\eta = 0$) form a parity and spin (± 1) doublet with topological mass [8] $m = |\kappa| eg/\pi$. With the above identification of parameters, and for $\kappa = 1$ (see below), this mass coincides with the plasma frequency $\sqrt{8E_C E_J}$ of the array. While the spectrum of the propagating modes in Josephson junction arrays is not necessarily relativistic, our relativistic model correctly reproduces the gap. Our model can thus be viewed as a relativistic, perfectly self-dual model encoding the essential aspects of the physics of Josephson junction arrays, exactly as the familiar relativistic Abelian Higgs model reproduces the essential features of superconductivity. This will be prominently reflected in the phase structure of the model.

Let us now describe the additional terms proportional to η. The coefficient of the $F_\mu f^\mu$ term is chosen such that the bare and induced Chern-Simons terms cancel in the effective action for the matter, obtained by integrating out the vortex degrees of freedom A_μ. On the contrary, the vortex effective action, obtained by integrating out the matter degrees of freedom B_μ, contains a Chern-Simons term with coefficient $\kappa^2/4\pi\eta$ for the gauge field A_μ. The model with $\eta \neq 0$ describes thus, at least for a certain range of parameters, a *quantum Hall regime* for the vortices. Given that the Magnus force is the dual of the Lorentz force, and based on the analogy with effective gauge theories of the quantum Hall effect [2], we expect such a regime to be realized in Josephson junction arrays in the presence of n offset charges and ϕ magnetic fluxes per plaquette in the ratio $\phi/n \leq O(1)$. Contrary to the situation in the quantum Hall effect, this would be a purely planar quantum Hall regime, with logarithmic interactions [9].

The model with $\eta \neq 0$ is related to the model with $\eta = 0$ by a simple transformation of parameters:

$$\mathcal{L}(A_\mu, B_\mu; \eta, e, g) = \mathcal{L}\left(A_\mu + \frac{\eta}{\kappa} B_\mu, B_\mu; \eta = 0, e, g' = \frac{g}{\sqrt{1 - \eta^2 g^2/\kappa^2 e^2}}\right). \tag{4}$$

Here, $E'_J \equiv g'^2/2\pi^2$ has to be considered as a renormalized (by the external offset charges) Josephson coupling. Correspondingly, the topological mass becomes $m = |\kappa| eg'/\pi$ (one has to choose $\eta g < \kappa e$ in order to avoid $m^2 < 0$).

In the described application of planar gauge theories to the physics of Josephson junction arrays, the two Abelian gauge symmetries of (2) have to be taken as *compact;* i.e., the gauge groups are U(1) rather than R. Indeed, the independence of the two energy scales $E_C = e^2/4$ and $E_J = g^2/2\pi^2$ is a consequence of the compactness of the two gauge groups, which fixes the normalization of the gauge fields. In the rest of this paper we shall investigate the *phase structure* of a compact version of (2) formulated on the lattice. Specifically, we shall study the crucial role of the Euclidean *topological excitations* (instantons) due to the compactness of the gauge groups [10].

To this end we introduce a cubic lattice with lattice spacing l and lattice sites denoted by $\mathbf{x}$. In addition to the usual forward and backward lattice derivatives d_μ and $\hat{d}_\mu$ [11], we introduce the forward and backward lattice shift operators $S_\mu f(\mathbf{x}) \equiv f(\mathbf{x} + \hat{\mu} l)$ and $\hat{S}_\mu f(\mathbf{x}) \equiv f(\mathbf{x} - \hat{\mu} l)$, where $\hat{\mu}$ denotes a unit vector in direction μ. To each link $\{\mathbf{x}, \mu\}$ of the lattice we assign two real gauge fields denoted by $A_\mu(\mathbf{x})$ and $B_\mu(\mathbf{x})$. These are compact variables defined on a circle of radius $2\pi/l$: $-\pi/l < A_\mu$, $B_\mu < \pi/l$. Following [12] and [11], we define the following two lattice Chern-Simons operators:

$$K_{\mu\nu} \equiv S_\mu \epsilon_{\mu\alpha\nu} d_\alpha, \quad \hat{K}_{\mu\nu} \equiv \epsilon_{\mu\alpha\nu} \hat{d}_\alpha \hat{S}_\nu, \tag{5}$$

