Acoustic Transmission Spectra in the Penrose Lattice

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Employing third sound in a superfluid helium film as an acoustic wave, transmission spectra are measured experimentally in the Penrose lattice. The spectra are extremely sensitive to the impinging direction of third sound. The transmissivity is suppressed strongly at certain frequency ranges. A transparent direction of the Penrose lattice is found simultaneously. We attribute the origin of the prominent suppressions of transmissivity to the multiple diffraction of third sound in the Penrose lattice.

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Quasiperiodicity is one of the most revolutionary concepts of physics in the recent decade. The biggest motivation to study quasiperiodicity is the discovery of the "quasicrystal" [1]. Quasiperiodicity or a quasiperiodic system can be defined as the lattice structure derived from a higher-dimensional hypercubic lattice by the projection method [2,3]. It is curious or even mysterious how such an abstract structure appears in real condensed matter. The wave propagation in a quasiperiodic lattice is of fundamental importance not only in conjunction with the physical properties of the quasicrystals but also in its own right because of the nontrivial nature of waves in the quasiperiodic system. Since the interference phenomena in quasiperiodic systems are complicated and delicate, it is worth studying the wave phenomena in a well-defined system. Transmission spectra of waves in the Fibonacci lattice, which is a one-dimensional prototype of the quasiperiodic system, have been studied experimentally [4,5]. Theoretically predicted properties [6,7], for example, self-similarity, nesting structure, and scaling properties, have been verified. As to the Penrose lattice, however, little experimental work has been done so far associated with the acoustic properties [8]; the Penrose lattice is a prototype of the two-dimensional quasiperiodic system. In particular, no transmission spectrum of sound in the Penrose lattice has been observed. We have developed an experimental method using third sound of superfluid helium films so that we could measure the transmission spectra of sound in an artificially prepared Penrose lattice [9]. In contrast to the one-dimensional case, not only multiple scattering but also multiple diffraction play important roles in the two-dimensional case.

Third sound is a hydrodynamic wave mode in a superfluid helium film adsorbed on solid surfaces. It has been shown that third sound is one of the most appropriate tools for studying delicate interference phenomena of sound in modulated structures [4,10,11]. Third sound has the following advantages for carrying out the transmission measurement in the Penrose lattice. (1) Small attenuation at low temperature ensures phase coherence over the entire sample. (2) Since the edge of aluminum films scatters third sound, fabrication of the Penrose lattice is relatively easy by decorating a glass surface with aluminum. (3) Scattering strength can be altered by changing the helium film thickness.

Since the Penrose lattice is two dimensional, there is freedom in the impinging direction of sound with respect to the orientation of the lattice. Figure 1(a) shows a sample substrate, with which the transmission of third sound is measured. The Penrose lattice in the present work is based on the Penrose tiling [12], which consists of two kinds of rhombuses, "fat" and "skinny." All sides of both rhombuses are the same. The area of the fat rhombus is larger than that of the skinny one by the golden ratio

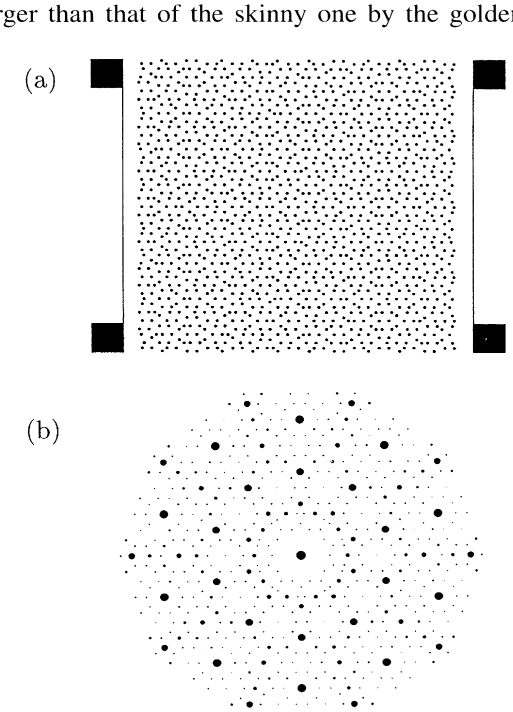

FIG. 1. The Penrose lattice and its structure factor. (a) The Penrose lattice used in the present experiment. What are drawn black indicate aluminum films on a glass substrate. The orientation of this lattice is defined as $\theta = 0$. Two parallel strips located on both sides are the emitter and detector of third sound, of which the width is about 50 μm. (b) The structure factor of the Penrose lattice.

