Gauge Invariance and Unstable Particles

Joannis Papavassiliou[1] and Apostolos Pilaftsis[2,*]
[1]Department of Physics, New York University, 4 Washington Place, New York, New York 10003
[2]Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, United Kingdom
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A gauge-independent approach to resonant transition amplitudes with nonconserved external currents is presented, which is implemented by the pinch technique. The analytic expressions derived with this method are $U(1)_{em}$ invariant, independent of the choice of the gauge-fixing parameter, and satisfy a number of required theoretical properties, including unitarity.

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Three decades after Veltman's pioneering work [1], the correct treatment of unstable particles in the context of renormalizable gauge field theories is still an open question. The interest in the problem resurfaced in recent years [2], mainly motivated by a plethora of phenomenological applications linked to machines, such as the CERN Large Electron Positron (LEP) collider, the LEP2, planned to operate at center of mass system (c.m.s.) energy $s = 200$ GeV, the Tevatron at Fermilab, and the CERN Large Hadron Collidor (LHC).

Even though the need for a resummed propagator is evident when dealing with unstable particles within the framework of the S-matrix perturbation theory, its incorporation to the amplitude of a resonant process is nontrivial. When this incorporation is done naively, e.g., by simply replacing the bare propagators of a tree-level amplitude by resummed propagators, one is often unable to satisfy basic field theoretical requirements, such as the gauge-parameter independence of the resulting S-matrix element, $U(1)_{em}$ symmetry, high-energy unitarity, and the optical theorem. This fact is perhaps not so surprising, since the naive resummation of the self-energy graphs takes into account higher order corrections, for *only* certain parts of the tree-level amplitude. Even though the amplitude possesses all the desired properties, this unequal treatment of its parts distorts subtle cancellations, resulting in numerous pathologies, which are artifacts of the method used. It is therefore important to devise a self-consistent calculational scheme, which *manifestly* preserves the aforementioned field theoretical properties that are *intrinsic* in every S-matrix element.

In this paper, we present a new gauge-independent (GI) approach to resonant transition amplitudes implemented by the pinch technique (PT) [3]. The PT is an algorithm that systematically exploits all the healthy properties of the S matrix and has numerous applications in electroweak physics. The crucial novelty we introduce here is that the resummation of graphs must take place only *after* the amplitude of interest has been cast via the PT algorithm into manifestly GI subamplitudes with distinct kinematic properties (propagators, vertices, boxes) order by order in perturbation theory. The application of the PT remedies all the aforementioned field-theoretical problems existing at present in the literature. In particular, the following are considered.

(i) The analytic results obtained within our approach are, by construction, *independent* of the gauge-fixing parameter, in *every* gauge-fixing scheme (R_ξ gauges, axial gauges, background field method, etc.). In addition, by virtue of the tree-level Ward identities satisfied by the PT Green's functions, the $U(1)_{em}$ invariance can be enforced, without introducing residual gauge-dependent terms of higher orders.

(ii) The PT treats bosonic and fermionic contributions to the resummed propagator of the W, Z boson, t quark, or other unstable particles, *on equal footing*. This feature is highly desirable for applications to extensions of the standard model (SM) at high-energy colliders, such as the LHC. For example, a heavy Higgs boson in the SM or new gauge bosons, such as, e.g., Z', W', Z_R, etc., predicted in models beyond the SM, can have widths predominantly originating from bosonic channels. In this way, it becomes even more obvious that prescriptions based on resumming *only* fermionic contributions as GI subsets of graphs, are insufficient.

(iii) The use of an expansion of the resonant matrix element in terms of a *constant* complex pole produces unavoidably spacelike threshold terms to all orders, while nonresonant corrections remove such terms only up to a given order. These spacelike terms, which explicitly violate unitarity, manifest themselves when the c.m.s. energy of the process does not coincide with the position of the resonant pole. On the contrary, the PT circumvents these difficulties by giving rise to an energy-dependent complex-pole regulator. For instance, possible unphysical absorptive parts originating from channels below their production threshold have already been eliminated by the corresponding kinematic θ functions.

(iv) Lastly, the amplitude obtained from our approach exhibits a good high-energy unitarity behavior, as the c.m.s. energy $s \to \infty$. In fact, far away from the resonance, the resonant amplitude tends to the usual PT amplitude, thus displaying the correct high-energy unitarity limit of the entire tree-level process.

