Zurek and Paz Reply: Before we discuss where we differ with Casati and Chirikov [1], let us start with one point on which we are in accord: The study of isolated quantum analogs of classically chaotic systems (to which their research has contributed so much) has led to mathematically elegant and physically appealing theory. Nevertheless, we believe that neither the *quantum time scale* [2] $t_\hbar = \lambda^{-1} \ln[(\sigma\chi)/\hbar]$ (where σ measures the dispersion in the initial conditions and χ is a scale of nonlinearity of the potential) nor a similar (but typically longer) random time scale t_r [1,3] over which quantum and classical evolutions can be hoped to coincide are long enough to assure classicality of quantum chaotic evolutions.

Classicality simply does not follow "as $\hbar \to 0$" in most *physically* interesting cases (including chaos). The Planck constant is $\hbar = 1.054\,59 \times 10^{-27}$ erg s and—*licentia mathematica* to vary it notwithstanding—it is a *constant.* Moreover, typical macroscopic values of the action I one would use in $t_r = \lambda^{-1} \ln(I/\hbar)$ are simply insufficient to assure semblance of classicality for long, even where classical behavior is expected and observed. To take an obviously macroscopic example, consider Hyperion, the chaotically tumbling moon of Saturn [4,5], for which the Lyapunov exponent is $\lambda \approx 5\ \mathrm{yr}^{-1}$. A generous overestimate of the relevant action will be given by the product of Hyperion's kinetic energy and its 21-day period. This yields $t_r \approx 100/\lambda \approx 20$ yr, which is obviously orders of magnitude less than Hyperion's age. One would, therefore, expect this moon to be in a very nonclassical superposition over macroscopically distinct orientations and to behave in a flagrantly quantum manner. Yet, Hyperion's state and its evolution seem perfectly classical.

Why? The answer is outlined in our Letter [2], as well as elsewhere [6]. Briefly, the loss of quantum coherence is caused by the *environment:* The incessant monitoring by the environment and the ensuing "reduction" of the quantum state of Hyperion (or any other open system) continually forces them to be classical. This is the essence of *decoherence,* the process which in turn leads to *environment-induced superselection* [6]. As a result, only a small subset of preferred *pointer states* in the Hilbert space of the system will be sufficiently immune to decoherence to be predictable and to belong to "classical reality."

The loss of coherence is accompanied by the increase of entropy: The information acquired by the environment is lost to the observer. In our Letter we explained why entropy production is so different for quantum open systems which are classically regular or chaotic: In the chaotic case, the exponential instability tends to create fine structure in the Wigner function W, but this process is stopped by the decoherence which results in momentum diffusion with a constant D. Thus W cannot squeeze beyond the critical width $\sigma_c = \sqrt{2D/\lambda}$. At this point entropy starts growing linearly in time at a rate fixed by the Lyapunov exponent $\dot{H} = \lambda$. This is how most of the entropy in an open chaotic system starting from a low entropy, localized (approximately classical) state will be produced. Eventually, close to equilibrium the effective support of W will fill in the available phase space, and the entropy production rate will decrease to halt at H_{eq}. This will occur near $t_{\mathrm{eq}} \simeq \lambda^{-1} H_{\mathrm{eq}}/H(0)$, where t_{eq} is the time scale for reaching equilibrium. By contrast, in a regular system trajectories diverge (or become squeezed) only with a power of time. Hence, the support of W in the presence of diffusion will increase polynomially, so that $\dot{H} \sim 1/t$. While we have argued for these conclusions with the help of an exactly solvable model—the unstable oscillator (which is of course not chaotic, but which represents well the local instability of chaotic evolution)—we believe that our conclusions concerning $\dot{H}$ will hold for $t_\hbar < t < t_{\mathrm{eq}}$ for chaotic systems. Indeed, we have conjectured that the entropy production rate in a slightly open system may be a good "diagnostic" to distinguish between chaotic and regular quantum systems [7].

Decoherence caused by the environment—deemed "unsatisfactory" by Casati and Chirikov [1]—is not a subterfuge of a theorist, but a fact of life: Macroscopic systems are exceedingly difficult to isolate from their environments for a time comparable to their dynamical time scale. Moreover, even if their energy is almost perfectly conserved, purity of their state may not be assured: As the examples studied in our Letter and elsewhere indicate, the boundary between the system and the environment may be nearly impenetrable to energy, but very "leaky" for information. This imperfect isolation is, we believe, the reason why classical behavior emerges from the quantum substrate.

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