Weak Levitation of 2D Delocalized States in a Magnetic Field

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The deviation of the energy position of a delocalized state from the center of a Landau level is studied in the framework of the Chalker-Coddington model. It is demonstrated that the introduction of weak Landau level mixing results in an upward shift of the delocalized state in energy. The mechanism of a levitation is a neighboring-Landau-level-assisted resonant tunneling which "shunts" the saddle points. The magnitude of levitation is shown to be independent of the Landau level number.

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It is commonly accepted that the electronic spectrum of a two-dimensional disordered system in a strong magnetic field contains only a single delocalized state per Landau level (LL). First conjectured by Halperin [1], this conclusion was later drawn from scaling ideas [2,3] and confirmed by numerical simulations [4–6].

To make this conjecture consistent with the absence of delocalized states at zero field ($B = 0$), Khmelnitskii [7] (see also [8]) suggested that with decreasing B the energy positions of delocalized states depart gradually from the centers of LL and float toward infinity as $B \to 0$. To obtain the positions of delocalized states, $E_n(B)$, Khmelnitskii pointed out that the system of scaling equations [2] for conductivities, σ_{xx} and σ_{xy}, should be solved together with the boundary conditions that at small scales the conductivities are given by their classical expressions: $\sigma_{xx}^{\mathrm{cl}} = \sigma_0(1 + \omega_c^2\tau^2)^{-1}$ and $\sigma_{xy}^{\mathrm{cl}} = \sigma_0\omega_c\tau(1 + \omega_c^2\tau^2)^{-1}$, where σ_0 is the Drude conductivity (proportional to the Fermi energy E_F), ω_c is the cyclotron frequency, and τ is the elastic scattering time. Since the scaling equations are periodic in σ_{xy}, it follows that $\sigma_{xx}(E_F)$ remains finite at large scales each time when $\sigma_{xy}^{\mathrm{cl}} = (n + 1/2)e^2/2\pi\hbar$. This gives $E_n = \hbar\omega_c(n + 1/2)[1 + (\omega_c\tau)^{-2}]$. The levitation of the delocalized states according to [7,8] is sketched in Fig. 1.

The above dependence $E_n(B)$, treated as a dependence of $\rho_{xx}^0 = \sigma_0^{-1}$ vs the inverse filling factor ν^{-1}, is, in fact, an essential constituent of the global phase diagram of the quantum Hall effect (QHE) [9]. The behavior of the $\rho_{xx}^0(\nu^{-1})$ boundary between the insulating and quantum Hall states, which follows from the selection rules derived in [9], represents, in fact, the generalization of the levitation scenario to the case of strongly disordered and interacting 2D gas. Recent experiments [10–13], in which an insulator–quantum Hall conductor–insulator transition was observed with increasing magnetic field were interpreted in terms of the global phase diagram [9] as follows: the horizontal line, $E_F(B) = \mathrm{const}$, crosses the curve $E_0(B)$ at two different values of B, so that for B between these values the system is in the QHE regime, while for smaller or larger B it is in the insulating phase. The latest experiment [14] allowed a direct measurement of the $E_0(B)$ dependence by tracing the positions of peaks in σ_{xx}, which were measured as a function of gate voltage for different magnetic fields. The measured dependence $E_0(B)$ exhibits a minimum in the region of B where spin levels are not resolved (the results of similar measurements on Si metal-oxide-semiconductor field-effect transistors are reported in [15]).

Numerical simulations [16] for short-range disorder indeed indicated an upward (from $E = \hbar\omega_c/2$) shift of the energy position of the extended state when the three lowest LL's were taken into account. However, no microscopic theory for such a levitation has been developed so far. In this Letter we develop such a theory for the region where the departure of the delocalized state from the center of the Landau band is relatively small. In this case one can treat the LL mixing as a perturbation. As a result of such a mixing, the resonant scattering of an electron with energy near the center of LL by localized states from neighboring LL becomes possible. We demonstrate that it is this scattering that leads to the levitation.

