Experimental Evidence for Differences in the Extended Self-Similarity Scaling Laws between Fluid and Magnetohydrodynamic Turbulent Flows

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It has been recently suggested that the various qth order velocity structure functions in turbulent flows are related to each other through well defined scaling laws, which extend outside the usual inertial domain. Even if theoretical models provide different scaling laws for fluid and magnetohydrodynamic turbulent flows, no attempt has been made up to now to furnish experimental evidence for these differences. By using measurements from the solar wind turbulence and from turbulence in ordinary fluid flows, we show that the differences can be observed only by looking at the high-order velocity structure functions.

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The most interesting aspect of fully developed turbulence is the existence of universal scaling behavior of small-scale fluctuations [1]. Indeed, turbulent flows are characterized by the presence of self-similarity in the inertial range, that is, on the spatial scale $\lambda \ll \ell \ll L$ (L is the energy injection scale and λ is the dissipative scale). In this range fluctuations reach an equilibrium state characterized by a continuous flux of energy from the scale L up to λ, which can be viewed in the real space as an energy cascade generated by the breakdown of eddies at different length scales (Richardson's energy cascade). The main statistical feature has been evidenced by Kolmogorov [2,3], who derived the well known relation

$$\delta V_\ell \sim \epsilon_\ell^{1/3} \ell^{1/3} \tag{1}$$

between the velocity differences $\delta V_\ell = |\vec{V}(\vec{x} + \vec{\ell}) - \vec{V}(\vec{x})|$ across the distance $\ell = |\vec{\ell}|$ [$\vec{V}(\vec{x})$ being the velocity field] and ϵ_ℓ, which represents the energy transfer rate per unit mass [4]. The same picture can be extended to magnetohydrodynamic (MHD) turbulence, even if, owing to the decorrelation effect of the large-scale magnetic field [5], the nonlinear interactions are lowered. This originates the Kraichnan scaling relation

$$\delta V_\ell \sim (c_A \epsilon_\ell)^{1/4} \ell^{1/4} \tag{2}$$

($c_A = B_0/\sqrt{4\pi\rho}$ is the Alfvén velocity of the large-scale magnetic field B_0 and ρ is the constant plasma mass density), which is valid for hydromagnetic flows. The interesting measurable quantities are the qth order velocity structure function $S_\ell^{(q)} = \langle \delta V_\ell^q \rangle$ (brackets denoting space average) and the scaling exponents ξ_q defined through

$$S_\ell^{(q)} \sim \ell^{\xi_q}, \tag{3}$$

which must hold in the inertial range [2,3]. In the absence of intermittency the linear scaling laws hold, $\xi_q = q/m$ ($m = 3$ for the Kolmogorov scaling law and $m = 4$ for the Kraichnan scaling law). If intermittency is taken into account, anomalous scaling laws are obtained where the scaling exponents ξ_q are nonlinear functions of q both in fluid flows [6,7] and in MHD flows [8–10].

Cascade models based on Richardson's picture have been developed in order to take into account intermittency corrections to the classical scaling laws, in both the fluid [11–14] and the MHD case [10,15–17]. Among others She and Lévéque [14] developed a model based on the hypotheses that the Kolmogorov refined similarity (1) is verified and that the moments of the energy transfer rate obey a given hierarchy

$$\epsilon_\ell^{(q+1)} \sim [\epsilon_\ell^{(q)}]^\beta [\epsilon_\ell^{(\infty)}]^{1-\beta} \tag{4}$$