where there is no summation over equal indices μ and ν. These operators are gauge invariant, $K_{\mu\nu} d_\nu = \hat{d}_\mu K_{\mu\nu} = 0$, $\hat{K}_{\mu\nu} d_\nu = \hat{d}_\mu \hat{K}_{\mu\nu} = 0$, and their product reproduces the Euclidean lattice Maxwell operator,

$$K_{\mu\alpha} \hat{K}_{\alpha\nu} = \hat{K}_{\mu\alpha} K_{\alpha\nu} = -\delta_{\mu\nu} \nabla^2 + d_\mu \hat{d}_\nu, \tag{6}$$

where $\nabla^2 \equiv \hat{d}_\mu d_\mu$ is the three-dimensional, Euclidean Laplace operator on the lattice. Note also that the two operators $K_{\mu\nu}$ and $\hat{K}_{\mu\nu}$ are interchanged upon a summation by parts on the lattice.

In order to take into account the periodicity of the link variables A_μ and B_μ we introduce four sets of integer link variables and we posit the following lattice Euclidean version of the Villain type [13] of (4):

$$Z = \sum_{\substack{\{n_\mu\},\{l_\mu\} \\ \{m_\mu\},\{k_\mu\}}} \int_{-\pi/l}^{\pi/l} \mathcal{D}A_\mu \, \mathcal{D}B_\mu \exp(-S),$$

$$\begin{aligned} S = \sum_{\mathbf{x},\mu} & \frac{l^3}{2e^2} \left[\left(F_\mu + \frac{2\pi}{l^2} n_\mu \right) + \frac{\eta}{\kappa} \left(f_\mu + \frac{2\pi}{l^2} k_\mu \right) \right]^2 \\ & - i \frac{l^3 \kappa}{\pi} \left[\left(A_\mu + \frac{2\pi}{l} l_\mu \right) + \frac{\eta}{\kappa} \left(B_\mu + \frac{2\pi}{l} m_\mu \right) \right] \\ & \times K_{\mu\nu} \left(B_\nu + \frac{2\pi}{l} m_\nu \right) + \frac{l^3}{2g'^2} \left(f_\mu + \frac{2\pi}{l^2} k_\mu \right)^2, \end{aligned} \tag{7}$$

where we have introduced the notation $\mathcal{D}A_\mu \equiv \prod_{\mathbf{x},\mu} dA_\mu(\mathbf{x})$ and we define the lattice dual field strengths as $F_\mu \equiv K_{\mu\nu} A_\nu$ and $f_\mu \equiv K_{\mu\nu} B_\nu$. Due to the property (6), the terms $\sum_{\mathbf{x},\mu} F_\mu^2$, and $\sum_{\mathbf{x},\mu} f_\mu^2$ reproduce the familiar lattice Maxwell action. The partition function (7) is clearly invariant under shifts $A_\mu \to A_\mu + 2\pi i_\mu/l$ and $B_\mu \to B_\mu + 2\pi j_\mu/l$ with integers i_μ and j_μ, since these can be reabsorbed by a redefinition of the integer link variables n_μ, m_μ, l_μ, and k_μ. With our normalizations, the parameter 2κ plays the role of the charge unit in the theory.

The integer link variables implementing the periodicity have an important consequence. Indeed, using the Poisson summation formula [13] one recognizes easily that the sum over $\{l_\mu\}$ enforces the constraint $l^2 2\kappa K_{\mu\nu}$ (B_ν +

$2\pi m_\nu/l) = 2\pi\beta_\mu$, $\beta_\mu \in Z$ for all values of B_μ and m_μ. This has the immediate consequence of requiring a quantization condition on the parameter κ: $\kappa = n/2$, $n \in Z$. In the following we choose $\kappa = p \in Z$, so that the charge unit is an integer multiple of 2, representing the charge of the Cooper pairs of Josephson junction arrays. Moreover, we shall specialize to the cases $\eta = 0$ and $\eta = \kappa$.

The exponential $\exp(-S)$ is invariant under gauge transformations $A_\mu \to A_\mu + d_\mu\Lambda$ only if Λ takes the values $\Lambda = (2\pi/2p)n$, $n \in Z_{2p}$ at infinity. The same holds true for gauge transformations $B_\mu \to B_\mu + d_\mu\Lambda$. This means that both *global* gauge symmetries are actually broken down to *discrete Z_{2p} symmetries.* This is a known phenomenon in compact Chern-Simons theories [14].