We consider the case of a smooth random potential and adopt the network model of Chalker and Coddington (CC) [17] (see also [18]) which we generalize in order to study the effects of neighboring LL. In this model, delocalization

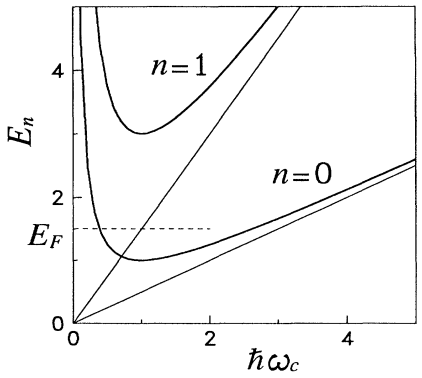

FIG. 1. Schematic picture of the levitation of delocalized states according to [7,8]. The straight lines correspond to $n = 0$ and $n = 1$ Landau levels.

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results from the tunneling of an electron through the saddle points of a potential which are connected by equipotential lines. The randomness of the potential is included by assuming the phase acquired by an electron traversing a link (equipotential line) to be random. The amplitudes of incoming waves Z_1 and Z_2 and outgoing waves Z_3 and Z_4 (see Fig. 2) are related as

$$\begin{pmatrix} Z_1 \\ Z_3 \end{pmatrix} = \mathbf{M}\begin{pmatrix} Z_4 \\ Z_2 \end{pmatrix}, \qquad \mathbf{M} = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}\begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}\begin{pmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{pmatrix}, \tag{1}$$

where the parameter θ characterizes the tunneling and ϕ_i are gauge phases. For the potential expanded near the saddle point, $V(x,y) = V_0 - m\Omega_x^2 x^2/2 + m\Omega_y^2 y^2/2$, where m is the electron mass, this parameter can be presented as [19]

$$\sinh\theta = \exp[(E - V_0)/\gamma], \qquad \gamma = \hbar\Omega_x\Omega_y/\pi\omega_c, \tag{2}$$

where E is the energy of the electron measured from the center of LL. Since the saddle-point heights, V_0, are distributed *symmetrically* around the value $V_0 = 0$ [corresponding to $\theta = \theta_c \equiv \ln(1 + \sqrt{2})$], the delocalized states occur at zero energy.

Now assume that the energy is close to the center of the $n = 0$ LL and study the change in the structure of electronic states caused by the $n = 1$ LL. The relevant $n = 1$ LL states are those with energies close to the center of $n = 0$ LL. The equipotentials corresponding to these states are depicted schematically in Fig. 2 by dashed loops. The prime role of these equipotentials is to shift the maximum of the density of states up from $E = 0$. However, such a shift does not affect the position of the delocalized state unless a coupling between the equipotentials of $n = 0$ and $n = 1$ LL is introduced. This coupling is illustrated in Fig. 2(a). The line connecting the equipotentials stands for a scattering matrix $\mathbf{S}$ defined as

$$\begin{pmatrix} Z \\ \tilde{Z} \end{pmatrix} = \mathbf{S}\begin{pmatrix} Z' \\ \tilde{Z}' \end{pmatrix}, \qquad \mathbf{S} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{i\tilde{\phi}} \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} e^{i\phi'} & 0 \\ 0 & e^{i\tilde{\phi}'} \end{pmatrix}. \tag{3}$$

Such a form of $\mathbf{S}$ insures the conservation of flux $|Z|^2 + |\tilde{Z}|^2 = |Z'|^2 + |\tilde{Z}'|^2$. The coupling strength is characterized by an angle α (gauge phases in matrix $\mathbf{S}$, as well as in $\mathbf{M}$, can be absorbed into the Z's). It is easy to see that the only effect of such a coupling to a loop is a phase shift between the amplitudes Z and Z'. Indeed, $\tilde{Z}$ and $\tilde{Z}'$ differ by the phase factor acquired by an electron traversing the loop: $\tilde{Z}' = \tilde{Z}e^{i\varphi}$. Then (3) yields $Z = Z'e^{i\delta}$, with

$$\tan\delta(E) = \frac{\sin\varphi\,\sin^2\alpha}{\cos\varphi(1 + \cos^2\alpha) - 2\cos\alpha}. \tag{4}$$

The energy dependence of the phase δ is determined by the energy dependence of φ. For a small coupling the resonances occur at $\varphi = \pm\alpha^2/2 + 2\pi p$ (p is an integer), the resonance width also being $\alpha^2/2$. However, these resonances are of no importance since the phase δ can also be absorbed into the random phase on the link.