We will now study some characteristic examples. Within the PT framework, the transition amplitude

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$T(s,t,m_i)$ of a $2 \rightarrow 2$ process, such as $e^- \overline{\nu}_e \rightarrow \mu^- \overline{\nu}_\mu$ with massive external charged leptons, can be decomposed as

$$T(s,t,m_i) = \widehat{T}_1(s) + \widehat{T}_2(s,m_i) + \widehat{T}_3(s,t,m_i), \qquad (1)$$

where the piece $\widehat{T}_1$ contains three *individually* GI quantities: the WW self-energy $\widehat{\Pi}^W_{\mu\nu}$, the WG mixing term $\widehat{\Theta}_\mu$, and the GG self-energy $\widehat{\Omega}$. Similarly, $\widehat{T}_2(s,m_i)$ consists of two pairs of GI vertices $We^-\overline{\nu}_e, Ge^-\overline{\nu}_e$ [$\widehat{\Gamma}^{(1)}_\mu$ and $\widehat{\Lambda}^{(1)}$], and $W\mu^-\overline{\nu}_\mu$ and $G\mu^-\overline{\nu}_\mu$ [$\widehat{\Gamma}^{(2)}_\mu$ and $\widehat{\Lambda}^{(2)}$]. Most importantly, in addition to being GI, the PT self-energies and vertices satisfy the following *treelike* Ward identities:

$$\begin{aligned} q^\mu \widehat{\Pi}^W_{\mu\nu} - M_W \widehat{\Theta}_\nu &= 0, \\ q^\mu \widehat{\Theta}_\mu - M_W \widehat{\Omega} &= 0, \\ q^\mu \widehat{\Gamma}^i_\mu - M_W \widehat{\Lambda}^i &= 0 \quad (i = 1,2). \end{aligned} \qquad (2)$$

These Ward identities are a direct consequence of the requirement that $\widehat{T}_1$ and $\widehat{T}_2$ are fully ξ independent [3]. If we assume that the PT decomposition in Eq. (1) holds to any order in perturbation theory (the validity of this assumption will be discussed extensively in Ref. [4]), and sum up contributions from all orders, we obtain for $\widehat{T}_1$ (suppressing contraction of Lorentz indices)

$$\begin{aligned} \widehat{T}_1 &= \Gamma_0 U_W \Gamma_0 + \Gamma_0 U_W \widehat{\Pi}^W U_W \Gamma_0 \\ &\quad + \Gamma_0 U_W \widehat{\Pi}^W \cdots \widehat{\Pi}^W U_W \Gamma_0 \\ &= \Gamma_0 \widehat{\Delta}_W \Gamma_0, \end{aligned} \qquad (3)$$

where $U_{W\mu\nu}(q) = t_{\mu\nu}(q)(q^2 - M_W^2)^{-1} + \ell_{\mu\nu}(q) M_W^{-2}$ [$t_{\mu\nu}(q) = -g_{\mu\nu} + q_\mu q_\nu/q^2$ and $\ell_{\mu\nu}(q) = q_\mu q_\nu / q^2$], and

$$\widehat{\Delta}_{W\mu\nu}(q) = \frac{t_{\mu\nu}(q)}{q^2 - M_W^2 - \widehat{\Pi}^W_T(q^2)} + \frac{\ell_{\mu\nu}(q)}{M_W^2 - \widehat{\Pi}^W_L(q^2)}. \qquad (4)$$

In Eq. (4), we have decomposed $\widehat{\Pi}^W_{\mu\nu} = t_{\mu\nu}\widehat{\Pi}^W_T + \ell_{\mu\nu}\widehat{\Pi}^W_L$.

