Polarizable Quantum Systems in Crossed Electric and Magnetic Fields

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It is found that a neutral particle acquires a nontrivial quantum phase independent of the particle velocity in crossed electric and magnetic fields. It is proved that the phase is independent of the particle trajectory shape if the fields are homogeneous along a certain direction. The phase induced by the crossed fields is shown to be responsible for dissipation-free flows in superfluid systems, which are similar to the Meissner currents in superconductors.

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The forceless action of crossed dc electric and magnetic fields upon neutral quantum particles was first observed in [1,2]. These papers describe the behavior in the magnetic field of a neutral particle with a dipole moment induced by the electric field. It was found that in crossed electric and magnetic fields the kinematic impulse of the particle did not coincide with its generalized impulse. This results in a number of striking effects; in particular, nondissipative mass flows appear in polarizable superfluids in crossed fields. Ginzburg shows [3] that the effects predicted in [1,2] may be interpreted as a direct consequence of the Abraham force acting upon the system. Effects close in idea to the predicted ones [1–3] were found independently six years later by Aharonov and Casher [4] who studied the behavior of a neutral quantum particle having its own magnetic moment in an electric field. The nontrivial character of these predictions [1–4] is emphasized by the fact that the notion of the Aharonov-Casher oscillations has appeared in the science. The oscillations are similar to those predicted earlier in [5].

However, in [1,2,5] the author did not pay attention to the close relationship between the predicted effects and the nontrivial quantum phase of the particles and to the topological nature of the effects. This relationship was first pointed out in [4]. The paper [4] provoked a considerable number of publications; e.g., see [6–11]. The paper by Wilkens [12] is one of the latest publications in this series. Wilkens studied the behavior of a neutral quantum particle with its own dipole moment in a magnetic field. Aharonov and Casher and Wilkens found a nontrivial quantum phase of particles having their own magnetic [4] and dipole [12] moments. The present paper reports the finding of a nontrivial quantum phase of a particle with a momentum induced by electric or magnetic fields. It turns out that the results obtained for such particles differ drastically from those in [4,12]. The reason is that the induced moments, like the inducing fields, decrease as the distance from the charge (for the electric field) and from the current (for the magnetic field) increases. It is found that the phase acquired by particles is essentially dependent on the problem of symmetry. In addition, new experiments are proposed, which can make the observation of the effects predicted in [1–5] much easier.

The nontrivial phase of the polarizable quantum particle in crossed fields is determined by the part of the Lagrange function which is dependent on the particle velocity. This part can readily be found if we know the force of the field action upon the particle and hence the equation of the particle motion. In the context of our interest in the effects induced by the nontrivial phase in condensed media, we find the velocity-dependent part of the Lagrange function for a homogeneous and isotropic liquid.

We proceed from the expression for the density of electromagnetic energy in the fluid

$$w = \frac{1}{8\pi}\left(\frac{D^2}{\varepsilon} + \frac{B^2}{\mu}\right). \tag{1}$$

It is assumed here that for immobile fluids the electric $\mathbf{D}$ and magnetic $\mathbf{B}$ inductions are related to the field strengths $\mathbf{E}$ and $\mathbf{H}$ as $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$. If the fluid is moving at the velocity $\mathbf{v}$, the induction-strength relationship becomes (e.g., see [13])

$$\begin{aligned} \mathbf{D} + \frac{1}{c}\mathbf{v}\times\mathbf{H} &= \varepsilon\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right), \\ \mathbf{B} + \frac{1}{c}\mathbf{E}\times\mathbf{v} &= \mu\left(\mathbf{H} + \frac{1}{c}\mathbf{D}\times\mathbf{v}\right). \end{aligned} \tag{2}$$

Now we calculate the derivative $\partial w/\partial t$ using the Maxwell equations. The derivatives $\partial\varepsilon/\partial t$ and $\partial\mu/\partial t$ are calculated taking into account that (cf. [14])

$$\begin{aligned} \frac{d\varepsilon}{dt} &\equiv \frac{\partial\varepsilon}{\partial t} + \mathbf{v}\cdot\nabla\varepsilon = \frac{\partial\varepsilon}{\partial n}\frac{dn}{dt} \\ &= \frac{\partial\varepsilon}{\partial n}\left(\frac{\partial n}{\partial t} + \mathbf{v}\cdot\nabla n\right) = -\frac{\partial\varepsilon}{\partial n}n\nabla\cdot\mathbf{v}, \end{aligned} \tag{3}$$

where n is the fluid density obeying the continuity equation $\partial n/\partial t + \nabla\cdot n\mathbf{v} = 0$. A similar equation exists for $d\mu/dt$. We then obtain

$$-\frac{\partial w}{\partial t} = \mathbf{j}_e\cdot\mathbf{E} + \mathbf{f}\cdot\mathbf{v} + \nabla\cdot\mathbf{S}. \tag{4}$$