($0 \le \beta \le 1$). The quantity $\epsilon_\ell^{(\infty)}$, obtained from the hierarchy in the limit $q \to \infty$, is associated with the most intermittent structures. It has been conjectured that this quantity has a divergent scaling $\epsilon_\ell^{(\infty)} \sim \ell^{-x}$. On the basis of relation (2), and assuming the same hierarchy (4), the model has been extended to MHD by Grauer, Krug, and Marliani [16] and independently by Politano and Pouquet [17]. An expression for the scaling exponents valid in both cases can be derived as

$$\xi_q = \frac{q}{m}(1 - x) + C\left[1 - \left(1 - \frac{x}{C}\right)^{q/m}\right], \tag{5}$$

where $C = x/(1 - \beta)$ represents the codimension of the most intermittent dissipative structures. In the "standard" cases the parameters x and β turn out to be functions of m [14,16,17], say $x = \beta = 2/m$. The fluid case is obtained choosing $m = 3$, i.e., $x = 2/3$, and the most intermittent structures are represented by filaments $C = 2$ [14]. In MHD we must assume $m = 4$ so that $x = 1/2$ and $C = 1$. In this case the most intermittent structures turn out to be planar sheets [16,17].

When relation (3) holds, i.e., in the inertial domain, the structure functions are not independent, rather the qth structure function is related to the pth one through

$$S_\ell^{(q)} = A(p,q)[S_\ell^{(p)}]^{\alpha_p(q)}, \tag{6}$$

with $\alpha_p(q) = \xi_q/\xi_p$. In fluid flows both Eqs. (3) and (5) furnish $\xi_3 = 1$, so that in the inertial domain the relation $\alpha_3(q) = \xi_q$ holds. Benzi *et al.* [18] verified that in fluid flows, almost unexpectedly, a linear relation between $\log S_\ell^q$ and $\log S_\ell^3$ extends well outside the inertial range within the dissipative range, showing that Eq. (6) has a more general validity than Eq. (3) (see Ref. [19]). This feature, which allows for a very good experimental determination of $\alpha_3^F(q)$ (here F is for fluid) has been called extended self-similarity (ESS). The presence of ESS has been shown, and the scaling laws have been derived in a lot of different situations, say fluid flows and simple models that mimic either fluid or MHD turbulence [20]. In MHD flows both Eqs. (3) and (5) give $\xi_4 = 1$, so we expect that $\alpha_4(q) = \xi_q$ in the inertial domain [10]. ESS has also been observed in the scaling laws derived from velocity measurements in the solar wind MHD turbulence [21]. In this last case the values of $\alpha_4^{\mathrm{MHD}}(q)$ are determined with a very good precision [21]. On the contrary the direct experimental determination of ξ_q through Eq. (2) in the solar wind turbulence is rather difficult [8,9,16].

Even if one usually determines the scaling exponents $\alpha_3^F(q)$ in the fluid case and $\alpha_4^{\mathrm{MHD}}(q)$ in the MHD case, Eq. (6) shows that it should be possible to compare the scaling laws obtained in the two cases by calculating, for example, $\alpha_4^F(q)$ also from data obtained in fluid flows, simply through

$$\alpha_4^F(q) = \frac{\alpha_3^F(q)}{\alpha_3^F(4)}. \tag{7}$$

So an interesting question is posed: are the scaling exponents and the coefficients $A(p,q)$ derived from experimental data through Eq. (6) really different for fluid and MHD turbulent flows, or rather does the relation (6) represent a universal law valid in both cases with the same set of values for $\alpha_p(q)$ and $A(p,q)$? The knowledge of the scaling exponents and of the coefficients $A(p,q)$ is equivalent to the knowledge of the probability distribution for the velocity differences through

$$f(\delta V_\ell) = \int_{-\infty}^{\infty} dk\, \exp\{ik\delta V_\ell\} \times \sum_{q=0}^{\infty} \frac{(ik)^q}{2\pi q!} A(p,q)[S_\ell^{(p)}]^{\alpha_p(q)}. \tag{8}$$

