Feedback-Controlled Dynamics of Meandering Spiral Waves

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(Received 10 May 1995)

The effect of a sequence of short light impulses forcing a meandering spiral wave in the Belousov-Zhabotinsky reaction is investigated. Each stimulus is applied at a moment that corresponds to the passage of the wave front through a particular measuring point of the excitable medium. It is shown that the introduction of such a feedback results in two new dynamical regimes, named entrainment and resonance attractors. For both regimes the trajectory of the spiral wave tip has a symmetry center located at the measuring point. The experimental results are consistent with numerical simulations performed with the two-component Oregonator model.

PACS numbers: 82.80.Ch, 05.70.Ln, 82.20.Wt, 87.90.+y

Spiral waves of excitation belong to the most intriguing spatiotemporal patterns in nonequilibrium reaction-diffusion systems. They are observed in such diverse systems as cardiac muscle tissue [1], aggregating slime-mold cells [2], CO oxidation on platinum surfaces [3], and the chemical Belousov-Zhabotinsky (BZ) reaction [4–6]. Control of these dynamic regimes is important in many applications including the prevention of cardiac arrhythmia [1]. The BZ reaction turned out to be the most suitable laboratory system to study the dynamics of spiral waves and to elaborate adequate means of control. If the light-sensitive catalyst $Ru(bpy)_3^{2+}$ is used the dynamics of the system can be effectively controlled by external illumination [7–11]. For instance, under constant illumination the shape of the trajectory of a spiral wave tip strongly depends on the applied light intensity [12]. Under harmonic modulation of the uniform illumination, a synchronization of the movement of the spiral tip by the external forcing within entrainment bands [13] and a resonance drift of the spiral core are observed [14,15].

In this work we used the light-sensitive BZ reaction as a tool to study the effect of a feedback-controlled forcing which has been shown to be very essential for oscillatory and wave processes in active media [16,17]. For this purpose a spiral wave rotating in the BZ solution was forced by a sequence of short "light pulses," and the time instant for applying each stimulus was determined by the activity level measured at a particular point of the system.

The experimental setup included a petri dish (diameter 7 cm) containing a thin layer (0.7 mm) of silica gel with 4.2 mM $Ru(bpy)_3^{2+}$ [18]. The BZ solution (18 ml) was poured onto this gel, and after chemical equilibrium was established the concentrations of the reactants (disregarding the bromination of malonic acid) were 0.2M $NaBrO_3$, 0.17M malonic acid, 0.39M H_2SO_4, and 0.09M NaBr. The experiments were carried out at a temperature of 23 ± 1 °C.

White observation light (halogen lamp, 250 W), which was used simultaneously to control the excitability, was applied to the petri dish through two polarization filters. One of these could be rotated by a computer-controlled stepper motor. Programmed rotation of the filter generated any desired function of light intensity in the range between 0.01 and 2.22 mW/cm^2. In our case the permanent "background intensity" of the illumination was fixed to 0.5 mW/cm^2. The stimulating light pulse (if triggered) was a short (5 s) increase of the global illumination intensity up to 1.5 mW/cm^2. Each intensity jump corresponded to 150 single steps of the stepper motor and took 0.54 s.

A charged-coupled device camera detected the transmitted light at 490 nm, which is near the maximum difference of absorption between the oxidized and reduced form of the catalyst (at 460 nm). The observed pictures of the catalyst concentration waves were digitized by an image acquisition card, analyzed on-line, and also stored on a video recorder [9].

To start an experiment a spiral wave was created in the center of the dish by breaking a circular wave front with a thin laser beam (diameter 1 mm) and shifting one of the two originating open ends to the boundary of the dish [9,10]. The remaining open end evolved into a single, unperturbed spiral wave. Its tip was determined automatically as the intersection of two overlaid contour lines of the spiral wave extracted from two consecutive frames (time step 1.44 s).