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$\tau = (\sqrt{5} + 1)/2$. We refer to the length of the sides as a. The vertices of the rhombuses are assigned to the lattice sites. The Penrose lattice used in the present work has rigorous fivefold rotational symmetry around the center of the lattice and five mirror axes. The orientation of the lattice is specified by the angle θ of a symmetry axis with respect to the impinging direction of third sound. The horizontal axis passing through the center of the lattice of Fig. 1(a) is one of the mirror axes. We chose this axis as the reference axis, hereafter. As explained in the following, the impinging direction of third sound is parallel to this axis; the orientation of the Penrose lattice shown in Fig. 1(a) is denoted as $\theta = 0$.

The two strips shown in Fig. 1(a) are the emitter and the detector of third sound. Third sound emitted from one of the strips has a linear wave front, and wave vectors perpendicular to the strips. We checked experimentally that the transmission spectra are identical regardless of the direction of propagation whether from right to left or from left to right. This implies that the transmission spectrum has tenfold rotational symmetry. This can be seen also from the structure factor of the Penrose lattice as shown in Fig. 1(b) [13]. In the structure factor of the quasiperiodic lattice an infinite number of δ-function peaks are distributed densely. If the lattice size and the phase coherence length of the wave are large enough, hardly any waves propagate through the quasiperiodic lattice. It is not true for the real experiment, however, because the sample size is finite. For a finite system the structure factor peaks of weak intensity can be neglected [Fig. 1(b)]. Taking into account the mirror symmetry as seen from the structure factor, the spectra within the range of incidence $0 \leq \theta \leq \pi/10$ are relevant. We have studied the Penrose lattices with five different orientations, *viz.*, $\theta = 0, \pi/40, 2\pi/40, 3\pi/40, 4\pi/40$.

The samples are made from aluminum-film evaporated glass substrates by means of photolithography. The thickness of the aluminum film is about 100 nm. The area of 8×8 mm^2 is occupied by about 1300 aluminum vertices. Accordingly, third sound propagates over the distance of at least $30a$ before reaching the detector. Details of the experimental procedure are written elsewhere [10]. The detector of third sound shown in Fig. 1(a) has maximum sensitivity for the wave components of which the wave front is parallel to the detector. That is, what we observe with the present setup is the components of third sound transmitted forward. Because third sound propagates only in the helium film adsorbed on the glass substrate, the scattering is purely two dimensional. There is no freedom for third sound to be scattered in the direction perpendicular to the plane of the Penrose lattice.

We show the transmission spectra of third sound in the Penrose lattice in Fig. 2. Here the abscissa is the effective wave number; $k_\omega = \omega a/\pi C_3$, where ω is the frequency of third sound and C_3 is the sound velocity. It can be seen immediately that the spectrum is sensitive to the impinging direction of third sound. The trace for $\theta = 0$, for example, shows dips at $k_\omega \sim 1.2, 2.0$, and 3.2, while one can hardly observe the corresponding structure in the trace for $\theta = \pi/40$. Similar sensitive behavior can be seen between the traces for $\theta = 3\pi/40$ and $\theta = 4\pi/40$. It is surprising that only a $\pi/40$ rotation results in such a drastic change in the spectrum. The Penrose lattice is very transparent for third sound impinging from the direction of $\theta = 2\pi/40$. Since $k_\omega = 1$ corresponds to $\lambda = 2a$, where λ is the wavelength of third sound, the propagation length is about 15λ. Therefore in the region $k_\omega > 1$ the observed structures are not due to the boundary effect of the finite sample size but due to the intrinsic structure of the Penrose lattice. In a one-dimensional periodic system the dispersion relation $\omega(k)$ is not a continuous function of the wave number k. There are discontinuities or gaps in certain frequency ranges whenever the half wavelength is commensurate with the periodicity. The wave mode having the frequency within the gap ranges cannot propagate in the system. The envelope of its wave function decays exponentially and the incident wave is reflected. The appearance of the gap is due to Bragg scattering or diffraction in one dimension. The phenomena in two dimensions are more or less the same.

