Time-Resolved Cuspidal Structure in the Wave Front of Surface Acoustic Pulses on (111) Gallium Arsenide

A. A. Maznev*
Fachbereich Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany

Al. A. Kolomenskii* and P. Hess
Institut of Physical Chemistry, University of Heidelberg, Im Neuenheimer Feld 253, 69120 Heidelberg, Germany

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The first experimental observation of the cuspidal structure in the surface acoustic wave (SAW) front propagating from a point source in a crystal is reported. Nanosecond SAW pulses are generated on the (111) surface of GaAs by focused nanosecond laser pulses and detected with a cw probe laser beam. Multiple SAW arrivals are observed in the cuspidal region near the $\langle 11\bar{2}\rangle$ direction. A peak of the SAW amplitude due to "phonon focusing" is observed in the cusp direction. The measured SAW wave forms are well described by a model calculating the surface response to a localized pulsed force.

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In an elastically anisotropic solid the group and phase velocities of surface acoustic waves (SAWs) are in general not collinear [1]. This leads to striking features in SAW propagation from a point source. The angular dependence of the SAW amplitude turns out to be strongly anisotropic with sharp maxima in certain directions. This phenomenon referred to as surface phonon focusing [2–5] has recently been observed experimentally with laser generation of SAWs [6,7]. Another closely related phenomenon is that the SAW wave surface, i.e., the wave front propagating from a point disturbance, may be folded forming cuspidal structures [2–4,8]. This should result [9] in multiple SAW arrivals in point-source and point-receiver measurements. For bulk acoustic waves, folds in the wave surface have been observed both with ballistic thermal phonons [10] and at ultrasonic frequencies [11,12]. However, no direct experimental observation of this phenomenon has been reported for SAWs.

This Letter reports the experimental observation of cusps in the SAW wave front propagating from a point source on the (111) surface of GaAs. We use optical generation and detection of nanosecond SAW pulses, a technique providing high temporal and spatial resolution and proven to be suitable for studying elastic and mechanical properties of materials [13], to observe multiple SAW arrivals in the cuspidal region. We believe this study to be of interest as a demonstration of the fundamental phenomena occurring in SAW propagation in crystals, and also for the application of point-source and point-receiver SAW techniques for the investigation of elastically anisotropic solids [14,15].

SAW wave front propagating from a point source corresponds within a scale factor to the polar plot of the SAW group velocity $v_g(\varphi)$. The value v_g and angle φ of the SAW group velocity vector $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$ are expressed in terms of the value v_f and angle θ of the phase velocity by

$$v_g = v_f[1 + (dv_f/d\theta)^2]^{1/2}, \tag{1}$$

$$\varphi = \theta + \arctan\left(\frac{1}{v_f}\frac{dv_f}{d\theta}\right). \tag{2}$$

For a given wave vector $\mathbf{k}$ the group velocity vector is normal to the constant frequency surface $\omega(\mathbf{k}) = \text{const}$ coinciding to within a scale factor with the so-called slowness surface, the polar plot of slowness $s = 1/v_f(\theta)$. If $(1/v_f)d^2v_f/d\theta^2 < -1$ for some interval of angles, then the corresponding section of the slowness surface is concave resulting in that several wave vectors may have group velocities in exactly the same direction. This gives rise to a folded wave surface with cusps corresponding to the inflection points of the slowness surface. The existence of the cusps is thus determined by the anisotropy of the phase velocity, which, in turn, depends both on the elastic constants of the medium and on the surface orientation [1].

SAWs on the (111) surface of GaAs yield a rather tiny cuspidal structure in the wave surface. Shown in Fig. 1(a) is the SAW slowness surface calculated with GaAs elastic constants from Ref. [16] and disregarding piezoeffect, which is fairly small for this material [17]. The slowness surface is concave near the $[11\bar{2}]$ and other symmetrical directions [18] resulting in small cuspidal structures in the wave surface as shown in Fig. 1(b).

To observe this feature experimentally we generated SAW pulses on the GaAs(111) surface by frequency-tripled (355 nm) Q-switched Nd:YAG laser irradiation, with the laser pulse duration and energy being 7 ns and 40 μJ, respectively. The laser beam was focused into a spot of 50 μm diameter to produce surface ablation accompanied by generation of SAWs propagating away from the laser spot. The duration of the generated SAW pulses was mainly determined by the ratio of the laser spot size to the SAW velocity, i.e., ~20 ns.