If the scaling exponents and the coefficients $A(p,q)$ for the fluid and MHD turbulent flows were indistinguishable, this could be an indication of the existence of a universal non-Gaussian probability distribution function valid for both fluid and MHD velocity differences δV_ℓ [22]. Theoretical models [12–17] provide different scalings. But when we plot $\alpha_4(q) = \xi_q/\xi_4$, obtained from Eq. (5), respectively, for $m = 3$ (fluid flows) and $m = 4$ (MHD flows), we can see that the curves are practically superposed as far as $q \leq 8$ and only for $q \geq 10$ do they separate out (see Fig. 1). From a physical point of view, according to the She and Lévêque model [14,16,17], a difference between ordinary fluid flows and MHD flows is due to the different topological properties of the most singular structures. This difference is contained in the different values for the parameter C in the models (5), and cannot be found in other intermittency models [10,12,13,15]. Obviously the difference becomes visible only when we look at the most singular structures, that is, when we examine the high-order scaling exponents, because higher values of q enhance the more singular structures.

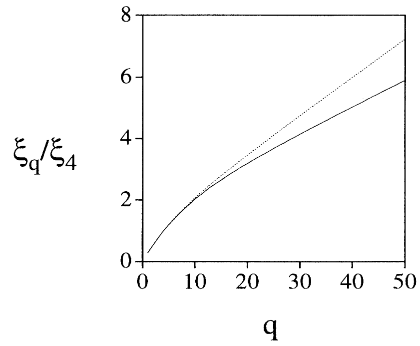

FIG. 1. We show the normalized scaling exponents ξ_q/ξ_4 vs q obtained from both the fluid model [14] (full line) and the MHD version of the same model [16,17] (dashed line).

In order to give an answer to the above-mentioned question, we have collected scaling exponents from both laboratory measurements (fluid flows) and space data (MHD flows). The scaling exponents ξ_q^F have been obtained in laboratory flows, say turbulent jets, duct flows, and wind tunnels. From these exponents the values of $\alpha_4^F(q)$ can be derived, and are reported in Fig. 2. The scaling exponents we used can be found in the literature (see the caption of Fig. 2). For the MHD flows we used the scaling exponents ξ_q^{MHD} obtained by Burlaga [8] from the solar wind turbulent velocity as measured by the Voyager satellite at 8.5 AU (astronomical units). Even in this case the values of $\alpha_4^{\mathrm{MHD}}(q)$ are plotted in Fig. 2. Moreover, we calculated the scaling exponents from measurements of the solar wind velocity field from the Helios 2 satellite in the inner heliosphere from 0.3 AU up to 1 AU. In this case [21] ESS allows for a very good direct determination of $\alpha_4^{\mathrm{MHD}}(q)$ (see Fig. 2). Looking at Fig. 2 it can be seen that for smaller values of q the scaling exponents $\alpha_4^F(q)$ and $\alpha_4^{\mathrm{MHD}}(q)$ follow the same curve, while as q increases it can be noted that the scaling exponents seem to belong to two distinct populations. To show that this behavior is statistically meaningful, we divide our data into two different samples. The first sample is built up with the scaling exponents $\alpha_4^{\mathrm{MHD}}(q)$ coming from the solar wind measurements, and the second sample is built up with the data $\alpha_4^F(q)$ coming from laboratory measurements on ordinary fluid flows. For each value of q we calculate the average values for both samples, say $\mu^{\mathrm{MHD}}(q)$ and $\mu^F(q)$. Through a t test we make inferences about the means of the two