Next we apply our formalism to the process $\gamma e^- \rightarrow \mu^- \overline{\nu}_\mu \nu_e$, in which two W gauge bosons are involved. This process is of potential interest at the LEP2. We concentrate on the part of the amplitude ($\widehat{T}_{1\mu}$) involving the γWW vertex, as given in Fig. 1. As discussed above, the PT method reorders the Feynman graphs into manifestly GI subsets. Resumming the PT self-energies one obtains the following resonant transition amplitude:

$$\begin{aligned} \widehat{T}_{1\mu} &= \Gamma_0 \widehat{\Delta}_W (\Gamma^{\gamma W^- W^+}_{0\mu} + \widehat{\Gamma}^{\gamma W^- W^+}_\mu) \widehat{\Delta}_W \Gamma_0 \\ &\quad + \Gamma_0 S_0^{(e)} \Gamma^\gamma_{0\mu} \widehat{\Delta}_W \Gamma_0 + \Gamma_0 \widehat{\Delta}_W \Gamma^\gamma_{0\mu} S_0^{(\mu)} \Gamma_0, \end{aligned} \qquad (5)$$

where $S^{(f)}$ is the free f-fermion propagator and $\Gamma^\gamma_{0\mu}$ is the tree γff coupling. In Eq. (5), contraction over all Lorentz indices except the photonic one is implied. Since the action of the photonic momentum (q) on the tree-level and one-loop PT γWW vertices gives

$$\frac{1}{e} q^\mu \Gamma^{\gamma W^- W^+}_{0\mu\nu\lambda} = U^{-1}_{W\nu\lambda}(p_+) - U^{-1}_{W\nu\lambda}(p_-)$$

and

$$\frac{1}{e} q^\mu \widehat{\Gamma}^{\gamma W^- W^+}_{\mu\nu\lambda} = \widehat{\Pi}^W_{\nu\lambda}(p_-) - \widehat{\Pi}^W_{\nu\lambda}(p_+),$$

respectively, the $\mathrm{U}(1)_{\mathrm{em}}$ gauge invariance of this resonant process is restored, i.e., $q^\mu \widehat{T}_{1\mu} = 0$. To any loop order, $\mathrm{U}(1)_{\mathrm{em}}$ and R_ξ invariance are warranted by virtue of the tree-type Ward identities that the PT vertex γWW satisfy [5] (all momenta flow into the vertex, i.e., $q + p_- + p_+ = 0$):

$$\begin{aligned} \frac{1}{e} q^\mu \widehat{\Gamma}^{\gamma W^- W^+}_{\mu\nu\lambda} &= \widehat{\Pi}^W_{\nu\lambda}(p_-) - \widehat{\Pi}^W_{\nu\lambda}(p_+), \\ \frac{1}{e}[p_-^\nu \widehat{\Gamma}^{\gamma W^- W^+}_{\mu\nu\lambda} - M_W \widehat{\Gamma}^{\gamma G^- W^+}_{\mu\lambda}] &= \widehat{\Pi}^W_{\mu\lambda}(p_+) - \widehat{\Pi}^\gamma_{\mu\lambda}(q) - \frac{c_w}{s_w} \widehat{\Pi}^{\gamma Z}_{\mu\lambda}(q), \\ \frac{1}{e}[p_+^\lambda \widehat{\Gamma}^{\gamma W^- W^+}_{\mu\nu\lambda} + M_W \widehat{\Gamma}^{\gamma W^- G^+}_{\mu\nu}] &= -\widehat{\Pi}^W_{\mu\nu}(p_-) + \widehat{\Pi}^\gamma_{\mu\nu}(q) + \frac{c_w}{s_w} \widehat{\Pi}^{\gamma Z}_{\mu\nu}(q). \end{aligned} \qquad (6)$$

We continue with some important technical remarks. We first focus on issues of resummation, and argue that the GI PT self-energy may be resummed, exactly as one carries out the Dyson summation of the conventional self-energy. The crucial point is that even though contributions from vertices and boxes are instrumental for the definition of the PT self-energies, their resummation does *not* require a corresponding resummation of vertex or box parts. In order to construct GI chains of self-energy bubbles, one can borrow pinch contributions from existing graphs, which are, however, not sufficient to convert each $\Pi_{\mu\nu}$ in the chain into the corresponding $\widehat{\Pi}_{\mu\nu}$. If we add the missing pieces by hand, and subsequently subtract them, we notice the following: (i) The regular string has been converted into a GI string, with $\Pi_{\mu\nu} \rightarrow \widehat{\Pi}_{\mu\nu}$; and (ii) the leftovers, due to the characteristic presence of the inverse bare propagator $(U^{\mu\nu})^{-1}$, are effectively one-particle *irreducible,* and together with the genuine vertex (V^P) and box pinch contributions (B^P) will convert the conventional self-energy (of order equal to the combined order of the string) into the GI PT self-energy of the same order. This procedure is generalizable to an arbitrary