Here $\mathbf{j}_e$ is the current density of free charges in the fluid. If we take $\mathbf{f}$ as the force with which the field acts on unit volume of the fluid and $\mathbf{S}$ as the energy flow, Eq. (4) attains the meaning of the law of energy conservation: a decrement in the field energy in unit time is equal to the work done by the field on the free charges $\mathbf{j}_e\cdot\mathbf{E}$ and on

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the fluid $\mathbf{v} \cdot \mathbf{f}$ plus the energy flow divergence $\mathbf{S}$. We write down the expressions for $\mathbf{S}$ and $\mathbf{f}$ when $\varepsilon - 1 \ll 1$ and $\mu - 1 \ll 1$, which is the only case considered here. Then the energy flow is

$$\mathbf{S} = \frac{1}{4\pi}\left\{c\mathbf{E} \times \mathbf{H} - \left[(\varepsilon - 1)\frac{E^2}{2} + (\mu - 1)\frac{H^2}{2}\right]\mathbf{v}\right\}, \quad (5)$$

and the force is

$$f_i = -\frac{\partial}{\partial t}\rho u_i - \frac{\partial}{\partial x_k}\rho v_k u_i + \rho v_k \frac{\partial u_k}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\frac{\varepsilon - 1}{8\pi}E^2 + \frac{\mu - 1}{8\pi}H^2\right). \quad (6)$$

Here the quantity

$$\rho\mathbf{u} = \frac{\varepsilon\mu - 1}{4\pi c}\mathbf{H} \times \mathbf{E} \quad (7)$$

is used where $\rho = mn$ is the mass density of the fluid. On adding the force from Eq. (6) to the right-hand side of the equation of motion for the kinematic momentum per unit volume of the fluid $\rho\mathbf{v}$, and using the continuity equation for the density ρ, we obtain the equation allowing for the forces with which the electromagnetic field acts on a neutral fluid,

$$\rho\left(\frac{\partial}{\partial t} + v_k\frac{\partial}{\partial x_k}\right)(v_i + u_i) \equiv \rho\frac{d}{dt}(v_i + u_i) = \rho v_k\frac{\partial u_k}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\frac{\varepsilon - 1}{8\pi}E^2 + \frac{\mu - 1}{8\pi}H^2 - p_0\right), \quad (8)$$

where p_0 is the pressure in the fluid at a given density ρ when the field is absent. It is seen now that the part of the Lagrange function dependent on particle velocity is

$$\int L d\mathbf{r} = \int\left(\frac{\rho v^2}{2} + \rho\mathbf{v} \cdot \mathbf{u}\right)d\mathbf{r}. \quad (9)$$

Correspondingly, the generalized momentum per unit volume of the fluid is given by

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = \rho\left(\mathbf{v} + \frac{\varepsilon\mu - 1}{4\pi nmc}\mathbf{H} \times \mathbf{E}\right). \quad (10)$$

Equations (8)–(10) are the basis for further calculations.

When a particle is moving along a closed path, the induced phase $\Delta\varphi$ is equal to the action increment divided by $\hbar$ during the time T of the motion on the closed path. Substitution of ρ in the form of $\sum_i m\delta(\mathbf{r} - \mathbf{r}_i)$ into expression (9) gives the nontrivial phase of the particle in crossed fields:

$$\Delta\varphi = \int_0^T \Delta L dt/\hbar = \frac{\varepsilon\mu - 1}{4\pi nc\hbar}\oint(\mathbf{H} \times \mathbf{E})\,dl = \frac{\varepsilon\mu - 1}{4\pi nc\hbar}\int\int \nabla \times (\mathbf{H} \times \mathbf{E})\,d\mathbf{S}. \quad (11)$$

In the general case, $\nabla \times (\mathbf{H} \times \mathbf{E})$ is intricately expressed in terms of the fields $\mathbf{E}$ and $\mathbf{H}$ and their derivatives. The situation can, however, be simplified if the problem is homogeneous is some direction. For example, let the field be independent of the coordinate z. Then the differentiation of the fields $\mathbf{E}$ and $\mathbf{H}$ and use of the Maxwell equation gives

$$[\nabla \times (\mathbf{H} \times \mathbf{E})]_z = 4\pi\left(\rho_e\mathbf{H} + \frac{1}{c}\mathbf{j}_e \times \mathbf{E}\right)_z. \quad (12)$$