The situation is completely different when a loop occurs in the vicinity of the saddle point so that it is coupled to both the incoming and outgoing links. It is important to note that such a loop is not coupled directly to the saddle point because there the gradient of the potential is precisely zero. Tunneling between the loop and the links occurs most favorably at a certain distance from the saddle point, as illustrated in Fig. 2(b). Since the characteristic tunneling distance at the saddle point is quite small (typically, the magnetic length), we neglect the effect of the loop on tunneling transparency at the saddle point.

Below, we demonstrate that the effect of such loops is to shift the position of delocalized state upward. The crucial observation is that the saddle point with the loops attached to the links can be viewed as some modified saddle point [see Fig. 2(b)] and thus is characterized by a matrix $\mathbf{M}'$ with the same unitarity properties as $\mathbf{M}$:

$$\begin{pmatrix} Z_1' \\ Z_3' \end{pmatrix} = \mathbf{M}'\begin{pmatrix} Z_4' \\ Z_2' \end{pmatrix}, \qquad \mathbf{M}' = \begin{pmatrix} e^{i\phi_1'} & 0 \\ 0 & e^{i\phi_2'} \end{pmatrix}\begin{pmatrix} \cosh\theta' & \sinh\theta' \\ \sinh\theta' & \cosh\theta' \end{pmatrix}\begin{pmatrix} e^{i\phi_3'} & 0 \\ 0 & e^{i\phi_4'} \end{pmatrix}. \tag{5}$$

The new parameter θ' can be expressed via θ and the elements of scattering matrices $\mathbf{S}_i$ [see Fig. 2(b)] using the following equations:

$$\begin{pmatrix} Z_2' \\ Z_5 \end{pmatrix} = \mathbf{S}_2\begin{pmatrix} Z_2 \\ Z_6 \end{pmatrix}, \quad \begin{pmatrix} Z_3 \\ Z_6 \end{pmatrix} = \mathbf{S}_3\begin{pmatrix} Z_3' \\ Z_5 \end{pmatrix}, \quad \begin{pmatrix} Z_1' \\ Z_8 \end{pmatrix} = \mathbf{S}_1\begin{pmatrix} Z_1 \\ Z_7 \end{pmatrix}, \quad \begin{pmatrix} Z_4 \\ Z_7 \end{pmatrix} = \mathbf{S}_4\begin{pmatrix} Z_4' \\ Z_8 \end{pmatrix}. \tag{6}$$

Consider for simplicity a case when all $\alpha_i = \alpha$ and there is only one loop (e.g., $\mathbf{S}_1 = \mathbf{S}_4 = \mathbf{1}$). Then the solution of (1), (5), and (6) reads

$$\sinh^2\theta' = \frac{2\sinh^2\theta\cos^2\alpha(1 - \cos\varphi)}{[\sin\varphi - \sin^2\alpha\cosh\theta\sin(\psi - \varphi)]^2 + [\cos\varphi - \cos^2\alpha + \sin^2\alpha\cosh\theta\cos(\psi - \varphi)]^2}. \tag{7}$$

Here φ [as in (4)] is the phase acquired by an electron traversing the loop while ψ is the phase acquired on the contour $Z_2 \to Z_3 \to Z_5$ [see Fig. 2(b)].

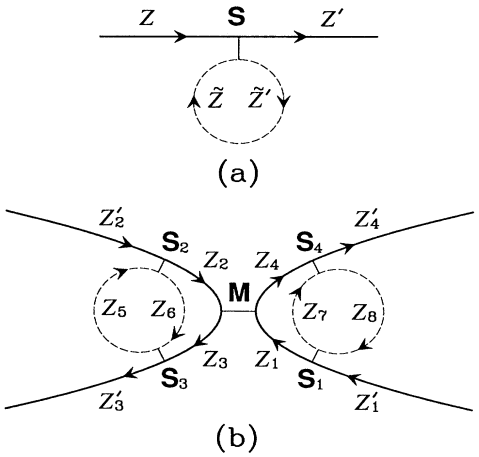

FIG. 2. Sketch of the saddle point. The full lines represent equipotentials corresponding to $n = 0$ LL. The dashed circles are equipotentials of $n = 1$ LL. (a) Scattering of an electron by isolated loop (**S** stands for the scattering matrix). (b) Passage of electron through the saddle point in the presence of loops connecting the links.

