Persistent Small Scale Anisotropy in Homogeneous Shear Flows

Alain Pumir,[1] and Boris I. Shraiman[2]
[1] Institut Nonlinéaire de Nice, CNRS, 1361 route des Lucioles, F-06560, Valbonne, France
[2] AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974
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The turbulent velocity fluctuations in a homogeneous shear flow (mean velocity $\vec{U} = Sy\hat{x}$) are studied numerically, in the range of Reynolds number $2600 \leq \mathrm{Re} \leq 11\,300$. The skewness of the z component of vorticity is independent of the Reynolds number in this range, suggesting that some small scale anisotropy remains at very high Reynolds numbers. This anisotropy is not seen in the two-point correlation functions. An analogy is drawn with earlier results on turbulent mixing of a passive scalar with a mean gradient. The relevance of this work for turbulent boundary layers is discussed.

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Fluid turbulence is known to involve a wide range of length scales. The largest scales of motion are controlled in real flows by boundary conditions which are specific to the system. However, according to Kolmogorov theory (K41) [1] small scale fluctuations should be independent of the structure of the large scale flow provided the Reynolds number Re is large enough. As a corollary, one expects the small scale fluctuations to be isotropic and have "universal" statistics. Decades of experiments on a number of flows have shown that the two-point functions, such as $\langle [u(x) - u(0)]^2 \rangle$ (where $\langle \cdots \rangle$ denotes a statistical average), closely follow K41's predictions [2–4], including the prediction for the isotropization on small scales [4–6]. The K41 theory, however, fails for the $2n$ moments of the structure function $\langle [u(x) - u(0)]^{2n} \rangle$ (for $n \geq 2$) which do not agree with the K41 predictions neither in the inertial [7] nor in the dissipative [8] range. This is attributed to the phenomenon of "intermittency" for which a variety of phenomenological theories [9] have been proposed. Yet the small scale fluctuations contributing to the high moments are always assumed to be isotropic.

The purpose of this Letter is to point out a violation of this isotropy assumption for homogeneous shear flow. Below we shall present the results of a numerical simulation demonstrating, within the accessible range of Re numbers ($\mathrm{Re} \leq 11\,300$), that although the two-point function of vorticity becomes more isotropic with increasing Re, the normalized third moment (skewness) of vorticity component consistent with large scale shear remains constant, indicating persistent anisotropy of the higher order correlators.

The above phenomenon is analogous to the behavior of a passive scalar in turbulent flow. It has been known for some time [10] that in a turbulent boundary layer over a heated wall the time derivative of temperature measured at one point is strongly asymmetric, resulting in a large skewness. More recently, a similar effect has been observed in numerical simulations [11,12] and experiments [13] on a passive scalar Θ, mixed by a homogeneous, isotropic random [11] or turbulent [12,13] flow, in the presence of an imposed mean scalar gradient: $\langle \Theta \rangle = \vec{G} \cdot \vec{x}$. Whereas the Kolmogorov type estimate [11] would predict longitudinal gradient skewness $s(\partial_\parallel \Theta) \equiv \langle (\partial_\parallel \Theta)^3 \rangle / \langle (\partial_\parallel \Theta)^2 \rangle^{3/2} \sim \mathrm{Re}^{-1/2}$ (where $\partial_\parallel \Theta \equiv G \cdot \nabla \Theta / |G|$), the observed value is $O(1)$. A snapshot of the scalar field [see Fig. 1(a)] reveals a "terraced" structure consisting of well-mixed "plateau" regions (of size comparable to the integral scale) separated by "cliffs" corresponding to sheets of large gradient, as if the imposed gradient has been expelled from the bulk of the flow and has gotten concentrated in sheets leading to the large positive $s(\partial_\parallel \Theta)$. This mechanism for the appearance of small scale anisotropy might be imagined to work also for vorticity in a shear flow and has motivated the present investigation.