Here ρ_e is the charge density and $\mathbf{j}_e$ is the current density. The fields $\mathbf{E}$ and $\mathbf{H}$ in Eq. (12) are the sum of the external fields and those excited by the charges ρ_e and the currents $\mathbf{j}_e$. It can be proved that if the currents exciting the field $\mathbf{H}$ are within the contour the integral of Eq. (12) over the space inside the contour is identically zero. If the currents are beyond the contour, only the first term contributes to the integral of Eq. (12). Since in the case considered the integrals of the xth and yth components of $\nabla \times (\mathbf{H} \times \mathbf{E})$ are equal to zero, then

$$\Delta\varphi = \frac{\varepsilon\mu - 1}{nc\hbar}H_z\int \rho_e\,dS_z. \quad (13)$$

Thus in crossed fields a neutral polarized particle which has accomplished a complete pass around the charges exciting the electric field gains a phase independent of the trajectory shape.

The phase induced by the crossed fields in a neutral quantum particle causes macroscopic effects when the particles condense into a superfluid liquid. Taking into account that the generalized momentum per unit volume of the superfluid liquid is $\hbar\nabla\varphi\rho_s$, where φ is the order parameter phase, Eq. (10) can give the circulation κ of the superfluid velocity along any closed contour around the uniformly charged line in the magnetic field homogeneous along this line

$$\kappa \equiv \oint \mathbf{v}_s \cdot dl = \frac{\hbar}{m}\oint\left(\nabla\varphi - \frac{\varepsilon\mu - 1}{4\pi nc\hbar}\mathbf{H} \times \mathbf{E}\right) \cdot dl = 2\pi\frac{\hbar}{m}\left(s - \frac{(\varepsilon\mu - 1)H_{\|}\tau}{2\pi nc\hbar}\right). \quad (14)$$

Here $H_{\|}$ is the magnetic field projection onto the charged line and τ is the charge per unit length of the line. The integer s can be found from the requirement for the minimum $\int d\mathbf{r}\,\rho_s v_s^2/2$, which, because of the arbitrariness of the integration contour in Eq. (14), is equivalent to the requirement for the minimum of the circulation modulus. A closer consideration of the condition shows convincingly that crossed electric and magnetic fields induce flows in the superfluid liquid whose velocity $\mathbf{v}_s$ is a periodic function of the charge in the line τ and of the magnetic field $\mathbf{H}$. The oscillation period is readily found from the relation $(\varepsilon\mu - 1)H_{\|}\tau = 2\pi nc\hbar$. These flows are dissipation free and correspond to the ground state of the neutral superfluid liquid in crossed fields. It is evident that, concurrently with the velocity $\mathbf{v}_s$, the energy and other thermodynamic characteristics also oscillate with the same period. These oscillations were predicted for $\mu = 1$ in [5].

Now we go from the system invariant with respect to translations in a certain direction to the system invariant against rotations about a certain axis. Let us consider the disk in Fig. 1, which has a current-carrying wire running through its orifice. Let the disk be covered with a superfluid liquid, e.g., He II, film. It is important for what follows that the radial steady-state current may flow in the film—e.g., from the center towards the edges in the upper film and the reverse in the lower film. On substitution of $\mathbf{v}_s = (\hbar/m)\nabla\varphi$ into the continuity equation $\nabla \cdot \mathbf{j}_s = 0$, the dependence of the phase φ upon the coordinate ρ can be found as

$$\varphi = \frac{2\pi s}{d/R + d/r + 2\ln R/r}\begin{cases} \ln \dfrac{\rho}{r} & \text{(upper film)}, \\ \dfrac{d}{R} + \ln \dfrac{R^2}{r\rho} & \text{(lower film)}. \end{cases} \tag{15}$$

If we charge the disk, it will excite the electric field $E_z = \pm 2\pi\sigma$ (σ is the two-dimensional charge density) normal to the disk. The current I_z through the wire will excite a magnetic field around the wire with a nonzero component H_φ equal to $2I_z/c\rho$. The crossed electric E_z and magnetic H_φ fields induce radial superfluid flows which add up to the radial flow found above. The energy of the total flow is

$$\mathcal{E} = 2\int_r^R \int_0^{2\pi} \frac{\rho_s}{2m^2}\left(\hbar\nabla\varphi - \frac{\varepsilon\mu - 1}{4\pi nc}\mathbf{H}\times\mathbf{E}\right)^2 \rho\, d\rho\, d\theta = \frac{2\pi\rho_s}{m^2}\left(\frac{\pi\hbar s}{\ln R/r} - \frac{\varepsilon\mu - 1}{n}\frac{\sigma I_z}{c^2}\right)^2 \ln R/r\,. \tag{16}$$

