Novel Spectroscopic Method for Analysis of Nonthermal Electric Fields in Plasmas

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In this Letter we present a simple and reliable method for the determination of the mean amplitude of the nonthermal fields and the ion Doppler temperature, based on the analysis of the Stark and Doppler broadening autocorrelation function. The method makes *no assumption* on the distribution function of the turbulent field or that the nonthermal fields are quasistatic. An independent "typical field" analysis may be performed to obtain information on the field frequency.

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The investigation of turbulent fields in a variety of plasma devices, for example, pulsed plasmas, is of much importance since the presence of turbulence may significantly affect the plasma behavior and the device operation. Turbulent fields should also be accounted for in interpreting spectroscopic measurements [1,2]. Nonintrusive spectroscopic methods can be highly useful if the spectroscopic information can be properly analyzed. In this Letter, we present a simple but reliable method for the determination of the mean amplitude $\langle E^2 \rangle$ of the nonthermal electric fields (NF's) from the spectral profiles of Stark-broadened lines. Treatments based on a standard impact/unified theory [1,3,4] or a simulation [5,6] for the analysis of line profiles in the presence of NF's require as an input the functional form of the time-dependent NF's, along with the distribution functions. Thus one needs to know the *functional form* of the stochastic nonthermal microfield

$$\mathbf{E}(t) = \mathbf{E}(t, [\alpha]), \tag{1}$$

where $[\alpha]$ is a set of parameters, as well as the *distribution function* for these parameters. For pure thermal fields, the set $[\alpha]$ consists of particle velocities, impact parameters, times of closest approach, and angles, where the total field is, in the independent particle model, a sum of Debye-shielded fields [6]. For the NF's, however, neither the functional form of $\mathbf{E}(t, [\alpha])$ nor the distribution functions are generally known, and simple models are used [7–9]. *In addition,* an impact/unified treatment is only valid for weak perturbations, which is usually an exception for NF's.

On the other hand, a quasistatic (QS) treatment is often used for the turbulent fields. Such treatments employ entire line profiles [10–12], or simply linewidths [13], and assume a one- or more dimensional Gaussian electric field probability distribution function (PDF). It is well known [5,6,14], however, that the QS approximation is quite bad for low- and medium-density plasmas, especially for lines with a strong unshifted component, such as H_α. Thus, a treatment that assumes QS turbulence for the *entire* line profile, although perhaps justifiable in specific cases, is in general inapplicable, unless thermal and Doppler broadening are large enough to produce linewidths that are much larger than the field fluctuation frequency. In addition, although the tail of the turbulent distribution function is expected to be Gaussian [15], a Gaussian approximation for the *entire* distribution function may be incorrect, as there are also other intervals where the shape of the distribution function is expected to be different. For example an E^{-5} dependence is predicted in the self-similar range [15].

The proposed method starts by Fourier transforming the experimental spectrum to obtain the experimental autocorrelation function (AF), denoted $C(t)$. All information will be determined from the short-time behavior of $C(t)$. This is essentially the same as using the line wings, rather than the widths. However, dealing with the AF rather than the profile appears to be clearer and simpler, in that no deconvolution of the thermal Stark profile is required. For short times, $C(t)$ is correctly reproduced by the QS approximation, i.e.,

$$C(t) = \sum_k L_k C_k(t), \tag{2}$$

where L_k are known constant, line-specific numerical coefficients, with the obvious property of $\sum_k L_k = 1$ and

$$C_k = \int_0^\infty \cos(a_k E) W(E)\, dE, \tag{3}$$

with $W(E)$ the total PDF, $a_k = skt$, and s a numerical constant. For example, for H_β, the QS AF is [16]

$$\begin{aligned} C_{H_\beta}(t) = \int dE\, W(E)[&153\cos(2x) + 912\cos(4x) \\ &+ 669\cos(6x) + 384\cos(8x) + 373\cos(10x) \\ &+ 16\cos(12x) + \cos(14x)]/2508, \end{aligned} \tag{4}$$

with $x = sEt$ and $s = 1.5ea_0/\hbar$, respectively. (In the case of lines with a quadratic Stark effect, this line of analysis yields $\langle E^4 \rangle$, which is also a useful quantity.) If we neglect Doppler and thermal Stark broadening, i.e., if $W(E)$ is due purely to NF's, then $C(t)$ is *quadratic* in time for short enough times. This fact *does not* depend on the precise functional form of $W(E)$, i.e., whether $W(E)$ has a Gaussian or more complicated decay; the only requirement is that $W(E)$ drops fast enough for large