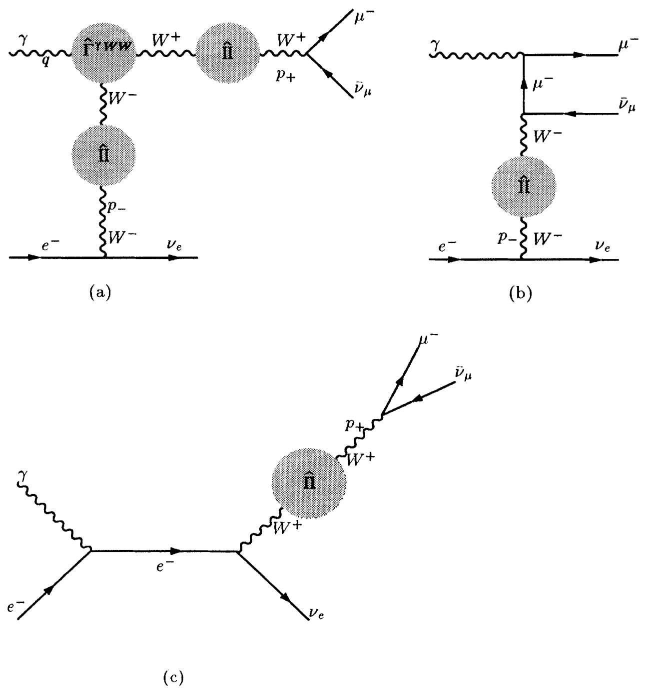

FIG. 1. The process $e^- \gamma \to \mu^- \overline{\nu}_\mu \nu_e$ in our PT approach.

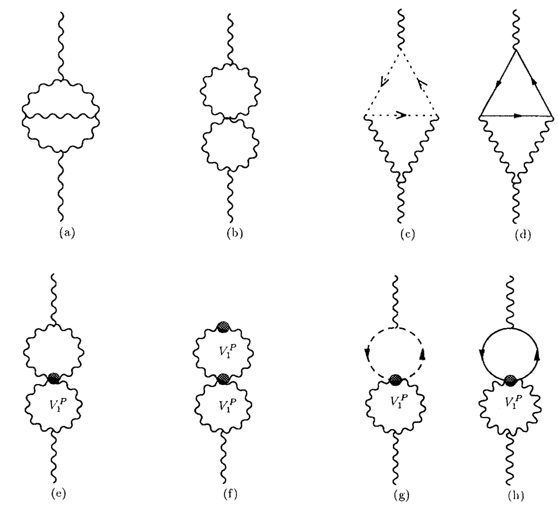

FIG. 2. Typical two-loop self-energy graphs (a)–(d), and some of the residual pinch contributions (e)–(h) contained in R_2^P.

order. So, the transverse propagatorlike pinch contributions in the Feynman gauge, to a given order n in perturbation theory, have the general form

$$\Pi_n^P(q^2) = (q^2 - m_0^2)V_n^P(q^2) + (q^2 - m_0^2)^2 B_n^P(q^2) + R_n^P(q^2), \tag{7}$$

where R_n^P are the residual pieces of order n. For $n = 2$, for example, it is easy to check that the string

$$\left(\frac{1}{q^2 - m_0^2}\right)\Pi\left(\frac{1}{q^2 - m_0^2}\right)\Pi\left(\frac{1}{q^2 - m_0^2}\right),$$

together with existing pinch pieces from graphs containing vertices, needs an additional amount $-R_2^P$, given by

$$-R_2^P(q^2) = \Pi V_1^P + \tfrac{3}{4}(q^2 - m_0^2)V_1^P V_1^P, \tag{8}$$

in order to be converted into the GI string

$$\left(\frac{1}{q^2 - m_0^2}\right)\widehat{\Pi}\left(\frac{1}{q^2 - m_0^2}\right)\widehat{\Pi}\left(\frac{1}{q^2 - m_0^2}\right).$$

However, R_2^P will be absorbed by the one-particle irreducible two-loop self-energy shown in Fig. 2. In general, the R_n^P terms consist of products of lower order conventional self-energies $\Pi_k(q^2)$, and lower order pinch contributions V_ℓ^P and (or) B_ℓ^P, with $k + \ell = n$ [4].