At a constant intensity of 0.5 mW/cm^2 the trajectory of the tip is approximately a four-lobed hypocycloid (dashed line in Fig. 1) with a wave period $T_\infty \approx 33$ s at a point far away from the tip and remains so for at least 2 h. An applied single light impulse deforms the trajectory. Nevertheless, the trajectory of the tip stabilizes soon after the stimulus and forms a similar four-lobed hypocycloid which is shifted and rotated with respect to the initial one (solid line in Fig. 1). In addition, the phase of the angular velocity, which is a periodical function, is shifted in time. All the parameters describing the effect of a single stimulus on the spiral wave dynamics depend on the phase of the tip motion at the instant when the stimulus is applied.

To realize the desired feedback mechanism the intensity of the transmitted light at a particular measuring point is determined on-line with a time interval of 0.02 s. Every time a wave front reaches the measuring point a

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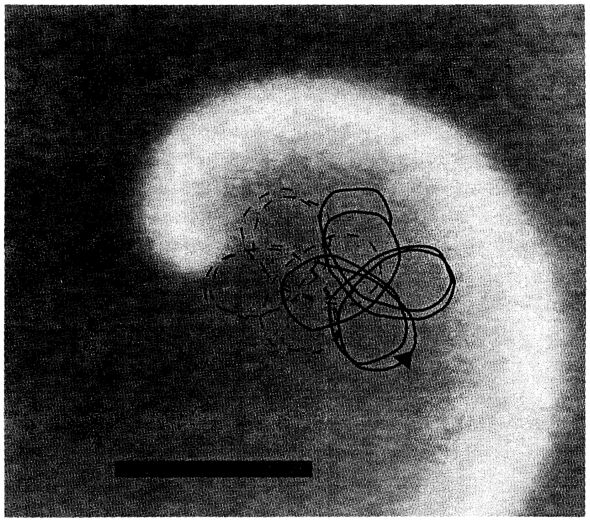

FIG. 1. The trajectory of the spiral wave tip observed for the constant background intensity of the illumination before (dashed line) and after (solid line) the application of the single light pulse. An intermediate position of the spiral wave is shown. Scale bar: 0.5 mm.

single stimulus is triggered immediately or with a certain time delay τ after the wave-front passage. When the feedback mechanism is switched on, the spiral is forced by a sequence of short impulses. The influence of each stimulus is overlayed with the effect of all the previous ones, which leads to the observation of two new regimes shown in Fig. 2.

If the measuring point is placed close to the center of the unperturbed trajectory, the feedback results in a synchronization of the tip motion by the external stimuli [see Fig. 2(a)]. After some transient process, the spiral tip approaches an asymptotic trajectory with a symmetry center located at the measuring point. In this case the spiral is forced by a periodic sequence of short stimuli leading to a deformation (as a rule to a blow-up) of the trajectory. Nevertheless, the distance between the tip position and the measuring point remains on the order of the size of one loop. The period of the light pulses is exactly the time during which one loop of the trajectory is described. Consequently, each stimulus is applied at the same phase of each loop.

If the measuring point is placed rather far from the initial trajectory, one observes a quite different evolution of the trajectory, as illustrated in Fig. 2(b). The tip of the spiral wave approaches a stable trajectory, which looks like a drift of the four-lobed hypocycloid along a large circle. The center of this circle coincides again with the measuring point. A significant difference to the former regime: The sense of the core rotation around the measuring point is opposite, the sequence of the stimuli is not exactly periodical, and the stimuli are applied at different phases of the lobes. The average period is larger than in the first case.