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E so that $\langle E^2 \rangle = \int_0^\infty dE\, W(E) E^2$ is finite, in contrast to the thermal case. Thus the requirements of the present method are even weaker than a Gaussian tail; the tail has only to decrease fast enough so that $\langle E^2 \rangle$ is finite. It is believed that $W(E)$ is such that *all* moments exist, whether there are corrections to the Gaussian decay or not. In that case, for short enough times, the Taylor approximation $\cos(ax) \approx 1 - (a^2x^2/2)$ yields

$$C_k(t) = 1 - \frac{a_k^2 t^2}{2} \langle E^2 \rangle + O(t^4). \tag{5}$$

Thus, for example,

$$C_{H_\alpha}(t) = 1 - 2s^2t^2\langle E^2 \rangle + O(t^4) \tag{6}$$

and

$$C_{H_\beta}(t) = 1 - \frac{62s^2t^2\langle E^2 \rangle}{3} + O(t^4). \tag{7}$$

Note that Yakovlev [17] had proposed to obtain the moments of the electric field by integrating the experimental profile multiplied by ω to the appropriate power, but Oks and Sholin [11] pointed out that the net $\langle E^2 \rangle$, obtained by integrating E^2 with the *total* microfield distribution, is still divergent due to the tail of the particle field distribution. All these convergence problems may be attributed to using $C(t)$ and its derivatives at exactly $t = 0$. If we use $C(t)$ at small, but nonzero times, the cosines provide convergence to the integrals (3), but then the thermal and NF contributions must be decoupled to obtain information on $\langle E^2 \rangle$.

By evaluating C_k for a joint distribution of a NF and a thermal field, and using a Holtsmark distribution for the thermal field, one may evaluate C_k and show that $C(t)$ has the short-time behavior

$$C(t) = 1 - b_1[\tfrac{5}{2}(sE_0t)^{3/2}] - \frac{t^2}{2}\left[b_2 s^2 \langle E^2 \rangle + \frac{k_B \Theta \omega_0^2}{Mc^2}\right] + O(t^3), \tag{8}$$

E_0 being the Holtsmark field [1], M the emitter mass, Θ its temperature, k_B the Boltzmann constant, ω_0 the unperturbed frequency, and c the speed of light. b_1 and b_2 are defined as $b_1 = \sum_k L_k k^{3/2}$ and $b_2 = \sum_k L_k k^2$. It should be pointed out that *no assumption* of isotropy is made for the NF. Also, a Holtsmark PDF for the thermal field is justified, not only because of our experimental situation, but because we are interested in the short-time behavior of the AF, which is determined by large fields, for which a Holtsmark PDF is valid. We note that for very short times the thermal $t^{3/2}$ contribution dominates. As a result, it is *in principle* possible to obtain the Holtsmark field E_0, and from it the density, as the limiting value of the quantity

$$R_1 = \frac{1 - C(t)}{\frac{5}{2}(st)^{3/2} b_1} = E_0^{3/2} \tag{9}$$

as $t \to 0$, where $C(t)$ is the *experimental* AF (the Fourier transform of the line profile). Having done so, and having deconvolved the thermal Stark profile, one then obtains C_N, which is the ratio of the experimental over the pure thermal Stark AF, which has the short-time behavior

$$C_N(t) \to 1 - \frac{t^2}{2}\left[b_2 s^2 \langle E^2 \rangle + \frac{k_B \Theta \omega_0^2}{Mc^2}\right]. \tag{10}$$

If the Doppler temperature is known or may be neglected, this yields directly $\langle E^2 \rangle$. In the opposite case, at least two lines are needed. Let us emphasize that if one deconvolves the pure Stark profile one should make sure that a theory that handles the line wings correctly has been used. A simple QS calculation of the short-time thermal AF is sufficient for our purposes. In principle, therefore, $E_0^{3/2}$ is determined by plotting R_1 vs t as $t \to 0$. The quantity

$$R_2 = \langle E^2 \rangle + \frac{\omega_0^2 k_B \Theta}{s^2 c^2 M b_2} \tag{11}$$

is determined by plotting for short times

$$\frac{2[1 - C_N(t)]}{(st)^2 b_2} \tag{12}$$

vs t and taking the limiting value as $t \to 0$.