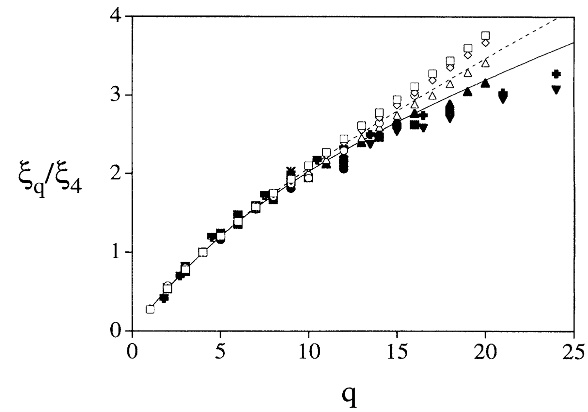

FIG. 2. We show the scaling exponents $\alpha_4^F(q)$ and $\alpha_4^{\mathrm{MHD}}(q)$ collected from different measurements both in ordinary fluid flows and in the solar wind turbulence. Open symbols refer to the solar wind measurements, while filled symbols refer to laboratory measurements. Open squares, diamonds, and circles refer to the analysis in the inner solar wind on the Helios data from 0.3 AU up to 1 AU [21], respectively, during the periods 81:00–83:00, 46:00–48:00, and 36:00–38:00. Open triangles refer to the data by Burlaga [8] obtained in the solar wind turbulence from the Voyager satellite measurements at 8.5 AU. Filled squares, diamonds, and circles refer to the measurements by Anselmet *et al.* [6] on a turbulent jet and on a turbulent duct flow. Filled triangles refer to the measurements by L. Zubair (private communication) in a wind tunnel. Filled crosses and reversed triangles refer to the measurements by Meneveau and Sreenivasan [7] by hot-wire measurements in wind tunnel, respectively, in the boundary layer and in the wake of the cylinder. Finally filled stars refer to the measurements by Benzi *et al.* [18] in wind tunnel. Superimposed, we reported the fluid model (full line) and the MHD model (dashed line).

populations, for each value of q, by testing the hypothesis H_0 that *the two populations have the same mean*

$$H_0(q) := \{\mu^{\mathrm{MHD}}(q) = \mu^F(q)\}. \tag{9}$$

By using Satterthwaite's procedure, we have calculated the probabilities $P[H_0(q)]$ that the hypothesis (9) is true. This probability is shown in Fig. 3. As can be seen $P[H_0(q)]$ is about 0.7 for $q \leq 7$. This is in agreement with what we expect, because the scaling exponents are almost the same for low values of q and the difference between the most intermittent structures in fluid and MHD flows is not achieved for low values of q. For $8 \leq q \leq 10$ the probability is about $P[H_0(q)] = 0.1$, while for $q \geq 12$ the probability of accepting H_0 falls down to $P[H_0(q)] = 10^{-3}$. This indicates that for high values of q the two populations are well separated, that is, the complementary hypothesis $H_1(q) := \{\mu^{\mathrm{MHD}}(q) \neq \mu^F(q)\}$ is accepted with the very high probability $1 - P[H_0(q)]$.

We are then led to the conclusion that there is a strong probability that the low-frequency MHD turbulence in the solar wind turbulence is physically different from turbulence in ordinary fluid flows. The physical difference

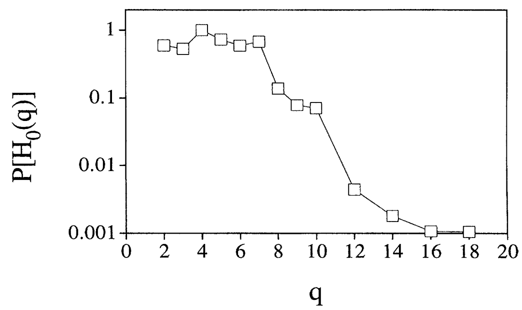

FIG. 3. We show the probability $P[H_0(q)]$ of the hypothesis $H_0(q)$ that the two populations in Fig. 2, made with the laboratory measurements and the solar wind measurements, belong to a single sample with the same mean.

seems to be due to the topological properties of the most intermittent structures, which appear to be filaments in fluid flows and planar sheets in MHD flows. Our results are in agreement with the different versions of the She-Lévêque model for intermittency in fluid flows, which is the only model that takes into account the physical difference. Concerning the solar wind turbulence our results show that measurements indicate a strong tendency to follow the MHD scaling laws, even if [23] the small-order scalings (including the usual spectral index) do not allow for a meaningful distinction between the Kolmogorov and the Kraichnan scaling laws.

