Comment on "Decoherence, Chaos, and the Second Law"

In a recent paper [1] Zurek and Paz have presented a theory for quantum decoherence and for the correspondence between quantum and classical dynamics in which the coupling of the quantum system to a (random) environment plays a crucial role. The main purpose of this Comment is to point out that, in order to reconcile quantum dynamics with classical Hamiltonian chaos, the interaction with the environment, being sufficient, is not necessary at all, nor is it satisfactory from the physical point of view. Namely, both quantum decoherence and the transition to classical chaos, including sensitive dependence on initial conditions, are quite possible, and typical, in the quasiclassical region, as $\hbar \rightarrow 0$, for a purely dynamical, classically chaotic, quantum system. As a matter of fact, this is an important recent achievement in the studies of so-called *dynamical chaos* (see, e.g., Ref. [2]).

The transition proceeds on some characteristic time scales of quantum dynamics. The shortest one—the *random time scale* t_r—is given by the estimate

$$\lambda t_r \sim \ln q , \tag{1}$$

where λ is the classical Lyapunov exponent, and $q = I/\hbar \rightarrow \infty$ stands for the dimensionless quasiclassical parameter (with a characteristic action I). This time scale was discovered and explained in Ref. [3], and it is essentially identical to estimate (2) in Ref. [1]. The quantum dynamics here is close to the classical one. However, as time grows, such a correspondence is eventually always destroyed (see, e.g., Refs. [4,5]).

We would like to emphasize that as to the *dynamical* correspondence, external noise considered in Ref. [1] does not help, since the noise indeed suppresses the squeezing which, on the other hand, is a typical feature of classical chaotic behavior. Concerning instead the *statistical* description, external noise is not necessary and the simplest (and standard) way to get rid of the dynamical fine structure (classical or quantum) is by making use of a coarse-grained phase-space density, e.g., of the Husimi distribution instead of the Wigner function.

Notice that the scale t_r grows indefinitely as $\hbar \rightarrow 0$, thus providing the transition to classical behavior in the double $(t, q \rightarrow \infty)$ conditional $(t/t_r = \text{const})$ limit. We would like to stress also that the permanent entropy growth ($\dot{H} \rightarrow \text{const} \neq 0$) found in Ref. [1] is due to the specific model used which is classically unstable but *nonchaotic*. Indeed, in the chaotic case, due to quantum diffusion, the entropy growth rate eventually decays:

$$\dot{H} \sim 1/t , \qquad t_r < t < t_R . \tag{2}$$

Besides the random time scale t_r, there is another, much longer and more important time scale—the *relaxation time scale* t_R,

$$t_R \sim q^p , \tag{3}$$

on which the quantum diffusion and relaxation remain close to the classical behavior even though the former are dynamically stable [6]. Here p is a model-dependent parameter (for example, $p = 2$ for the popular kicked-rotator model [7]). It is important to realize that the very existence of quantum diffusion, even on a finite time scale (3), as was found in the first numerical experiments [7], implies both dynamical decoherence and correlation decay. Notice that decoherence arises as a result of the standard quantum averaging over any chaotic pure state.

The external noise modifies, of course, quantum dynamics, and it can provide classical *statistical* behavior as was found also in Ref. [8]. However, the existence of the two time scales t_r and t_R allows a purely dynamical quasiclassical transition and therefore allows one to get rid of the unsatisfactory inclusion into the theory of the external noise.

In conclusion, it may be worth mentioning that the theory of dynamical chaos can be successfully applied also to so-called *mesoscopic* phenomenon with mixed classical-quantum behavior [9].

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