Turning now to practical questions, there are two essential requirements as to the times that may be used to determine the quantities of interest $E_0^{3/2}$ and R_2. The first requirement is to restrict oneself to times for which the QS approximation is valid. This criterion [18] reads $t \ll \langle \rho \rangle / v_0$ where $\langle \rho \rangle$ is the mean interparticle spacing and $v_0 = \sqrt{2k_B\Theta_p/\mu}$ with μ the reduced emitter-perturber mass and Θ_p the perturber temperature. In the case of nonthermal broadening, the right-hand side of the above criterion must be replaced by the inverse of the typical fluctuation frequency of the field, for which an *upper bound* may be given by the electron plasma frequency. Whereas in the thermal case a factor of 10 may not be sufficient for the applicability of the above criterion [14] due to the small impact parameter part of the distribution, in the nonthermal case there is no part of the distribution oscillating faster than the electron plasma frequency, and hence this criterion is much easier to use.

In addition to this requirement, the times used for the parameter determination must be such that higher order corrections within the QS approximation are negligible. In practice, this is the case if the quantities plotted approach a constant value as $t \to 0$, but it can also be easily verified from a simple evaluation of the expansion parameters.

These requirements may be too stringent as far as the Holtsmark field determination from R_1 is concerned, since, in order to preserve the validity of the QS approximation for the thermal Stark broadening *and* to ensure that the thermal term is the dominant one (compared to Doppler plus nonthermal), one is forced to restrict oneself to very short times, for which the experimental AF is only very slightly smaller than unity. Such an accuracy may be hard to achieve. However, not determining E_0 *does not* preclude an accurate determination of $\langle E^2 \rangle$ and Θ. If the mean NF amplitude $\langle E^2 \rangle^{1/2}$ is much larger than the typical particle field (of the order of the Holtsmark field), then the decay of $C(t)$ is very quickly dominated by the NF. (This also applied to Doppler broadening if it dominates thermal Stark broadening, and of course also to electron broadening, whose contribution to the AF for any time of practical interest is linear in time.) $\langle E^2 \rangle$ and Θ will, in that case, be determined in a region where the drop in $C(t)$ due to the thermal field is negligible compared to the nonthermal/Doppler drop and thus the thermal $C(t)$ need not be deconvolved at all (i.e., one may neglect the $t^{3/2}$ term); hence exact knowledge of E_0 is *not* required. The point is that if the turbulent field is dominant $\langle E^2 \rangle$ is determined from a spectral region that is not in the too distant wings.

If $L(\omega)$ is the experimental profile and $L(\omega) + \delta L(\omega)$ the true profile, then one may show that the relative error $\Delta C(t)$ in $1 - C(t)$ and hence all quantities linear in it is

$$\Delta C(t) = \frac{r_0[r_1(t) - C_E(t)]}{1 - C_E(t) + r_0[1 - r_1(t)]}, \qquad (13)$$

where $r_0 = \int \delta L(\omega)\, d\omega / \int L(\omega)\, d\omega$ and $r_1(t) = \int \delta L(\omega) e^{\iota\omega t}\, d\omega / \int \delta L(\omega)\, d\omega$, with $C_E(t) = \int L(\omega) \times d\omega e^{\iota\omega t} / \int L(\omega)\, d\omega$ the experimental AF. The point is that the numerator in $\Delta C(t)$ is, for short times, the product of two small numbers, while the denominator is the sum of two small numbers; hence ΔC should be small. The experimental error $\delta L(\omega)$ arises from the "missing frequency range" (MFR), i.e., the fact that the experimental profile does not extend to infinity, electrical and shot noise (N), and possibly other causes. The net r_0 is simply the sum of the MFR and Nr_0. For the net $r_1(t)$,

$$r_1(t) = \frac{\int \delta L_{\mathrm{MFR}}(\omega) e^{\iota\omega t}\, d\omega + \int \delta L_N(\omega) e^{\iota\omega t}\, d\omega}{r_0 \int L(\omega)\, d\omega}. \qquad (14)$$