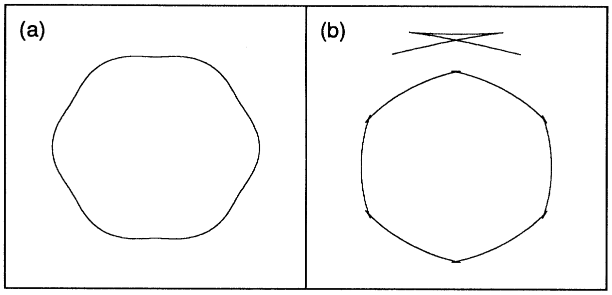

FIG. 1. (a) Slowness surface and (b) wave surface of SAWs in the (111) plane of GaAs. The $\langle 11\bar{2}\rangle$ direction is vertical. The inset is a magnified view of the cuspidal structure in the wave surface.

The corresponding SAW frequency spectrum was thus limited by ~50 MHz. For these frequencies, which were 3 orders of magnitude lower than those of ballistic thermal phonons [10], both the room temperature SAW absorption and the surface-roughness scattering on the optical quality polished sample were negligible for the propagation distances of a few centimeters.

SAW wave forms were detected using the probe beam deflection technique [19]. The probe beam of a cw diode-pumped frequency-doubled Nd:YAG laser (532 nm) of 30 mW power was focused on the surface to a spot of 10 μm diameter at a distance of 3.21 cm from the excitation point. The deflection angle of the reflected probe beam was detected by an arrangement of two fast photodiodes sensitive to the beam position. The signal, proportional to the surface slope and hence to the vertical surface velocity in a SAW pulse, was preamplified and recorded by a digital oscilloscope. The setup was able to detect a surface slope of 5×10^{-6} rad in the single-shot regime, which corresponded to about 0.1 nm surface displacement in a SAW pulse [20]. The temporal resolution was 4 ns, being mainly limited by the finite size of the probe laser spot.

We used fluences slightly above the ablation threshold for excitation so that the SAW wave forms exhibited only minor changes over the first 100 laser shots used for averaging to improve the signal-to-noise ratio. To carry out measurements at different observation angles the GaAs sample was rotated about an axis normal to the surface while the excitation and detection points were fixed. Thus a new point on the sample was used for SAW excitation at each measurement.

The SAW wave forms measured at different observation angles to the $\langle 11\bar{2}\rangle$ direction are presented in Fig. 2. Two or three pulses corresponding to different branches of the cuspidal structure are observed at angles $|\varphi| \leq 4.5°$. Note that it is because the ratio of the SAW time-of-flight to the SAW pulse duration is as large as ~1000 that the tiny cuspidal structure is well resolved. At $\varphi = 0°$ the intersection of two branches of the wave front results in a doubling of the amplitude of the second SAW arrival. This fact was used by us for the accurate determination of the $\langle 11\bar{2}\rangle$ direction. When the observation direction deviates from $\langle 11\bar{2}\rangle$ by more than 0.2°, three SAW arrivals can be discriminated. With a further increase in the observation angle, the second arrival merges with the first one resulting in a pulse of enhanced amplitude with a maximum at about 2.5°. Subsequently, its amplitude falls off rapidly, and for $\varphi \geq 5°$ only a single SAW arrival can be observed.

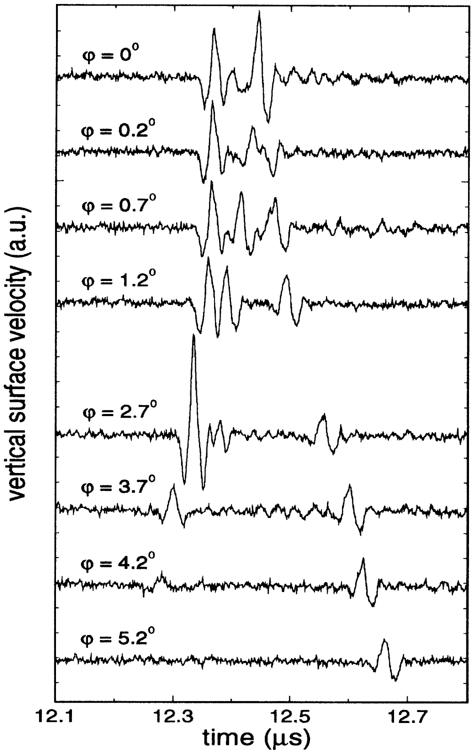

FIG. 2. Measured SAW wave forms at different angles to the $\langle 11\bar{2}\rangle$ direction on GaAs(111).