Below we present the results of a direct numerical simulation of a homogeneous shear flow. The imposed mean flow is $U = (Sy, 0, 0)$, where S is the shear. The size of the computational domain is $L = 2\pi$ in all directions. The Navier-Stokes equations imply that the velocity fluctuations $\vec{u} \equiv (u, v, w)$ obey

$$\partial_t \vec{u} + Sy \partial_x \vec{u} + Sv\hat{x} + (u \cdot \nabla) u = -\nabla p + \nu \nabla^2 u, \quad (1)$$

$$\nabla \cdot u = 0. \quad (2)$$

Because of the advection of velocity fluctuations by the mean flow, the field cannot be periodic in the normal, y, direction. However, it is possible to impose periodic boundary conditions in the variables $x' = x - Syt$, $y' = y$, and $z' = z$, and therefore use efficient pseudospectral methods [14]. To prevent too wide a difference between the computational and the physical mesh, the moving coordinates (x', y', z') are periodically remeshed, with a period S^{-1}. By analogy with the definition used in Ref. [15], the Reynolds number is defined by $\mathrm{Re} \equiv SL^2/\nu$.

Equation (1) implies the following equation for the kinetic energy:

$$\partial_t \langle \vec{u}^2 \rangle / 2 + S \langle uv \rangle = -\nu \langle \vec{\omega}^2 \rangle, \quad (3)$$

where $\vec{\omega}$ is vorticity. In a system of finite size one expects to reach a statistically steady state [16], characterized

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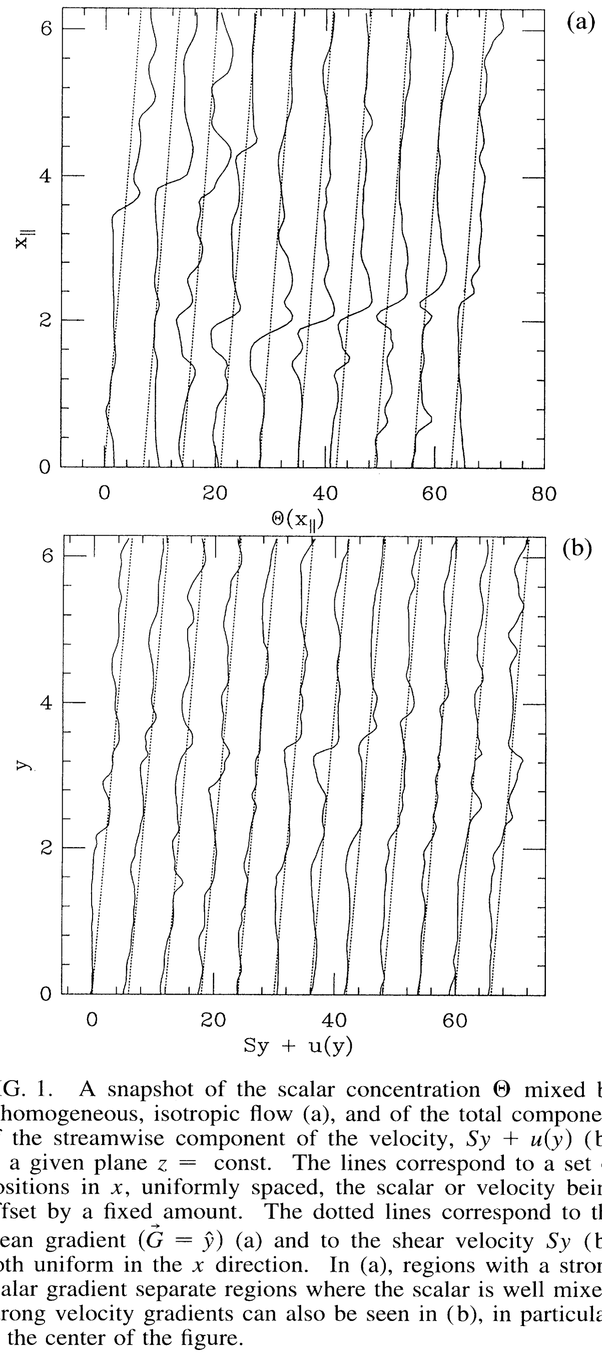

FIG. 1. A snapshot of the scalar concentration Θ mixed by a homogeneous, isotropic flow (a), and of the total component of the streamwise component of the velocity, $Sy + u(y)$ (b), in a given plane $z =$ const. The lines correspond to a set of positions in x, uniformly spaced, the scalar or velocity being offset by a fixed amount. The dotted lines correspond to the mean gradient ($\vec{G} = \hat{y}$) (a) and to the shear velocity Sy (b), both uniform in the x direction. In (a), regions with a strong scalar gradient separate regions where the scalar is well mixed. Strong velocity gradients can also be seen in (b), in particular, in the center of the figure.