We now rewrite (7) in a fashion that exposes explicitly the topological configurations and their interactions. To this end we decompose n_μ and k_μ as $n_\mu \equiv lK_{\mu\nu}l_\nu + a_\nu$, $k_\mu \equiv lK_{\mu\nu}m_\nu + b_\nu$, with a_μ and b_μ integers. Correspondingly, the sum over all configurations $\{n_\mu\}$ and $\{k_\mu\}$ in (7) can be replaced by a sum over all configurations $\{a_\mu\}$ and $\{b_\mu\}$. By changing variables $A_\mu \to A_\mu + (2\pi/l)l_\mu$ and $B_\mu \to B_\mu + (2\pi/l)m_\mu$ in the integration and performing the sum over all configurations $\{l_\mu\}$ and $\{m_\mu\}$, we can extend the integration domain of the variables A_μ and B_μ from $[-\pi/l, +\pi/l]$ to $[-\infty, +\infty]$. In a last step we perform the Gaussian integration of the model in the form $Z = Z_0 \cdot Z_{\rm top}$, where Z_0 is the partition function for the noncompact version of the model (describing only the propagating modes) and $Z_{\rm top}$ is the contribution of the topological excitations:

$$Z_{\rm top} = \sum_{\substack{\{a_\mu\}\\ \{b_\mu\}}} \exp\Bigg\{\sum_{\mathbf{x},\mu}\Bigg[-\frac{2\pi^2}{le^2}\left(a_\mu + \frac{\eta}{p}b_\mu\right)\frac{m^2\delta_{\mu\nu} - d_\mu\hat{d}_\nu}{m^2 - \nabla^2}\left(a_\nu + \frac{\eta}{p}b_\nu\right) - \frac{2\pi^2}{lg'^2}b_\mu\frac{m^2\delta_{\mu\nu} - d_\mu\hat{d}_\nu}{m^2 - \nabla^2}b_\nu - i\frac{4\pi p}{l}b_\mu\frac{\hat{K}_{\mu\nu}}{m^2 - \nabla^2}\left(a_\nu + \frac{\eta}{p}b_\nu\right)\Bigg]\Bigg\}. \tag{8}$$

The stringlike topological excitations a_μ and b_μ originate as the "integer parts" of F_μ and f_μ, respectively, and have thus the obvious interpretation of *magnetic flux strings* and *electric flux strings.* The strings can be closed, in which case $\hat{d}_\mu a_\mu = 0$ and $\hat{d}_\mu b_\mu = 0$, or open, in which case they terminate on monopole-antimonopole pairs. In our Euclidean formalism, these monopoles describe tunneling events corresponding to the creation and destruction of $2p$ localized charges or fluxes. Charges and fluxes are indeed conserved only modulo $2p$ due to the discrete global gauge symmetry Z_{2p}.

The nature of the ground state of our original planar quantum model is thus determined by the three-dimensional statistical mechanics of a coupled gas of strings with "Hamiltonian" βH given by $S_{\rm top}$ in (8). In order to establish what are the possible ground states, we shall use the same free energy arguments adopted in the analyses of related three-dimensional models [15]. One assigns a free energy

$$\beta F = \left[\frac{2\pi^2}{le^2}(ml)^2G(ml)\left(a + \frac{\eta}{p}b\right) + \frac{2\pi^2}{lg'^2}(ml)^2G(ml)\,b^2 - \mu\right]N, \tag{9}$$

to a string of length $L = lN$ carrying magnetic and electric quantum numbers a and b. Here, $G(ml)$ is the diagonal element of the lattice kernel $G(\mathbf{x} - \mathbf{y})$ representing the inverse of the operator $l^2(m^2 - \nabla^2)$. The last term in (9) represents the entropy of the string: The parameter μ is given roughly by $\mu = \ln 5$, since at each step the string can choose among five different directions. In a dilute instanton approximation in which all values $a_\mu, b_\mu \geq 2$ are neglected, it can be proved that the correct value of μ is the same for open and closed strings [16]. In (9) we have neglected all subdominant functions of N, like a $\ln N$ correction to the entropy and a constant term due to the monopole contribution to the energy for open strings. Moreover, we have neglected the imaginary term in the action (8). This is justified self-consistently, since the contribution of this term vanishes in all phases of the model, as we now show.