The measured angular dependence of the group velocity of the SAW pulses is presented in Fig. 3(a) along with the calculated one. A remarkable finding is that the first SAW arrival is detected well beyond the group velocity cusps lying at $|\varphi| = 2.9°$. The reason for this is the restricted nature of the group velocity calculations corresponding to the geometrical acoustics approximation in which $\lambda/r \rightarrow 0$, where λ is the SAW wavelength and r is the traveling distance. More rigorous calculations are shown below to account well for this feature. Apart from this, measured and calculated group velocity data are in good agreement. A slight deviation of the calculated curve from the experimental points is well within the uncertainties in the reference data of the elastic constants of GaAs amounting to about 1%, which by an order of magnitude exceeds the piezoelastic stiffening effect [16,17]. It should be noted that the measurements of the bulk acoustic wave group velocities in the cuspidal region have been employed for

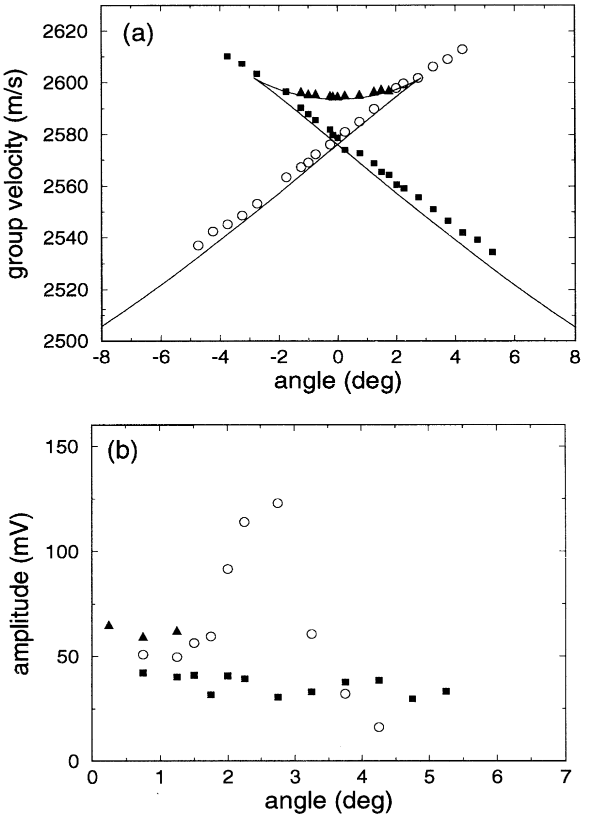

FIG. 3. (a) measured angular dependence of the SAW group velocity. Triangles, open circles, and squares refer to different arrivals. The solid line is a theoretical curve calculated with Eqs. (1) and (2). The following values of the GaAs elastic constants [16] and density were used: $C_{11} = 118$ GPa, $C_{12} = 53.5$ GPa, $C_{44} = 59.4$ GPa, $\rho = 5.316$ g/cm^3. (b) angular dependence of the peak-to-peak amplitude of the SAW pulses. The same symbols as in (a) are used for different arrivals. For $\varphi = 0°$ half of the measured amplitude of the second SAW arrival, resulting from the superposition of two pulses, is used.

the accurate determination of elastic constants [11]. The use of SAW group velocity measurements for this purpose has been suggested in Ref. [14], though no measurement in the cuspidal region has been reported up to now. We believe that measurements in the cuspidal region are more informative, e.g., even a single measurement at a particular observation angle yields three different group velocity values, which should suffice to determine three independent elastic constants of a cubic crystal.

The angular dependence of the peak-to-peak amplitude of the SAW pulses is presented in Fig. 3(b). The focusing effect can clearly be seen in the enhancement of the first pulse amplitude in the cusp direction. This confirms our previous observation of SAW focusing on GaAs(111) reported in Ref. [7], where the angular dependence of the SAW amplitude was visualized using the shake-off of fine powder particles by SAWs. In the geometrical acoustics approach, surface phonon focusing is described by the focusing factor [4] $A = |d\varphi/d\theta|^{-1}$, characterizing the enhancement of the ray density due to the deviation of the ray or group velocity direction from the wave vector. At the cusp point we have $d\varphi/d\theta = 0$, and the focusing factor becomes infinite yielding a caustic with infinite SAW intensity. Going beyond the ray approximation and taking into account the finite SAW wavelength λ shows [2,3,5] that in the caustic direction SAW amplitude falls off with distance as $r^{-1/3}$ in contrast to the usual $r^{-1/2}$ dependence in other directions. This results in a sharp maximum of the amplitude at large r.