by a balance between the production term $-S\langle uv\rangle$, the Reynolds stress tensor, and the dissipation $\nu\langle\vec{\omega}^2\rangle$, at least for high enough Reynolds numbers. The transient regime is characterized by a violent growth of the kinetic energy $\langle\vec{u}^2\rangle/2$ and enstrophy $\langle\vec{\omega}^2\rangle$. While this growth eventually stops, as Fig. 2 demonstrates, the turbulent regime that follows exhibits large fluctuations of spatially averaged quantities (much larger than for simulations of

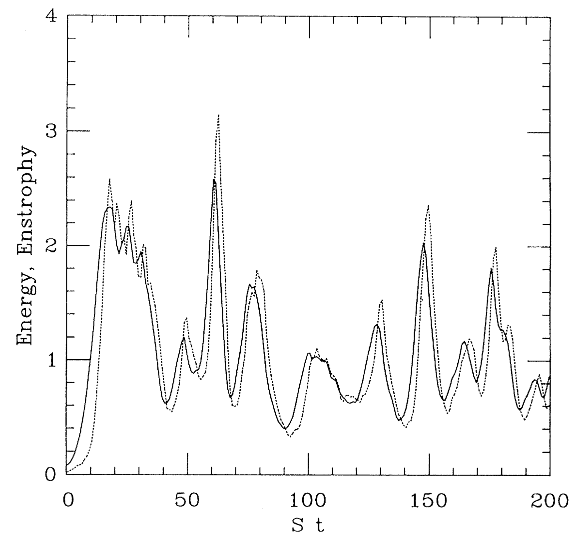

FIG. 2. Energy fluctuations (full line), and enstrophy fluctuations (dotted line), as a function of time for our run 3. Both quantities have been divided by their rms: $\langle u^2/2\rangle = 1.85$ and $\langle\omega^2\rangle = 49.26$. Spikes in kinetic energy are followed by spikes in enstrophy (i.e., dissipation). The level of the fluctuations is about 50% and roughly independent of the Reynolds number.

homogeneous, isotropic turbulence), at least for Re $\gtrsim$ 2600. The initial phase of the run (for $St \lesssim 40$) was systematically discarded, when computing the statistical averages discussed below. Because of the unusually large level of fluctuations, very long runs are necessary to get steady averages, which explains why we chose to work at moderate resolution (144^3). The convergence of the results was studied for the lowest Reynolds number. Table I contains a list of runs, with the sampling time T_s, measured in units of the shear time scale (S^{-1}), the effective resolution $\Delta x/\eta$, where $\eta \equiv (\nu^2/\langle\vec{\omega}^2\rangle)^{1/4}$ is the Kolmogorov length scale. Comparison of runs 1 and 2 shows that our resolution is adequate. Some of the main statistical results are contained in Table II and are discussed below.

A measure of the small scale anisotropy is provided by the norm of the following tensor: $v_{ij} \equiv \langle\omega_i\omega_j\rangle/\langle\vec{\omega}^2\rangle - \delta_{ij}/3$. This quantity decreases with increasing Reynolds number; see Table II. Although the range of scales available in our numerical simulations cannot match Ref. [4],

TABLE I. A list of our runs showing the number of grid points (n), the effective resolution achieved ($\Delta x/\eta$), and the sampling time, in units of the shear time (T_sS^{-1}).

Run No.	Re	n	$\Delta x/\eta$	ST_s
1	2631.9	48	2.7	210
2	2631.9	64	2.0	210
3	3432.9	64	2.4	160
4	4386.5	80	2.2	110
5	6073.6	100	2.2	70
6	8224.7	108	2.5	80
7	11279.5	144	2.2	72

TABLE II. The variance of the vorticity fluctuations, the vorticity anisotropy tensor ($\langle v_{ij} v_{ij} \rangle$), and the skewness of the z component of the vorticity and of the partial derivative of u in the z direction.