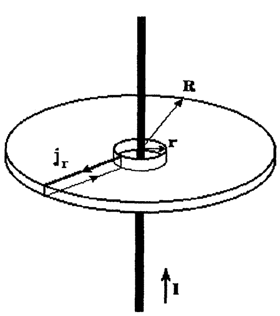

FIG. 1. A disk covered with a superfluid film in the magnetic field of current. On charging the disk, the electric field E_z normal to the disk surface and the magnetic field H_φ circulating around the wire induce dissipation-free radial currents j_r in the film.

In this expression the integer s can be found from the requirement for the minimum of energy $\mathcal{E}$. As the product σI_z increases, the integer s takes on the values $0, 1, \ldots$. The energy is an oscillating function of the density of charge σ and current I_z. The oscillation period can be found from the condition $(\varepsilon\mu - 1)\sigma I_z/nc^2 = \pi\hbar/\ln R/r$.

Now we shall discuss experiments in which we can measure directly the circulation of a superfluid liquid in crossed electric and magnetic fields. The first of the experiments proposed is similar to that known as Vinen's experiment [15] on measuring circulation of He II in a rotating vessel (see also [16]). Let us assume that a thin flexible charged wire of radius r is stretched in a vessel with a superfluid liquid, and a homogeneous magnetic field H is switched on parallel to the wire (the axis z). As the wire is moving in the plane xy, four forces will act on its unit length: tension $T\partial^2\mathbf{u}/\partial z^2$ (here and henceforth $\mathbf{u}$ is the displacement of the wire and T is the tensile force applied along the wire), Lorentz force $(\tau H/c)(\partial\mathbf{u}/\partial t) \times \mathbf{z}$, Magnus force $\rho_s\kappa\mathbf{z} \times (\partial\mathbf{u}/\partial t)$, where κ is the circulation from Eq. (14), and, finally, friction. If vibration frequency of the wire ω obeys the inequality $\omega \gg \nu_n/r^2$ (ν_n is the kinematic viscosity of the normal component of the liquid) and the vibration amplitude of the wire a is small as compared to r, the friction force is [17] $2\pi\rho_n r\sqrt{2\nu_n\omega}\,\partial\mathbf{u}/\partial t$. The equation of motion of the wire is

$$\pi r^2(\rho_0 + \rho)\frac{\partial^2\mathbf{u}}{\partial t^2} = T\frac{\partial^2\mathbf{u}}{\partial z^2} + \left(\frac{\tau H}{c} - \rho_s\kappa\right)\frac{\partial\mathbf{u}}{\partial t} \times \mathbf{z} - 2\pi\rho_n r\sqrt{2\nu_n\omega}\,\frac{\partial\mathbf{u}}{\partial t}\,. \tag{17}$$

Here ρ_0 is the density of the wire material and $\rho = \rho_s + \rho_n$ is the total density of the liquid. The coefficient before $\partial^2\mathbf{u}/\partial t^2$ shows the effective mass of the unit length of the wire, which is the sum of the wire itself and the added mass equal to the mass of the liquid expelled by the wire (for a cylindical wire).

The eigenfunctions of Eq. (17) are $\mathbf{u} = \mathbf{u}_0\exp[i(kz - \omega t)]$, where the components u_{0x} and u_{0y} are related as $u_{0x} = \pm i u_{0y}$. The signs $-$ and $+$ correspond to the right- and left-polarized modes, respecitvely. The corresponding eigenfrequencies are

$$\omega_\pm = \Omega_\pm + [\Omega_\pm^2 + Tk^2/\pi r^2(\rho_0 + \rho)]^{1/2}. \tag{18}$$

Here

$$\Omega_\pm = \left[\pm\left(\frac{\tau H}{c} - \rho_s\kappa\right) - i2\pi\rho_n r\sqrt{2\nu_n\omega}\right] \Big/ 2\pi r^2(\rho_0 + \rho)\,. \tag{19}$$