The conversion of the parameter θ into θ' can be viewed as an effective change in the height of the saddle point, δV_0, caused by $n = 1$ LL:

$$\delta V_0 = -\gamma \ln(\sinh\theta'/\sinh\theta). \quad (8)$$

The distribution function of δV_0, which comes from averaging over the random phases φ and ψ, is plotted in Fig. 3 for different values of α. The narrow peaks at small δV_0 originate from the values of φ that are not too close to $2\pi p$. Then it is easy to see from (7) that for such φ and $\alpha \ll 1$ we have $\theta' \simeq \theta$, and, consequently, δV_0 is small. At the same time, the tails of the distribution come from the resonances $\varphi \simeq 2\pi p$. For such φ we may have both $\theta' > \theta$ and $\theta' < \theta$. It is important to realize, however, that the tails are *asymmetric*, i.e., the distribution function falls off slower toward large δV_0. As a result, the average $\overline{\delta V_0}$ appears to be *positive*. For

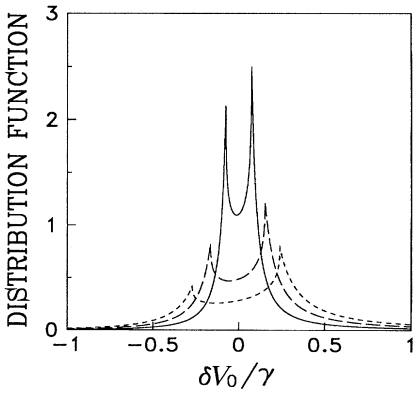

FIG. 3. The distribution function of δV_0 is shown at $E = V_0$ ($\theta = \theta_c$) for $\alpha^2 = 0.1$ (solid line), $\alpha^2 = 0.2$ (long-dashed line), and $\alpha^2 = 0.3$ (dashed line).

small α this average can be easily calculated analytically:

$$\overline{\delta V_0}(\theta) = (\alpha^2\gamma/\pi)[\sinh\theta + \arcsin(1/\cosh\theta)], \quad (9)$$

where θ is related to V_0 by Eq. (2). We see that $\overline{\delta V_0}(\theta)$ is proportional to the width of the resonance, α^2, since it is determined by resonant loops. Since $\overline{\delta V_0}(\theta)$ is finite, the average saddle-point height, $\overline{V_0}$, moves from $\overline{V_0} = 0$ to a *finite* value $\overline{V_0} = \langle\overline{\delta V_0}(\theta)\rangle_{V_0}$, where $\langle\rangle_{V_0}$ stands for averaging over V_0. The values of V_0, relevant for delocalization, are of the order of γ, the relevant $E - V_0$ being also of the order of γ, so that the relevant θ in (9) is $\theta \sim 1$. This leads to the following estimate for the energy shift of the delocalized state:

$$\delta E_0 \sim \langle\overline{V_0}\rangle_\alpha \sim \overline{\alpha^2}\gamma, \quad (10)$$

where $\overline{\alpha^2}$ is the coupling strength averaged over the loops. It can be seen that the shift is much smaller than a typical saddle-point height: $\delta E_0/\gamma \sim \overline{\alpha^2} \ll 1$.

From a physical standpoint, the levitation (positive δE_0) originates from the loops providing a direct transmission between links [20], bypassing the saddle point. Thus, in order to compensate the leakage of electrons to the opposite link via the loop, the energy of the delocalized state is raised upward.

In the derivation of (9) we neglected the effect of the loop $Z_7 \to Z_8$ on the other side of the saddle point [see Fig. 2(b)]. It is easy to see, however, that the two loops cause essentially an additive effect. Indeed, an electron, traversing the link Z_1', can also bypass the saddle point due to a resonant transmission to the opposite link Z_4'.

The key point in the consideration above was that the loops can occur only in directions of the descent of a saddle-point potential (valleys), i.e., to the right and to the left from the saddle point in Fig. 2(b). This is the case when the effect of the $n = 1$ LL on the $n = 0$ delocalized state is studied. For the $n = 1$ delocalized state the situation is more complicated: the loops from the $n = 2$ LL occur in the directions of the valleys while the loops from the $n = 0$ LL occur in the directions of the hills. The latter loops cause an *opposite* trend, pulling the delocalized state *downward*. The resulting shift can be presented as $\delta E_1 \sim \gamma(\overline{\alpha_{12}^2} - \overline{\alpha_{10}^2})$. Note that the coupling, $\overline{\alpha_{12}^2}$, of $n = 1$ to $n = 2$ LL is stronger than the coupling, $\overline{\alpha_{10}^2}$, of $n = 1$ to $n = 0$ LL, due to a larger size of the $n = 2$ wave function. Thus, δE_1 is also positive. Another effect which we have neglected is coupling of the $n = 0$ links via loops corresponding to $n = 0$ equipotentials. Since these loops occur with equal probability in *all* directions from the saddle point, it is obvious that positive and negative contributions to δE_0 from these loops cancel each other out.