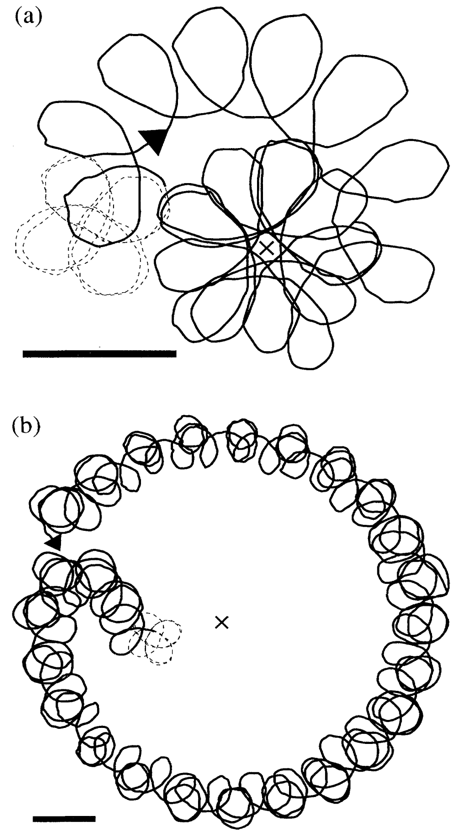

FIG. 2. Two attractors observed in the light-sensitive BZ reaction forced by a sequence of short light pulses applied at every moment when a wave front passes through the measuring point (+). The distance between the center of the unperturbed trajectory (dashed line) and the measuring point was (a) 0.49 mm, (b) 0.57 mm. Scale bars: 0.5 mm.

Both asymptotic trajectories are stable with respect to a small shift of the measuring point and can be considered as attractors for the studied dynamical system including a feedback.

So far, the experiments described above were performed without any artificial time delay τ in the feedback loop. We also investigated the role of a delay between the registration of a wave front at the measuring point and the triggering of the stimulus. We found that the synchronization also occurs if the delay is small with respect to the period of one loop [Fig. 3(a)]. The diameter of the synchronized trajectory increases with the delay [compare Figs. 1(a) and 3(a)]. This growth of diameter is accompanied by an increase of the number of the loops in the pattern. If the delay reaches a certain value, the synchronization breaks down, and only the regime of the drift around the measuring point is observed [Fig. 3(b)].

We complemented the experiments by numerical simulations of the observed phenomena. For this purpose we used an extension of the Oregonator model [19,20], which includes an additional term ϕ describing the light-induced

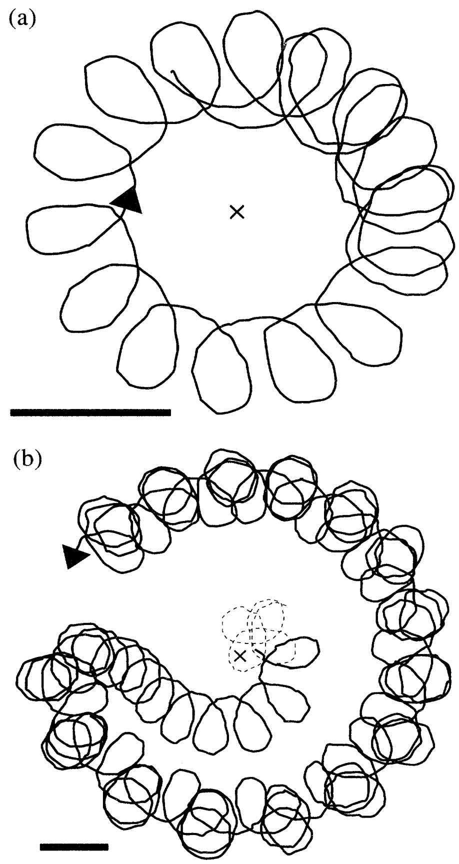

FIG. 3. Tip trajectories measured in the light-sensitive BZ reaction for different values of the time delay τ in the feedback loop: (a) $\tau = 5$ s and (b) $\tau = 7$ s. Scale bars: 0.5 mm.

bromide production [13,14].

$$\frac{\partial u}{\partial t} = \nabla^2 u + \frac{1}{\epsilon}\left[u - u^2 - (fv + \phi)\frac{u - q}{u + q}\right], \quad (1a)$$

$$\frac{\partial v}{\partial t} = u - v. \quad (1b)$$

The variables u and v describe the concentrations of the autocatalytic species $HBrO_2$ and the catalyst, respectively. The parameters $\epsilon = 0.05$, $q = 0.002$, and $f = 2.0$ were fixed. The term $\phi = \phi(t)$ describes an additional bromide production induced by the external illumination of the system. It consists of a sequence of impulses with amplitude A and duration D that are applied with a time delay τ after the passage of the wave fronts through a particular measuring point. These impulses are added to a background flow $\phi_0 = 0.01$.