Worst case MFR estimates are easy to do, since the slowest decay rate is given by the Holtzmark asymptote. Improved bounds may be obtained by extrapolation of the observed spectrum into the wings. Bounds for the noise contribution may be obtained from its variance, which is in turn estimated [19] from its power spectrum. Alternatively, such bounds may be deduced from noise level estimates [20].

To illustrate the use of the method, Fig. 1 shows an $\langle E^2 \rangle$ determination using the Balmer α and β profiles from a recent experiment with a plasma opening switch [20,21]. For this experiment, a 2 eV Doppler temperature was determined from measurements at an earlier time, and estimates based on collision and charge exchange rates. Thus, after deconvolving a Doppler profile corresponding to a temperature of 2 eV, one finds $\langle E^2 \rangle$ to be 227 (kV/cm)^2 from H_α and about 207 (kV/cm)^2 based on H_β, as Fig. 1 shows.

An improved version is not to assume the Doppler temperature, but to calculated it self-consistently. Thus,

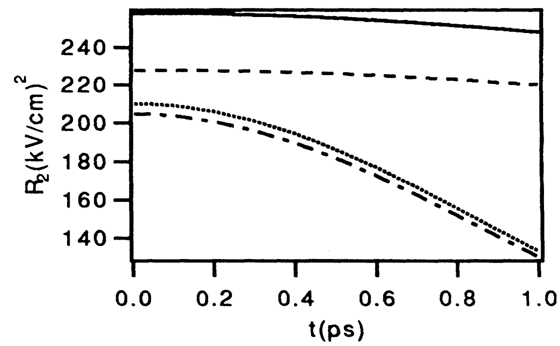

FIG. 1. Determination of $\langle E^2 \rangle$ from the short-time behavior of the AF for the experiment described in Refs. [20,21]. The solid line is R_2 determined by using the H_α experimental AF, the dashed line uses the H_α experimental AF after deconvolving a 2 eV Doppler contribution, the dotted line uses the H_β experimental AF, and the dash-dotted line uses the H_β experimental AF after deconvolving a 2 eV Doppler contribution. It may be seen that for the $\langle E^2 \rangle$ determination from H_α and H_β, times shorter than about 0.6 and 0.2 ps, respectively, are used.

a self-consistent $\langle E^2 \rangle$ and Θ determination from H_α and H_β, i.e., determining R_2 for each line and solving the system of these two equations and two unknowns $\langle E^2 \rangle$ and Θ, yields $\langle E^2 \rangle = 200$ (kV/cm)^2 and $\Theta = 1.9$ eV.

Such a calculation of $\langle E^2 \rangle$ may be supplemented by a "typical" field analysis. Recall that the AF is an average of product of U matrices describing the propagation under a single realization of the stochastic E field [14]. A typical field is a single field with an *assumed oscillatory* form, which would result in an unaveraged AF (UAF) that is close to the experimental AF for not too long times. There are two physical effects involved in this definition and illustrated in Fig. 2: The first is that for a given fluctuation frequency the initial decay of the UAF is faster for stronger fields. The second effect is that for a given field amplitude this decay is slower for a faster oscillating field. This second effect is very simple to see from the Schrödinger equation, for a rapidly changing sign of the interaction. Since the plasma frequency represents an upper bound for the field fluctuation frequency, and, furthermore, since a QS field is ineffective in broadening the central component of H_α, the typical field amplitude is between the value that most closely matches the decay of the experimental AF at zero (or any QS) frequency and the value that matches the experimental AF at the plasma frequency. The combination of these two effects results in a relative insensitivity to the short and intermediate time UAF with functional form. For each experimental profile one may thus construct a field amplitude vs frequency curve that most closely matches the experimental AF. If one has two profiles, the intersection of these curves gives the typical field amplitude and frequency. It must be emphasized that this approach completely neglects the statistics. The typical field is meaningful to the extent