Long strings with quantum numbers a and b condense in the ground state if the coefficient of N in (9) is negative. When two or more condensates are possible, one has to choose the one with the lowest free energy. Upon defining the new parameter $\delta(ml) \equiv \mu/(ml)^2G(ml)$, the condensation condition describes thus the interior of an ellipse with semiaxes $le^2\delta/2\pi^2$ and $lg'^2\delta/2\pi^2$ on a nonrectilinear lattice of integer magnetic and electric charges. The phase diagram is thus obtained by investigating which points of the integer lattice lie inside the ellipse as its semiaxes (i.e., the parameters in the theory) are varied. We find it convenient to present the result in terms of the parameters lm and e/g'. For $\eta = \kappa$ we obtain

$$\begin{aligned}
&\frac{\delta lm}{2\pi p} > 1 \to \begin{cases} e/g' < 1, & \text{oblique confinement (quantum Hall)},\\ e/g' > 1, & \text{confinement (insulating)},\end{cases}\\
&\frac{\delta lm}{2\pi p} < 1 \to \begin{cases} e/g' < \delta lm/2\pi p, & \text{oblique confinement (quantum Hall)},\\ \delta lm/2\pi p < e/g' < 2\pi p/\delta lm, & \text{Chern-Simons},\\ e/g' > 2\pi p/\delta lm, & \text{confinement (insulating)},\end{cases}
\end{aligned} \tag{10}$$

In the oblique confinement phase [17] the ground state consists of a dyonic condensate of strings carrying both magnetic flux $a = \pm 1$ and electric charge $2pb = \mp 2p$. The confinement phase is characterized by a ground state consisting of a condensate of magnetic flux strings. Finally, there is no condensation of topological excitations in the Chern-Simons phase. This phase is the analog of the Coulomb phase of the related three-dimensional models [15]. However, in $(2 + 1)$ dimensions, the propagating modes are massive also in this phase due to the Chern-Simons mechanism [8]: hence the name Chern-Simons phase. In all these phases the product $b(a + \eta b/p)$ vanishes for the condensed strings. This is why the imaginary term in (8) does not contribute.

The order parameters distinguishing the various phases are the *Wilson loop* [10] expectation values for the two gauge fields in the corresponding ground states. We shall present the details of this computation elsewhere [18]. Here we report only the results obtained in the weak coupling limit $ml \ll 1$. These follow a general rule [15], namely, the only nonconfined excitations carry magnetic and electric quantum numbers in the same ratio as in the condensate. Thus, both electric and magnetic excitations are nonconfined in the Chern-Simons phase. In the confinement phase, characterized by a magnetic condensate, electric charges are linearly confined. In the oblique confinement phase, the physical excitations carry magnetic flux $a = \pm 1$ and electric charge $2pb = \mp 2p$. In analogy with the physics of the quantum Hall effect [19], we identify this oblique confinement phase as a quantum Hall regime for the vortices. The flux and charge carrying excitations are the analogs of Laughlin's fractional statistic quasiparticles [4]. The parameter $1/2p$ plays thus the role of the filling fraction in this purely planar quantum Hall regime.

For $\eta = 0$, the phase diagram has the same structure, with the only difference being that the oblique confinement phase turns into a Higgs, or superconducting, phase, characterized by an electric condensate in the ground state and confinement of magnetic fluxes. In this case, the structure of the phase diagram reflects the self-duality of our model.

We conclude this paper by stressing that an insulating-superconducting quantum phase transition is actually observed experimentally in Josephson junction arrays at extremely low temperatures [7]. Our results suggest that the superconducting phase might turn into a quantum Hall regime for the vortices in the presence of n offset charges and ϕ magnetic fluxes per plaquette in the ratio $\phi/n = 1/2p$. Correspondingly, the insulating phase would turn into a standard quantum Hall regime for the charges for $n/\phi = 1/2p$. Within our formalism this can be described by a bare Chern-Simons term for A_μ in the original action.

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