In our calculations we fixed the duration to $D = 0.3$ and studied the trajectories of the spiral wave tip for different values of the amplitude A and the delay τ. The computations were performed by the explicit Euler method, using the five-point approximation of the Laplacian on a 380×380 array with a grid spacing $\Delta x = 0.1$ and time steps $\Delta t = 0.001$.

A single spiral was induced by a special choice of initial conditions for the system (1). For zero amplitude A of the stimulus one can observe the unperturbed trajectory, which is close to a five-lobed hypocycloid with a wave period $T_\infty = 3.6$ Oregonator time units measured far away from the symmetry center.

If the center of the unperturbed spiral wave is located rather close to the measuring point, it is attracted to this new position after same transient process [Fig. 4(a)]. If this initial distance is large enough, one can observe an evolution to another attractor: The center of the unperturbed hypocycloid moves and describes a big circle around the measuring point [Fig. 4(b)].

Increasing the delay τ results in a large number of lobes for the first type of attractor [compare Figs. 4(a) and 5(a)]. The synchronization breaks down for a rather big delay, and the system then tends to the second attractor [Fig. 5(b)]. The observed deformation of the unperturbed trajectory becomes more pronounced if a stimulus with higher amplitude A is applied. The radius of the big circle which characterizes the trajectory for the second type of attractor decreases with the time delay τ in the feedback

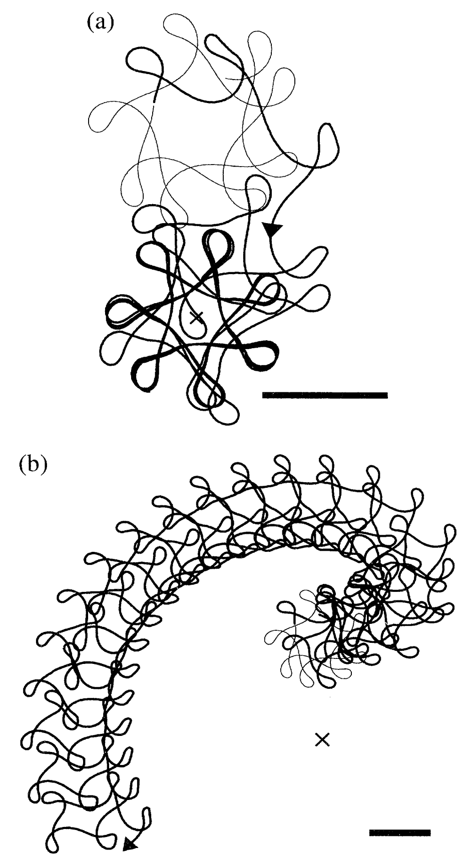

FIG. 4. Two attractors computed for the model (1) with $A = 0.01$ and $\tau = 0.5$. The distance between the center of the unperturbed trajectory (thin line) and the measuring point was (a) 6 Oregonator space units (su), (b) 7 su. Scale bars: 4 su.