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[1] D.C. Leslie, *Developments in the Theory of Turbulence* (Clarendon Press, Oxford, 1973).

[2] A.N. Kolmogorov, Dokl. Akad. Nauk. SSSR **30**, 301 (1941).

[3] A.N. Kolmogorov, J. Fluid Mech. **13**, 82 (1962).

[4] R.H. Kraichnan, J. Fluid Mech. **26**, 305 (1974).

[5] P.S. Iroshnikov, Sov. Astron. **7**, 566 (1963); R.H. Kraichnan, Phys. Fluids **8**, 1385 (1965); M. Dobrowolny, A. Mangeney, and P. Veltri, Phys. Rev. Lett. **45**, 144 (1980).

[6] F. Anselmet, Y. Gagne, E.J. Hopfinger, and R.A. Antonia, J. Fluid Mech. **140**, 63 (1984).

[7] C. Meneveau and K.R. Sreenivasan, J. Fluid Mech. **224**, 429 (1991).

[8] L.F. Burlaga, J. Geophys. Res. **96**, 5847 (1991).

[9] L.F. Burlaga, J. Geophys. Res. **98**, 17 467 (1993); E. Marsch and S. Liu, Ann. Geophys. **11**, 227 (1993).

[10] V. Carbone, Ann. Geophys. **12**, 585 (1994).

[11] U. Frisch and G. Parisi, in *Turbulence and Predictability in Geophysical Fluid Dynamics,* edited by M. Gil, R. Benzi, and G. Parisi (North-Holland, Amsterdam, 1984), p. 84.

[12] R. Benzi, G. Paladin, G. Parisi, and A. Vulpiani, J. Phys. A **17**, 3521 (1984).

[13] C. Meneveau and K.R. Sreenivasan, Phys. Rev. Lett. **59**, 1424 (1987).
[14] Z.-S. She and E. Lévêque, Phys. Rev. Lett. **72**, 336 (1994).
[15] V. Carbone, Phys. Rev. Lett. **71**, 1546 (1993).
[16] R. Grauer, J. Krug, and C. Marliani, Phys. Lett. A **195**, 335 (1995).
[17] H. Politano and A. Pouquet, Phys. Rev. E **52**, 636 (1995).
[18] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Massaioli, and S. Succi, Phys. Rev. E **48**, R29 (1993).
[19] B. Dubrulle, Phys. Rev. Lett. **73**, 959 (1994).
[20] G. Stolovitzky and K.R. Sreenivasan, Phys. Rev. E **48**, R33 (1993); V. Carbone, Phys. Rev. E **50**, R671 (1994); R. Benzi, S. Ciliberto, C. Baudet, G. Ruiz Chavarria, and R. Tripiccione, Europhys. Lett. **24**, 275 (1993); M. Briscolini and P. Santangelo, J. Fluid Mech. **270**, 199 (1994); M. Briscolini, P. Santangelo, S. Succi, and R. Benzi, Phys. Rev. E **50**, R1745 (1994); F.F. Abraham, Europhys. Lett. **26**, 247 (1994); P. Frick, B. Dubrulle, and A. Babiano, "Scaling Properties of a Class of Shell Models" (to be published).
[21] V. Carbone, R. Bruno, and P. Veltri, "Evidences for Extended Self-Similarity in Hydromagnetic Turbulence" (to be published).
[22] It is worthwhile to remark that, when the velocity differences δV_ℓ calculated from the observations in the solar wind are suitably normalized to the average velocity within each observational period (see Ref. [21]), the coefficients $A(p,q)$ in Eq. (6) turn out to be independent of q, and, in particular, $A(p,q) = 1$.
[23] E. Marsch, in *Solar Wind Seven,* edited by E. Marsch and R. Schwenn, Cospar Colloquia Series Vol. 3 (Pergamon Press, New York, 1992).