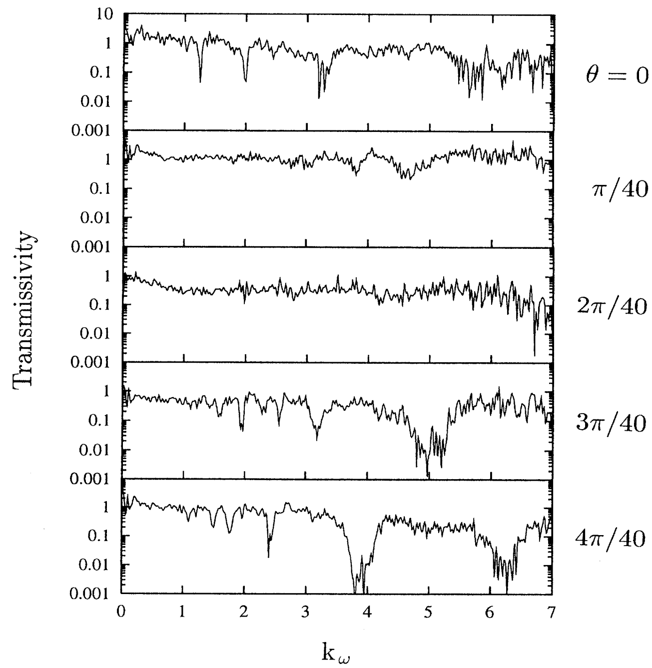

FIG. 2. Transmission spectra of third sound in the Penrose lattice. The traces correspond to the lattice orientations, $\theta = 0, \pi/40, 2\pi/40, 3\pi/40, 4\pi/40$, from top to bottom, respectively. The abscissa is the effective wave number, $k_\omega = \omega a/\pi C_3$, where ω is the frequency of third sound and C_3 is the velocity of third sound. Thickness of the helium film is 1.96 nm for $\theta = 0$ and 1.86 nm for the rest. All spectra in this Letter are measured at 0.73 K.

The incident wave in the gap region is diffracted not only backward but also to other directions. Suppression of the forward transmission is expected, if the diffraction is strong enough. Because the structure factor of the Penrose lattice consists of δ-function peaks, we expect the same phenomena in the Penrose lattice as in the periodic case.

In order to verify whether the diffraction is associated with the observed suppression of the transmissivity, we discuss the structure factor $S(\mathbf{k})$ of the Penrose lattice and Ewald's construction. Ignoring the form factor of the aluminum vertex, the Fourier transform of the Penrose lattice which is used in the experiment is calculated numerically. It is close to $S(\mathbf{k})$ of the Penrose lattice of the infinite size. Figure 3(a) shows only prominent δ-function peaks in $S(\hat{\mathbf{k}})$ that correspond to the orientation $\theta = 0$, where the caret means normalization by π/a. The radius of the spot is proportional to the structure factor. Because $S(\hat{\mathbf{k}})$ is symmetric with respect to the $\hat{k}_x$ axis in the case of Fig. 3(a), the region of $\hat{k}_y < 0$ is omitted. Third sound is impinging on the lattice along the $\hat{k}_x$ axis from right to left. According to the kinematical diffraction theory, the incident wave having the wave vector $\mathbf{k}$ may be diffracted to $\mathbf{k}'$, if the conditions $|\mathbf{k}'| = |\mathbf{k}|$ and $\mathbf{k}' = \mathbf{k} + \mathbf{G}$ are satisfied, where $\mathbf{G}$ is one of the wave vectors of the δ-function peaks of $S(\mathbf{k})$. Ewald's construction shows these conditions graphically. In Fig. 3(a) the Ewald "circles" (sphere in three dimensions and the lower halves are omitted) are drawn. The incident wave vector $\hat{\mathbf{k}}$ corresponds to the vector from the center of the circle to the origin. The diffracted wave vector $\hat{\mathbf{k}}'$ is the vector from the center of the circle to one of the δ-function peaks of $S(\hat{\mathbf{k}})$, namely, $\hat{\mathbf{k}}' = \hat{\mathbf{k}} + \hat{\mathbf{G}}$. In addition, if $\hat{\mathbf{G}}$ is located on the Ewald circle, $|\hat{\mathbf{k}}'| = |\hat{\mathbf{k}}|$. The conditions necessary for diffraction to occur are satisfied. Scanning the radius of the Ewald circle, we can find the radius with which a prominent δ-function peak or peaks of $S(\hat{\mathbf{k}})$ are located on the Ewald circle. Under such conditions third sound may be diffracted and forward transmission will decrease.