Another issue is whether the GI PT complex pole is identical to the GI physical pole of the amplitude. Here we concentrate on the case of a stable particle, and demonstrate how its mass does not get shifted by the PT. The masses m and $\widehat{m}$ are, respectively, defined as the solution of the equations $m^2 = m_0^2 + \Pi(m^2)$ and $\widehat{m}^2 = m_0^2 + \widehat{\Pi}(\widehat{m}^2)$. In perturbation theory, $m^2 = m_0^2 + \sum_1^\infty g^{2n} C_n$ and $\widehat{m}^2 = m_0^2 + \sum_1^\infty g^{2n} \widehat{C}_n$, and one has hence to show that $C_n - \widehat{C}_n = O(g^{2n+1})$. To zeroth order $m^2 = \widehat{m}^2 = m_0^2$. Similarly, from Eq. (7), using the fact that $B_1^P = 0$ (in the Feynman gauge), and $R_1^P = 0$ (in any gauge), we have that $C_1 = \widehat{C}_1$, because the pinch contribution $(q^2 - m_0^2)V_1^P$ is of $O(g^4)$. The nontrivial step in generalizing this proof to higher orders is to observe that not all pinch contributions of Eq. (7) contribute terms of higher order. To be precise, the terms of R_n^P which do not have the characteristic factor $q^2 - m_0^2$ in front are *not* of higher order, and are instrumental for our proof. We will illustrate this point at the two-loop order. The second order m^2 and $\widehat{m}^2$ are given by

$$m^2 = m_0^2 + \Pi_1(m^2) + \Pi_2(m^2),$$
$$\widehat{m}^2 = m_0^2 + \Pi_1(\widehat{m}^2) + \Pi_2(\widehat{m}^2) + \Pi_1^P + \Pi_2^P,$$

where the subscripts 1 and 2 denote loop order, and

$$\begin{aligned}\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) = {}& (\widehat{m}^2 - m_0^2)[V_1^P(\widehat{m}^2) + V_2^P(\widehat{m}^2)] \\ & + (\widehat{m}^2 - m_0^2)^2 \\ & \times [B_1^P(\widehat{m}^2) + B_2^P(\widehat{m}^2)] \\ & + R_2^P(\widehat{m}^2).\end{aligned} \tag{9}$$

It is not difficult to show that $\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) = O(g^6)$. Substituting $\widehat{m}^2 - m_0^2 = \Pi_1(\widehat{m}^2) + O(g^4)$ into Eq. (9), and neglecting terms of $O(g^6)$ or higher, we find

$$\begin{aligned}\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) &= R_2^P(\widehat{m}^2) + \Pi_1(\widehat{m}^2)V_1^P(\widehat{m}^2) \\ &\quad + O(g^6) \\ &= 0 + O(g^6),\end{aligned}$$

where we have also used Eq. (8) at $q^2 = \hat{m}^2$. The generalization of the proof to an arbitrary order n in perturbation theory proceeds by induction and will be given in Ref. [4], together with the case of an unstable particle—both mass and width remain unshifted.

Another point, important for unitarity, is whether the PT self-energy contains any unphysical absorptive parts. In particular, the propagatorlike part $\hat{T}_1$ of a reaction should contain imaginary parts associated with physical Landau singularities only, whereas the unphysical poles related to Goldstone bosons and ghosts must vanish in the loop. Explicit calculations (see Ref. [4]) show that, indeed, our GI procedure does not introduce any fixed unphysical poles. Here we offer only a qualitative argument in that vein, namely, that the PT results may be obtained equally well if one works *directly* in the unitary gauge, where only physical Landau poles are present [3].

Although our discussion has been restricted to the W gauge boson, our considerations are also valid for the Z boson and the heavy top quark, thus providing a self-consistent framework for investigating the CP properties of the t quark at LHC. Moreover, our approach can be straightforwardly extended to possible new-physics phenomena induced by non-SM gauge bosons, such as the bosons W_R, Z', etc., predicted in SO(10) or E_6 unified models [6]. Since our method treats bosonic and fermionic contributions equally, it can be implemented in the study of the resonant dynamics of a heavy Higgs boson, and of a strong Higgs sector at the LHC.

*Electronic address: pilaftsis@v2.rl.ac.uk

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