Run No.	$\langle \omega^2 \rangle / S^2$	$(v_{ij} v_{ij})^{1/2}$	$S(\omega_z)$	$S(\partial_y u)$
1	38.4 ± 1.4	0.17 ± 0.006	−0.54 ± 0.05	0.84 ± 0.06
2	36.6 ± 1.5	0.17 ± 0.006	−0.54 ± 0.05	0.86 ± 0.07
3	49.2 ± 2.4	0.15 ± 0.007	−0.53 ± 0.05	0.88 ± 0.07
4	53.4 ± 3.5	0.14 ± 0.009	−0.54 ± 0.07	0.87 ± 0.07
5	59.2 ± 3.1	0.14 ± 0.008	−0.54 ± 0.05	0.82 ± 0.08
6	82.9 ± 4.2	0.12 ± 0.007	−0.54 ± 0.06	0.79 ± 0.08
7	86.3 ± 3.5	0.12 ± 0.006	−0.58 ± 0.06	0.87 ± 0.07

the decrease of $(v_{ij} v_{ij})^{1/2}$ with Re is roughly consistent with Corrsin-Lumley's argument [5,6] that the anisotropy on any small scale r is of the order of the ratio of the large scale strain to the strain on scale r, which for the dissipative range implies $(v_{ij} v_{ij})^{1/2} \sim S/\langle \vec{\omega}^2 \rangle^{1/2} \sim \mathrm{Re}^{-1/2}$. One may note that in contrast to vorticity the local velocity covariance tensor (the Reynolds stress) $\langle u_i u_j \rangle$ is dominated by the large scales and remains anisotropic [20].

The above estimate of small scale anisotropy applies equally well to the skewness. However, instead of decreasing [5,6] as $s/\langle \vec{\omega}^2 \rangle^{1/2}$ the numerical data yield

$$s(\omega_z) \equiv \frac{\langle \omega_z^3 \rangle}{\langle \omega_z^2 \rangle^{3/2}} \approx -0.53\,, \tag{4a}$$

$$s(\partial_y u_x) \equiv \frac{\langle \partial_y u_x^3 \rangle}{\langle \partial_y u_x^2 \rangle^{3/2}} \approx 0.85 \tag{4b}$$

independent of Re (Table II). Note that we took here $S > 0$; changing the sign of S would result in opposite signs in Eqs. (4a) and (4b). The probability distribution function (PDF) of ω_z, shown in Fig. 3(a) for runs 2 and 7, does not change much in the range of Re covered here.

Thus the shear flow exhibits a similar violation of small scale isotropy, quantified by the derivative skewness, as the passive scalar problem. However, the skewness in the former is comparatively smaller than the skewness of the scalar derivative $\partial_\parallel \Theta$, which was found [11–13] to be 1.5–2. The large skewness of the scalar derivative was shown to be related to the "cliff and plateaux" structure, clearly seen in physical space in Fig. 1(a) [11,12]. The situation is similar in the shear flow problem, although less pronounced [compare Figs. 1(a) and 1(b)]. Strong vortex sheets, corresponding to large, positive values of $\partial_y u_x$, can be found, but their size in the (x, z) plane is comparatively smaller than the size of the scalar sheets. Also, whereas the effective expulsion of the scalar gradient from large regions results in the sharp maximum of the PDF [11,12,21] of the derivative of the scalar fluctuations at $\partial_\parallel \Theta = 0$, in the case of the velocity derivative $\partial_y u_x$ (or ω_z) the maximum of the PDF is located at about $-0.6S$ (or $0.6S$), which correspond to weaker "expulsion" (here, complete expulsion would correspond to the peak of the PDF of $\partial_y u_y$ at $-S$). However, vorticity is not a passive field, and one cannot expect more than a qualitative analogy between the two cases. In fact, in the steady state, the sheets always coexist with vortex tubes, which are at least as intense as the vortex sheets.

The fact that a large scale anisotropy seems to affect the moments of order $n > 2$ of the velocity derivative suggests that the study of intermittency effects should not be limited to the longitudinal correlation functions. One possible experimental system for studying the phenomenon in question is the logarithmic region of a turbulent boundary layer. These flows are characterized by a constant momentum flux, $|\langle uv \rangle| \approx u_*^2$. In boundary layers, the averaged shear varies like $\partial U/\partial y \approx u_*/y$, where y is the distance to the wall, and u_* is the scale of the velocity fluctuations [22]. The vorticity fluctuations behave as $\langle \omega^2 \rangle \sim u_*^3/\nu y$. The dimensionless ratio $\langle \omega^2 \rangle^{1/2}/\partial_y U \sim y^{1/2}$, implying that the vorticity fluctuations are getting bigger compared to the mean shear as one gets further out from the wall, corresponding to the increase of the effective Re number. If the scenario presented here is correct, one expects the anisotropy of the two-point function of vorticity to decrease with y but the vorticity skewness to be of $O(1)$, independent of y. This, in fact, is consistent with the existing measurements [23].