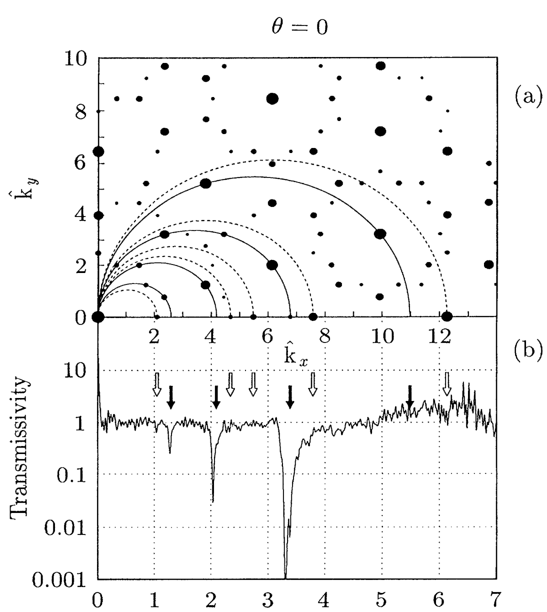

FIG. 3. (a) The structure factor of the Penrose lattice for the orientation $\theta = 0$. Semicircles represent the Ewald circles. See text. (b) The transmission spectrum for $\theta = 0$. Thickness of the helium film is 1.68 nm. The abscissa is elongated twice with respect to (a) for the convenience of the explanation. See text.

To compare the experimental results with the analysis above, we plot the transmission spectrum for $\theta = 0$ in Fig. 3(b) under Fig. 3(a). This spectrum is taken on a helium film thinner than that of the corresponding trace in Fig. 2. The thinner the film, the weaker the scattering of third sound [14]. Although the normalization process of the abscissa causes a slight shift of the position, the structures of the spectra have good correspondence with each other and hence are stable in the region $k_\omega < 5$. In the region $k_\omega > 5$, however, the structure in the spectrum becomes fragile. For the convenience of comparison, the horizontal scale of the spectrum, Fig. 3(b), is expanded twice compared to Fig. 3(a). Then the *diameter* of the Ewald circle corresponds to the wave number of the spectrum. On the Ewald circles drawn by solid lines one finds more than one peak of $S(\hat{\mathbf{k}})$. On the circles drawn by broken lines, on the other hand, only one peak is on the circle. All peaks of the latter case locate on the $\hat{k}_x$ axis implying backward scattering. Solid arrows in Fig. 3(b) correspond to the diameters of the solid-line Ewald circles, while open arrows correspond to the broken-line Ewald circles. We find a good correspondence between the dips at $k_\omega \sim 1.3, 2.1,$ and 3.3 in the spectrum and the solid arrows. These numbers are scaled by the golden ratio, namely, $1.3 \times \tau \sim 2.1$, etc. The interesting fact is that the open arrows hardly find the corresponding suppression of transmissivity except for one at $k_\omega \sim 1.05$.