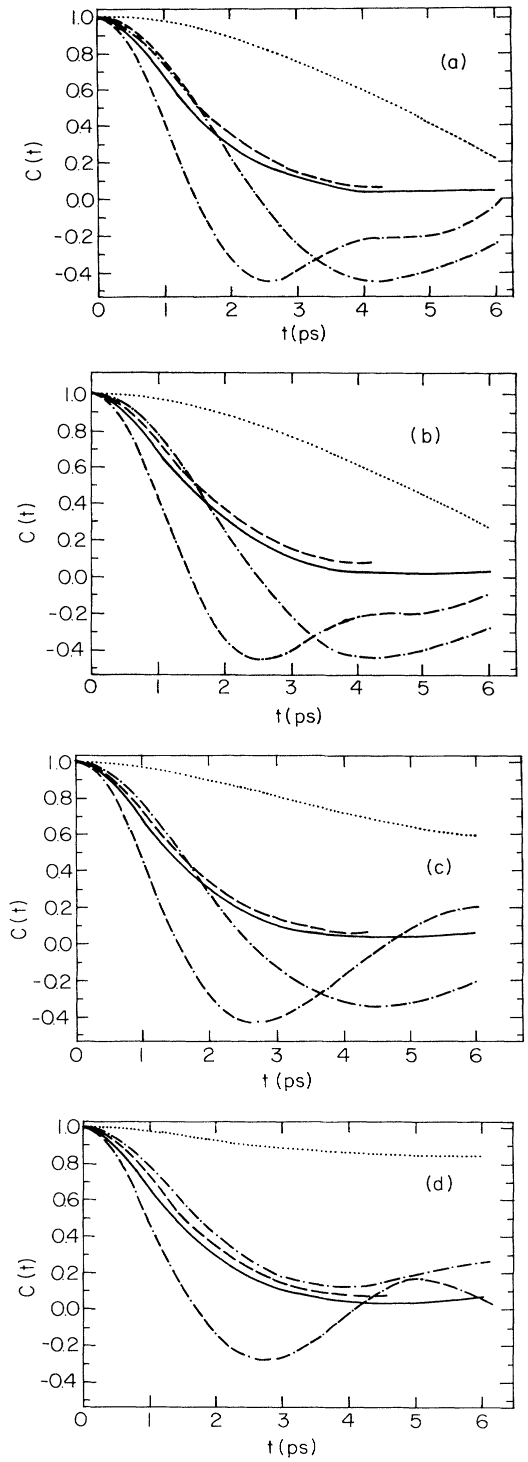

FIG. 2. Experimental and calculated AF's for H_β for the experiment of Refs. [20,21]. The solid line is the experimental AF and the dashed line the experimental AF divided by the thermal AF. The field amplitude is, respectively, 2.16 (dotted), 10.8 (dash-dotted), and 21.6 kV/cm (double-dash-dotted). The fluctuation frequency is (a) $\Omega = 3 \times 10^{10}$ Hz, (b) $\Omega = 9 \times 10^{10}$ Hz, (c) $\Omega = 3 \times 10^{11}$ Hz, and (d) $\Omega = 5.6 \times 10^{11}$ Hz.

that the NF may be described by amplitude and frequency and then only in the sense that a distribution of fields with a dominant probability density region significantly displaced from the above-mentioned curve will result in too fast or too slow an AF decay that will be inconsistent with the experiment.

In summary, a new nonperturbing method for the determination of $\langle E^2 \rangle$ is proposed, which makes *no assumption* on the turbulent PDF and does not assume weak or quasistatic turbulence. The method requires good quality line profiles covering a fair part of the line wings. In addition, one may obtain the "typical" field amplitude and fluctuation frequency from a solution of the Schrödinger equation for a *single* field. The method has been demonstrated obtaining the mean electric field amplitude in our plasma opening switch experiment using the observed H_α and H_β profiles.

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