The effect of the finite acoustic wavelength on SAW focusing for the cw single-frequency SAW excitation was studied theoretically by Tamura and Yagi [21]. As expected, strong oscillations in the angular dependence of the SAW amplitude in the cuspidal region were predicted as a result of the interference of SAWs corresponding to different parts of the cuspidal structure and propagating in the same direction. This effect termed "internal diffraction" [12,15] is not observed in our experiment as we deal with single-pulse rather than harmonic SAW excitation, and hence there is no interference of SAW pulses propagating with different group velocities.

The SAW wave forms generated in our experiment can most simply be simulated by assuming that the surface ablation results in a pulsed localized vertical force acting on the surface. We consider a semi-infinite anisotropic elastic continuum with the displacement field $\mathbf{u}(\mathbf{r}, t)$ being described by the equations of motion [1]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k}, \tag{3}$$

where ρ is the material density and C_{ijkl} is the elastic constant tensor. The boundary conditions at the free surface $z = 0$ are given by

$$\sigma_{3i} = C_{3imn} \frac{\partial u_m}{\partial x_n} = \delta_{i3} F_0 \exp(-t^2/\tau^2) \exp(-r^2/a^2). \tag{4}$$

Here σ_{ij} is the stress tensor, and the right-hand side of the equation represents the external force whose temporal and spatial distribution is assumed to be Gaussian. The calculation procedure used to compute the surface displacement response will be presented in detail elsewhere and is in some features similar to that described in Ref. [21]. The main difference is that we used a pulsed rather than a harmonic source term.

Figure 4 shows the calculated SAW wave forms for $\tau = 4$ ns corresponding to 7 ns FWHM laser pulse duration, $a = 25$ μm, and the source-to-receiver distance used in our experiment. Good agreement between measured and calculated wave forms is obtained. As in the experiment, the first SAW arrival is present well beyond the group velocity cusps. Different shapes of SAW pulses corresponding to different parts of the cuspidal structure [9] are reproduced by the model calculations. The calculated enhancement of the first pulse amplitude near the

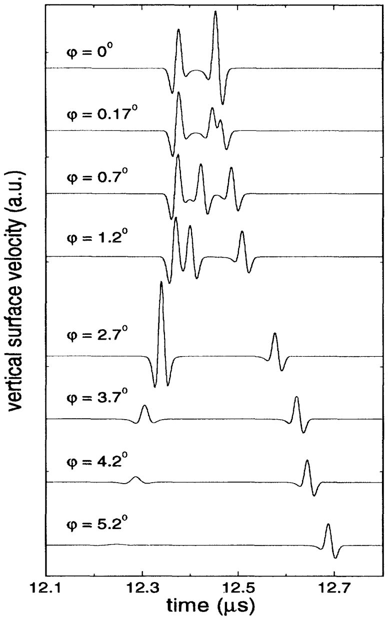

FIG. 4. Calculated SAW wave forms generated by a pulsed localized vertical force acting on the surface.

caustic direction is also consistent with the experiment. Thus, despite the very simple choice of the source term in modeling the ablation regime of SAW generation, the calculations appear to simulate the SAW wave forms in the cuspidal region very well.

In conclusion, the first experimental observation of folds in the SAW wave front propagating from a point source in a crystal has been carried out with optical generation and detection of nanosecond SAW pulses. Cuspidal features in the SAW wave front are a fundamental phenomenon of SAW propagation in anisotropic media and can be observed on surfaces of many crystals [3,5,8]. In some cases the calculated cuspidal structures are much more pronounced than on GaAs(111). That this phenomenon has not been revealed so far in the SAW measurements results from the fact that most SAW studies utilize line sources generating quasiplane waves, with the SAW phase velocity being measured. On the other hand, folds in the group velocity should feature prominently whenever a pointlike source is involved. In addition to laser ultrasonics, we would like to refer here to the angular scanning technique with ultrasonic immersion transducers, developed by Vines, Tamura, and Wolfe [15]. Miniature piezotransducers and the capillary fracture technique used by Kim and Sachse [11] for bulk acoustic wave measurements can also be used for SAWs. Finally, for the study of ballistic pulses of thermal surface phonons [22], the phenomenon considered in this paper will be of great importance.

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*On leave from General Physics, 117942 Moscow, Russia.

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