Finally, we mention that our simulation of the homogeneous shear layer also provides information about the PDF of velocity and momentum flux (or Reynolds stress tensor), which have been studied in considerable detail in the boundary layer experiments [24]. In these flows, the wall breaks the inversion symmetry $(u, v) \rightarrow (-u, -v)$, as well as translational invariance in y. Remarkably, we find that the coefficient of correlation [24] $R \equiv \langle uv \rangle / \langle u^2 \rangle^{1/2} \langle v^2 \rangle^{1/2} \approx -0.44$ is very close to what is observed experimentally in the logarithmic layer [4,22,24]. The possible "universality" of R may be rationalized on the basis of its relation to the energy balance [Eq. (3)]. The ratio $\langle v^2 \rangle / \langle u^2 \rangle \approx 0.37$ falls within the range of experimentally measured values. We find that the PDF of u is almost Gaussian and the PDF of v and w have exponential tails for $|v|$, $|w|$ larger than ~ 3 times the rms. This suggests further analogy with a passive scalar [25] (see [26] for a comparable study in turbulent boundary layers). The PDF of the momentum flux, uv, is shown in Fig. 3(b) for our run 7. This PDF hardly varies with Re.

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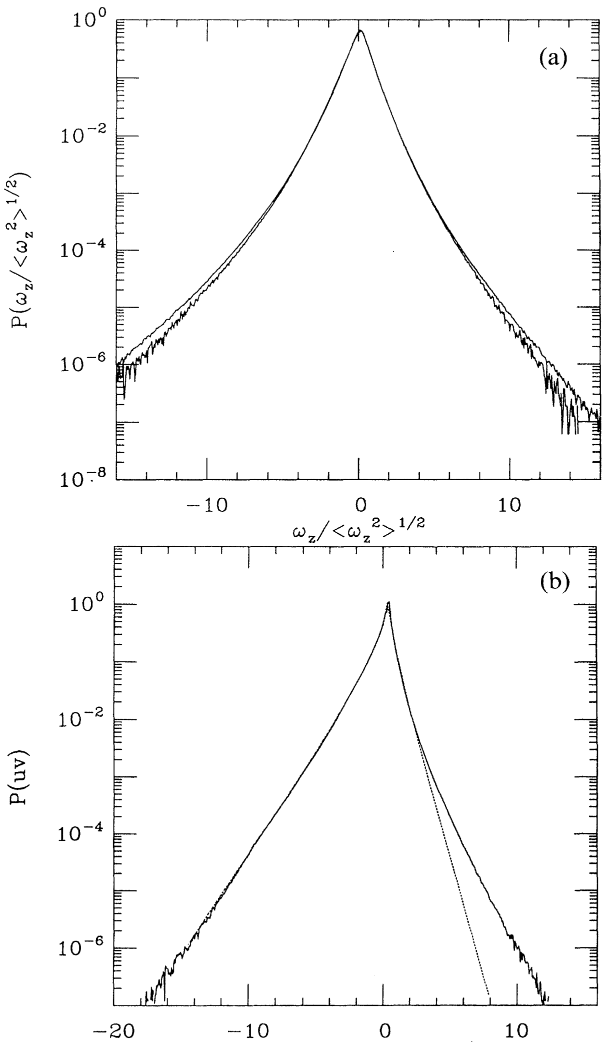

FIG. 3. Probability distribution function of the spanwise component of the vorticity (a) and of the Reynolds stress tensor $\langle uv\rangle$ (b). In (a), the curve corresponding to run 7 is above the curve corresponding to run 2. The negative skewness of the PDF of ω_z is easily seen in (a). In (b), the fit, shown by the dashed line, corresponds to a joint Gaussian distribution of u and v. The fit is excellent near the center of the distribution (compare with Ref. [25]), and on the negative side. Significant differences can be seen on the positive side.

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