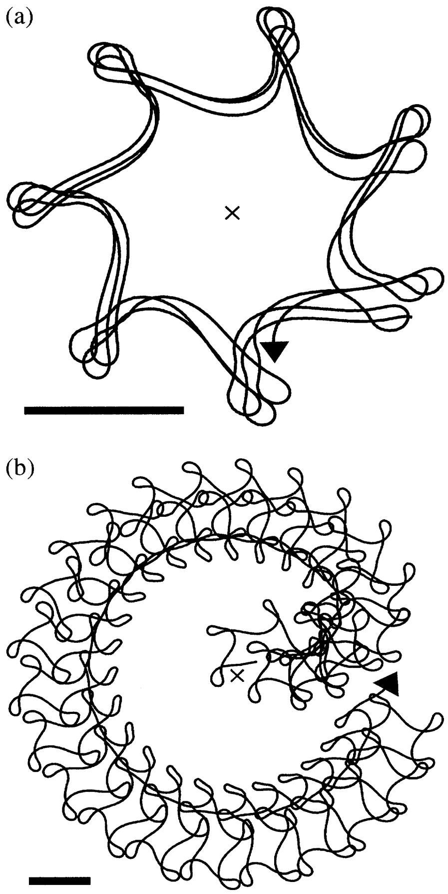

FIG. 5. Tip trajectories computed for the model (1) with $A = 0.01$ and different values of the time delay τ in the feedback loop: (a) $\tau = 0.8$ and (b) $\tau = 1.5$. Scale bars: 4 Oregonator space units.

loop. The increase of the amplitude A results in an increase of the drift velocity along this circle but does not influence its radius.

Thus, the numerical simulations very well confirm the existence of the two kinds of attractors observed in the experiments with the BZ reaction controlled by a feedback mechanism. The nature of these attractors becomes more clear if one takes into account earlier results obtained for the external forcing of spiral waves [13,14]. If the measuring point is located in the symmetry center of a meandering trajectory, the period of the stimuli is exactly equal to the period of one lobe of the trajectory, and the phase of the applied stimulus is the same for each lobe. We call this regime "entrainment attractor" because it exhibits the basic properties of frequency entrainment as it occurs in similar systems with a periodic modulation of the excitability, but without feedback [13,14]. Either way, the increase of the number of lobes with the time delay can be interpreted as an effect of the phase of the applied stimulus and corresponds to the variety of the trajectories induced by different phase relationships within an entrainment band [14].

In the second case the measuring point is far away from the tip trajectory. A different period of excitation ($T \approx T_\infty$) is registered at the measuring point. As observed earlier for the system without feedback [14], the external forcing of the spiral wave with such a period results in a so-called resonance drift. For this reason we call this regime "resonance attractor." A similar regime occurs in the case of nonmeandering spiral waves [17]. The location of a separatrix between these two attractors was determined very roughly yet. It depends on the amplitude and the delay of the stimulus as well as on the initial orientation of the unperturbed pattern.

Both these attractors are stable with respect to a small shift of the measuring point. Moreover, the center of the meandering pattern follows any slow displacement of this particular point. Hence, the described feedback mechanism yields a powerful means to control the position of a spiral wave using only comparatively small global perturbations.

V. S. Z. acknowledges support from the WE-Heraeus-Stiftung, Hanau.

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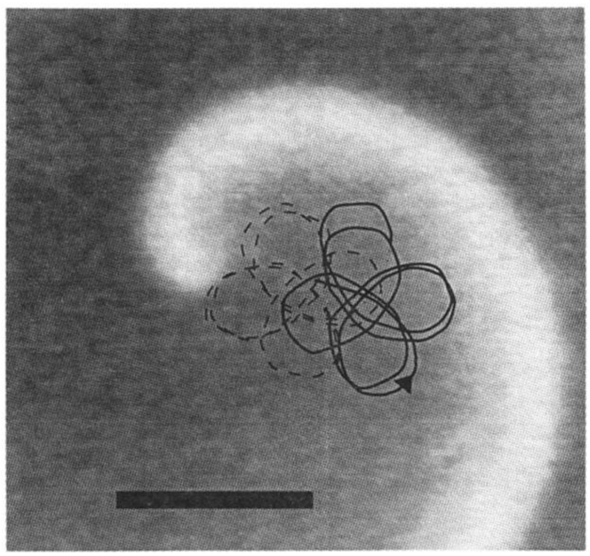

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