The effective coupling constants $\overline{\alpha_{n,n+1}^2}$ are small if the LL width is small, i.e., $W \ll \hbar\omega_c$. But even if $W \gg \hbar\omega_c$, the coupling can still be small since it is determined by tunneling between equipotentials. A crude estimate for a tunneling amplitude in this case, $W \gg \hbar\omega_c$, can be obtained as follows. The spatial distance between

equipotentials is of the order of $s = \hbar\omega_c/(W/R_c)$, where R_c is the correlation radius of a smooth potential. This corresponds to the momentum transfer $q = s/l^2$, where l is the magnetic length. The coupling is efficient only if $q < R_c^{-1}$; otherwise, the smooth potential would not be able to provide the necessary momentum, and the tunneling amplitude would be exponentially small. The latter condition, $qR_c \lesssim 1$, can be rewritten as

$$\hbar\omega_c \lesssim [W(\hbar^2/mR_c^2)]^{1/2} \equiv \hbar\omega_c^* . \quad (11)$$

Equation (11) requires that the magnetic field should not be too strong; however, the quasiclassical picture of electron motion along equipotentials still applies at $\omega_c \sim \omega_c^*$: for such ω_c we have $l/R_c \sim [(\hbar^2/mR_c^2)/W]^{1/4} \ll 1$, which is just the condition that the potential is smooth. On the other hand, the use of the CC model is justified only if the saddle point "discriminates" between neighboring LL, i.e., $\hbar\omega \gtrsim \gamma$. Remarkably, the two conditions, $\omega_c \sim \omega_c^*$ and $\hbar\omega_c \sim \gamma$, coincide. Indeed, following [21], the parameter γ in (2) can be estimated as $\gamma \sim \hbar(W/mR_c^2)/\omega_c \sim \hbar\omega_c^{*2}/\omega_c$. Thus, the CC model is inadequate for $\omega_c \ll \omega_c^*$, when the saddle point at $\hbar\omega_c(n + 1/2)$ becomes transparent also for several neighboring to nth LL's; for such fields one might expect strong levitation. Our consideration applies for magnetic fields $\omega_c > \omega_c^*$, when the levitation is weak.

Let us briefly discuss the dependence of the coupling on LL number n. This dependence is determined by the overlap of nth and $(n + 1)$th Landau wave functions shifted in space by a distance s. This overlap is known to be proportional to $e^{-u/2}L_n^1(u)[u/(n + 1)]^{1/2}$, where $u = s^2/2l^2$ and L_n^1 is the Laguerre polynomial. If the ratio $\omega_c/\omega_c^* = (qR_c)^{1/2} = (R_c s/l^2)^{1/2}$ is not too large (the coupling is not exponentially small), we have $s < l$. Then it is easy to see that $\overline{\alpha_{n,n+1}^2} \propto [L_n^1(0)]^2/n \propto n$. As a result, the levitation δE_n, which is proportional to $\overline{\alpha_{n,n+1}^2} - \overline{\alpha_{n,n-1}^2}$, does not depend on n.

In conclusion, we have shown how the LL mixing gives rise to an upward shift of the energy of the delocalized state from the position $\hbar\omega_c(n + 1/2)$. In a smooth potential the shift may be relatively small even when the peaks in the density of states corresponding to different LL are not well resolved (it can be seen from (11) that $\hbar\omega_c^*/W = [(\hbar^2/mR_c^2)/W]^{1/2} \ll 1$).

The picture considered above was a single-particle picture of delocalization. We have completely neglected the effects of screening caused by electron-electron interactions. At the same time, it is known that in a smooth potential the electron-electron interactions drastically affect the distribution of electrons within the plane. In fact, when the potential is very smooth, the equipotentials we have dealt with are separated by incompressible strips [22]. In this situation one should speak not about the energy position of delocalized state but rather about the critical filling factor at which delocalization occurs. It was argued in [18] that by replacing equipotentials with edge excitations, the many-electron problem can be effectively reduced to the CC model. In the framework of this scenario our theory predicts that the critical filling factor is larger than $n + 1/2$ due to LL mixing.

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