Thus the normal modes of the charged wire in the magnetic field are circularly polarized; the frequencies of the rotation in opposite directions differ by $\Delta\omega = \omega_+ - \omega_-$. At low temperatures the imaginary part of Ω is much smaller than the real one; i.e., the attenuation time of the vibrations is large as compared to their periods. The circulation κ in crossed fields can therefore be measured by measuring $\Delta\omega$. If κ is replaced by its value from Eq. (14), then $\Delta\omega$ is

$$\Delta\omega = \left[1 + \frac{\rho_s}{\rho}(\varepsilon - 1)\right]\frac{\tau H}{c}\frac{1}{\pi r^2(\rho_0 + \rho)} \qquad (20)$$

in weak fields, when the integer s in Eq. (14) is zero. The addition proportional to ρ_s in Eq. (20) is due to nondissipative flows which induce crossed electric and magnetic fields in the superfluid liquid. For He II at $\rho_s \simeq \rho$ this addition is about 0.05 of the frequency difference $\Delta\omega$ in the normal phase (i.e., at $\rho_s = 0$).

To conclude, an experiment is described in which the superfluid currents induced by crossed fields can be detected through measuring the static characteristics of the system. Let oppositely charged cylinders of radius r be immersed in a superfluid liquid and the magnetic field H be applied in parallel to the cylinder axes (see Fig. 2). In the presence of flow with velocity $\mathbf{v}_0$, the cylinders experience oppositely directed Magnus forces $\rho_s\kappa_\pm\ell(\mathbf{v}_0 \times \hat{\mathbf{z}})$, where $\kappa_\pm$ is the velocity circulation about the positively and negatively charged cylinders. The couple of Magnus force turns the thread OB (see Fig. 2). The rotation angle of the thread in its bottom part is

$$\varphi = \frac{2L}{\pi a^4}\frac{\ell}{\mu}(\mathbf{R}\cdot\mathbf{v}_0)\rho_s(\kappa_- - \kappa_+). \qquad (21)$$

Here μ is the shear modulus of the thread, a is the thread radius, and R is the length of the arm AB. The other symbols are given in Fig. 2. If the absolute values of the charges on the cylinders coincide and are equal per unit length τ, the maximum modulus of the angle φ is $(4\pi LR\ell/m\mu a^4)\rho_s v_0$. This value will be achieved at $\gamma = \frac{1}{2} + s$, where $\gamma = 2\alpha\tau H/c\hbar$ and s is an integer. The relation $\gamma = \frac{1}{2} + s$ dictates the values of the field H and the charge τ at which the circulation κ around each of the cylinders changes by jumping to the neighboring quantized value. The jumps are accompanied by the change of the φ sign. This process is periodically repeated when γ changes. The oscillations of φ should have the period $\gamma = 1$.

Now let us consider the numerical estimates. Let $L = 50$ cm, $a = 10^{-3}$ cm, $\ell = 10$ cm, $r = 1$ cm, $v_0 = 1\ \mathrm{cm\,s^{-1}}$, $\mu = 3 \times 10^{11}\ \mathrm{erg\,cm^{-3}}$, $\rho_s \simeq \rho = 1.5 \times 10^{-1}\ \mathrm{g\,cm^{-3}}$, $m = 6.6 \times 10^{-24}$ g (the latter two values are for ^{4}He). Then $\varphi_{\max} = 0.75$ rad. In the magnetic field $H = 10^5$ G this $\varphi_{\max}$ is first achieved for ^{4}He ($\alpha \simeq 2 \times 10^{-25}\ \mathrm{cm^3}$) in the electric field $E \simeq 2.5 \times 10^5$ V/cm at the cylinder surfaces. The field can be decreased if we increase the radius r of the cylinders (the field $E \sim r^{-1}$). These estimates show that, even though the observation of oscillations v_s with H and τ is no simple problem, the fact of the superfluid currents appearing in crossed fields can be established without trouble since, e.g., at $H = 10^5$ G, $E = 10^4$ V/cm, the angle $\varphi \simeq 2°$.

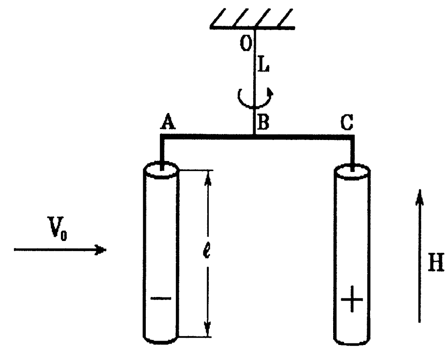

FIG. 2. Schematic view of the system for measuring superfluid currents induced by crossed electric and magnetic fields.

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