As for the spectrum for $\theta = 4\pi/40$, this is another symmetric orientation; the structure is more prominent than that for $\theta = 0$, as shown in Figs. 2 and 4(b). We find again that the solid-line Ewald circles in Fig. 4(a) are accompanied by dips. The suppressions at around $k_\omega \sim 1.1, 1.5, 1.8, 2.5, 4.2,$ and 6.6 correspond to the solid arrows. The weaker structures at around $k_\omega \sim 1.2, 2.0,$ and 3.2 may be claimed to correspond to the open arrows and to the broken-line Ewald circle. According to the above observation, we find as a rule that most of the prominent suppressions of transmissivity occur when more than one peak of $S(\hat{\mathbf{k}})$ is on the same Ewald circle. This rule is not sufficient, however. Since the structure factor has mirror symmetry with respect to the $\hat{k}_x$ axis, many other such circles can be drawn associated with the δ-function peaks not on the $\hat{k}_x$ axis.

Neglecting the multiple scattering, the condition that more than one δ-function peak is located on the Ewald

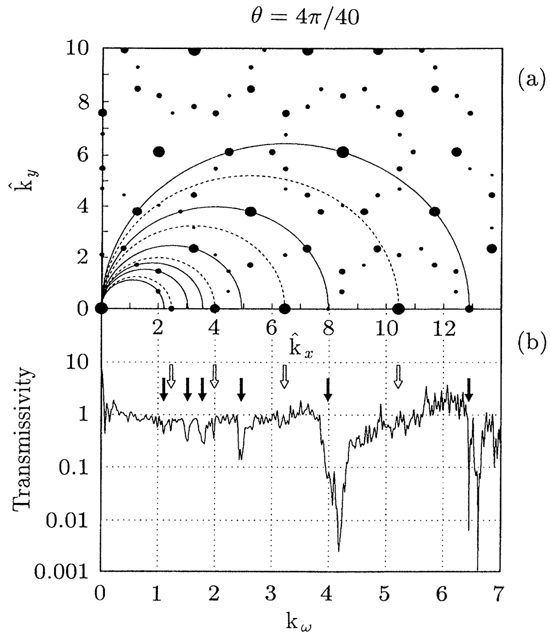

FIG. 4. The same diagram as Fig. 3 for the orientation $\theta = 4\pi/40$. Thickness of the helium film is 1.67 nm.

circle results in simultaneous reflection. For this case the contribution from each peak is additive, contradicting the observation that the Penrose lattice is quite transparent in the direction $\theta = 2\pi/40$. Therefore multiple scattering cannot be neglected. When multiple scattering is important, diffraction should be treated in the framework of the dynamical theory of diffraction. The diffraction under these conditions is known as multiple diffraction. Since multiple diffraction is due to the interference of many waves, the phenomenon may be very sensitive to the tuning condition. This is consistent with the fact that the spectra are extremely sensitive to the angle of incidence. We find also that the wave diffracted by one of the peaks on the Ewald circle sees still the equivalent lattice orientation to that before being diffracted. This is essential for multiple diffraction. In the Penrose lattice one can find the multiple diffraction condition very frequently, even in the direction of $\theta = 3\pi/40$. Although this orientation does not look symmetrical at first sight, a close inspection of the structure factor shows us that it has a novel symmetry. The suppressions of transmissivity at $k_\omega \sim 1.9, 3.2,$ and 5.0 are due to multiple diffraction.

To conclude, we succeeded in observing the transmission spectra of third sound in the Penrose lattice. The spectrum changes very sensitively according to the impinging direction of third sound with respect to the lattice orientation. In some directions of incidence suppressions of the transmissivity at several frequencies are observed. The suppressions are due to multiple diffraction. Emergence of multiple diffraction is very frequent in the Penrose lattice. There are directions of incidence from which the lattice is transparent for third sound. It should be noted that the properties observed here conflict with the general belief for infinite systems. The finite system size and relatively weak scattering of third sound from each vertex give rise to the present results. Theoretical investigation of multiple diffraction is desirable in the Penrose lattice. The discussion of the electronic properties of quasicrystals in the context of multiple diffraction might be interesting.

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